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Notice

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FOREWORD

The interest that geophysical prospecting holds for men in the mining and petroleum industries is well shown by the many papers concerned with its problems which continue to be submitted to the Institute and by the lively discussion at the meetings at which they are presented. The subject has proved to be one that challenges various specialists and yet still holds the attention of operating men. The physicist and the mathematician are attracted by the need to state basic theories in accurate terms and to devise and improve apparatus for measuring the various fields that are employed. The geologist, to whom the methods offer new means of mapping structures, both for their own geological significance and for the control they exert over the occurrence of ore and oil, is given a range of fresh problems arising from the demand for more accurate data concerning critical physical properties of geologic bodies, and from the necessity of adjusting the ideal assumptions of the physicists to geological realities. And even in these days of overproduction, operators and managing boards are still eager to reduce costs of maintaining supplies of raw material and are not unmindful of the possibilities offered by geophysical methods. These various interests are reflected in the contributions included in this volume and testify to the life and vigor that the subject possesses.

Since the appearance of the first geophysical prospecting volume of the TRANSACTIONS in 1929, papers have accumulated from which twenty-four have been selected for publishing together in this second volume. No particular unity or completeness can be claimed for the group, of course, except that each is concerned with some aspect of geophysical prospecting and each is the generous contribution of a busy man who is closely identified with some phase of the subject and is actively engaged in the utilization of geophysical methods in his professional work or is concerned with investigations of related problems in the laboratory or office.

The profession is to be congratulated that physicists and mathematicians of high skill continue to give attention to these practical applications of geophysics. The need for clear and precise statement of the physical problems involved in each method is fundamental, for without it results are largely empirical and in danger of the many errors and uncertainties that develop when short-cuts are attempted before the objective and the difficulties of the path are properly understood. It is with satisfaction, therefore, that we include a number of papers in this volume in which theoretical studies are presented with sufficient completeness of mathematical statement to be of service to one who is investigating a specific problem in a thorough way.

In a technique as young as geophysical prospecting, standard practices are still far from being attained. New instruments appear each year, old instruments are improved and tried in new ways, and experimentation is steadily in progress toward perfecting methods of measurement. Several papers from men in close contact with field procedure are devoted to these aspects of the subject.

For members of the profession who are not specialists in geophysical prospecting, the papers of greatest importance and interest are probably those which describe actual surveys. "Case records" of this sort are of utmost importance to establish clearly what can be done under specific conditions. Theoretical conceptions must be supplemented and modified by field facts, and a rich background of experience must be built up to enable operating men to decide when geophysical surveys are warranted and when confidence can be placed in the results. Each paper with adequate statement of the essential physical and geological data and of results is helpful in building up the common store of experience.

Geophysical prospecting methods are, in final analysis, simply means of geological surveying. Almost every variation in physical properties of rocks, minerals and ground waters gives rise to anomalies which under favorable conditions might be measured and used to trace formations, contacts and structures. It is, of course, obvious that geophysical methods are impractical or unnecessary where the disturbing effect is too small to cause a significant anomaly, where too many intricate effects are blended in the resultant field, or where reliable interpretation of underground conditions can be made from good exposures, but cases are probably more abundant than geologists generally realize where valuable aid can be obtained by a geophysical survey of one type or another. It is to be regretted that no comprehensive study of the subject primarily from the geological point of view has been made, but, in many of the examples that are discussed, the geological setting is summarized sufficiently to give a fair idea of its relation to the problem.

Credit for the success of the sessions devoted to geophysical prospecting, which have been held at the annual meetings of the Institute since 1928, must be given to the many workers both in applied and in pure geophysics who have freely contributed their results and have taken part in the discussion. To each of them, the thanks of the Committee and of our members who have received the benefits from their generosity are abundantly due.

Donald H. McLaughlin, *Chairman*,
Committee on Geophysical Methods of Prospecting.

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* Mr. Bain resigned as Secretary, in the fall of 1931, and Mr. Parsons was appointed by the Directors to that office. The change went into effect on November 1.

Choice of Geophysical Methods in Prospecting for Oil Deposits*

By E. DeGolyer, New York, N. Y.

The only known direct method of discovering oil deposits is by the drilling of test wells. Such exploration is always hazardous and generally very costly. The problem of the prospector, therefore, is to select test sites that are favorably located and so to reduce the odds against his venture.

Early in the history of the oil industry, it was realized that oil and gas deposits, other conditions being favorable, were usually found in anticlinal types of structure. This very understandable and rational habit of occurrence furnished the first substantial clue to the prospector. Oil occurs in sedimentary rocks, and beds of such rocks are deposited originally in approximately parallel layers. An anticline at the surface, therefore, could be accepted as an indication of an anticline in lower beds. Structural geology became the tool of the prospector, and by its use in the past two decades the majority of the known important oil deposits of the world have been discovered.

This is but a partial, and the simplest, statement of the problem, Nine-tenths and more of the important oil pools of the world belong to the great class of accumulations controlled by structural traps—anticlines, faulted structures, salt domes, etc.—but by no means all of these structures are clearly or adequately expressed at the surface. As the more obvious anticlines were explored and the search for new deposits led to the testing of progressively deeper beds, it became more and more apparent that marked favorable structures involving petroliferous rocks can, and do, exist without any marked surface expression. There are great areas where the surface, covered by some masking material such as water, alluvium, glacial deposits, or sand dunes, affords no clue to structural conditions in the subsurface, and there are also regions where the surface formations are unconformable with the oil-bearing formations. The structure in which the prospector is fundamentally interested is that of the oil-bearing formation itself, together with roof and floor beds. No other geologic information is of interest or importance to him except as it affords clues to the stratigraphy and structure of the supposed oil-bearing formations.

* A chapter prepared for the volume on Choice of Methods of The A. I. M. E. Series. New York, 1931. McGraw-Hill Book Co.

These areas of obscure geology in the United States became increasingly important as testing by the drill reduced the number of possible prospects in the areas amenable to geologic work. Much progress was madê by the study of subsurface conditions from the logs and cuttings of wells drilled. The location of such information was generally accidental and casual, however, and the drilling of a test well for geologic information, even at a critical point, is somewhat too expensive an operation to be considered lightly.

The next development, and a logical step in the solution of this baffling problem in many areas, was the mapping of structure by core-drilling to some recognizable bed. This method, while eminently successful and still in use, is also slow and expensive. Since the drilling is relatively shallow, it is also subject to the possibility of giving a wrong or incomplete picture of the structure in oil-bearing formations lying unconformably under it at depth.

Furthermore, there is a group of oil deposits, numerically few but of the greatest economic importance, which are without controlling structure; the trap function resulting from stratigraphic variations such as old shore lines, sand lenses, the feathering out of blanket sands against old land masses, buried stream channels, etc. Notable examples of pools of this type are the Shoestring pools of Kansas, Burbank, the Maracaibo Basin fields of Venezuela and the new Rusk-Gregg field of East Texas.

For many years, one of the most difficult problems of the oil prospector has been the search on the Gulf Coast of the United States for salt domes, the peculiar and interesting type of structure with which oil pools are generally associated in the Gulf coastal plain of Texas and Louisiana, in the Tehuantepec region of Mexico, in Germany, Rumania, and in other regions. Many of the salt domes of this area are clearly expressed by topographic mounds at the surface. Although some of these mounds are almost imperceptible, the drilling campaign which followed the discovery of the first American salt-dome oil field at Spindletop in 1901 resulted in the discovery before the end of 1905 of some 34 salt domes or oil fields believed to be associated with salt domes in the immediate Gulf Coast.

In the continued search for structures during the next 19 years, all sorts of obscure indications were tested. Suspicious topography, physiography, gas seeps, "paraffin dirt," "sour dirt," geographic position in the general pattern of salt domes—almost no abnormality was of too slight consequence to be drilled. As a result of the consequent expenditure of millions of dollars, six additional salt domes and two oil fields not known to be associated with domes had been discovered up to the end of 1923. Since then, to the end of 1930, two additional salt domes and two additional oil fields have been found by similar methods—a total

of forty-two salt domes and four additional oil fields discovered by the drilling of prospects, obvious or poorly expressed, but without the aid of any definite prospecting method except the drill.

Introduction of Geophysical Methods

Geophysical methods were introduced experimentally into the Gulf Coast late in 1922. In 1924, the first salt dome was discovered as a result of geophysical surveys. To the end of 1930, some 43 additional salt domes and structures believed to be associated with domes had been discovered as a result of geophysical surveys and had been checked by drilling. In addition to these there is a considerable but undetermined number of undrilled prospects, many of which are practically certain to be salt domes.

This was the real substantial bow of geophysics to the oil industry, and as an aid to the oil prospector it was brilliantly successful. Naturally, with this record of success, much thought and effort has been expended in an attempt to extend the usefulness of the method to areas other than the Gulf Coast, and while these have met with substantial successes, the development is still in its early stages, and it will probably be years before the record of success will equal that established for the Gulf Coast.

Geophysics today is the tool of the geologist, and only a superior method for the determination of subsurface structure or for the securing of clues to structure which cannot otherwise be obtained, but the methods are hardly more than on the threshold of their development. It is undoubtedly the hope of the far-seeing geophysicist that he will some day be able to determine definitely in advance of the drill whether or not petroleum in quantity is present in the subsoil.

Geophysics

Geophysics, in its broader sense, is the science of earth physics, and includes geology, meteorology, geodesy, seismology, oceanography, and a host of other sciences. In the more restricted sense in which the term is used in this paper, and in which it is generally used by economic geologists, however, it is the science of determining the existence, structure, and/or composition of rocks buried in the earth's crust by the measurement of physical effects resulting from them.

There are numerous geophysical methods, but in oil prospecting the methods which have been developed to the point of having any claim to practicability are four: the magnetic, gravimetric, electric and seismic methods. In actual operation all geophysical exploration consists of two distinct and equally important steps—the measurement of physical effects, and the translation of such data into geologic terms. The first step is the business of the competent and skilled geophysicist; the second

is the business of the geologist with the assistance of the physicist. Interpretation of physical data of the new type is the great problem. In the following statement of the more important geophysical methods, it is assumed that the reader has knowledge of the technique of the methods, or can secure them elsewhere.[1]

In trying to value, as an aid to the oil prospector, the various geophysical methods, the writer is concerned chiefly with their technical utility and soundness, and not at all with their casual and possibly accidental record to date in barrels of oil discovered. Barrels of oil are the ultimate aim of prospecting, but no tested geophysical method as yet pretends to be more than a geologic tool, and it is on this basis that its value must be judged. From the viewpoint of achievement in advancing technique and usefulness, it is of infinitely greater importance that the 5000-ft. depth to the Viola limestone in an unsuccessful wildcat well in Oklahoma should check within the limits indicated by a seismic survey than that the Van oil field should have been discovered on the basis of an area of high geo-velocities as indicated by a seismic survey.

Briefly, and as a practical matter, the various geophysical methods in oil finding are rated as follows:

Magnetic Method

The magnetic method, oldest of the geophysical methods, and with an honorable record of success in the iron fields, is the most rapid and cheapest method of geophysical surveying. Unfortunately for the oil prospector, variations in the earth's magnetic field often appear to be more intimately associated with variations in the distribution of different types of igneous rocks in the basement complex, or of volcanic and intrusive rocks in the sedimentary rocks, than with the structure of the sedimentary rocks themselves. Unless the possible oil-bearing structure is of a type likely to be associated with buried topographic features or with major deformation of the basement rocks—the buried Amarillo Mountains of the Texas Panhandle or the buried Nemaha Mountains (granite ridge) of Kansas, for examples—or with igneous intrusives, as at Monroe and Richland, La., and Jackson, Miss., the results of magnetic surveys are likely to be uncertain and, if tested by the drill, disappointing. This is true not because structures of an oil-bearing type are not associated with magnetic anomalies, but because of the very common occurrence of equally important anomalies which are the results of other causes.

This suggests the possibility of an improvement in our technique of interpretation which will enable us to differentiate between important

[1] The chief easily available general sources of such information are: Ambronn: Elements of Geophysics, Trans. by M. C. Cobb. New York, 1928; C. A. Heiland, Geophysical Methods of Prospecting Principles and Recent Successes. *Quarterly:* Colorado School of Mines (1929) No. 1; and Geophysical Prospecting, A. I. M. E. (1929).

and nonimportant magnetic phenomena, and thus make the magnetic survey a more dependable indicator of the presence or absence of structure in sedimentary rocks than it is at present.

The magnetic method, because of its cheapness and speed, is chiefly valuable as a reconnaissance method. It is not positive, either as indicating or denying the existence of structure in the sedimentary rocks, and generally its findings, in so far as they invite attention, should be checked by another method before drilling.

Gravity Methods

Variations in the earth's normal or regional gravitational fields are the result of unequal distribution of masses of rock of various and substantially different specific gravities in the earth's crust.

Two methods are available for determining gravity variations. The older pendulum method, depending upon determination of the period of oscillation of a pendulum, determines absolute gravity. The newer and more widely used torsion balance method is a differential method which determines the direction and magnitude of the gravity gradient or the horizontal rate of change of gravity and "differential curvature," a complex mathematical concept of a function of gravity—a quantity proportional to the difference of curvature of the equipotential surface in the direction of greatest and least curvature—often of value in differentiating between shallower and deeper effects. Pendulum determinations of gravity have been made over a widely spaced net covering most of the earth, and upon them have been based the determination of the form of the geoid, the development of the theory of isostasy, and the beginning of the science of determination of distribution of rock masses from the interpretation of gravity anomalies. Intensive surveys of limited areas are only beginning to be made by this method.

The torsion balance has been and is being widely used for intensive surveys. Its outstanding success in oil prospecting has been in the discovery of salt domes in the Texas-Louisiana coastal plain, but it has substantial achievement to its credit in the finding and mapping of buried hills, structural ridges, faults, buried structural troughs filled with relatively heavy rocks, buried stream valleys and deeply buried features of the basement complex.

Since the interpretations of torsion balance surveys only too often admit of several readings, the method has made greatest and most rapid advancement in the development of skill in interpretation, and its future advancement is likely to be along the same line.

Much money and thought has been expended by commercial organizations within the past few years upon the improvement in speed, accuracy, and cost of survey by the pendulum method. Whether surveys by this method will be marked improvement over torsion balance surveys

for geologic purposes is still debatable, but it seems probable that a combination of the two methods whereby, say, one-quarter to one-third of the stations in a torsion balance net are repeated with pendulum observations will result in a decided increase in the usefulness of gravity surveys as a geologic method. Such scant information as has come to the writer's knowledge regarding the use of gravity surveys by the pendulum alone indicates that it may give results in areas where the torsion balance is not serviceable.

While the gravity methods are not, in the writer's opinion, subject to anything like the same limitations which hold in magnetic surveys, their usefulness is limited by the possibility of multiple interpretations, by the mistaking of interference effects for those caused by geologic phenomena, by the not uncommon eccentricity of surface effects with subsurface causes, and by the mistaking of deep-seated for shallower geologic phenomena. These difficulties and limitations become less serious with our constantly improved ability to interpret.

Electrical Methods

The electrical methods, of which there are a great variety, have met with their greatest successes in the mining industry. They have been less used in oil prospecting than either of the three other major methods, although they have demonstrated their usefulness in mapping salt domes, faults in the mapping of salt-water tables, and in determining the structure of certain sedimentary beds of marked conductive or resistance qualities.

The electrical methods are the only ones of those in use which have any promise of producing a direct or primary method—of giving information on whether or not oil is present. The chief drawback of these methods at present is that no great depth has been attained, and since the more important oil deposits to be found in the future are likely to be deep, the type of structural information obtainable by electrical investigations is distinctly less satisfactory than that which may be secured by other methods.

Seismic Methods

The sonic or seismic method, in its present state of development, consists essentially of the generation of elastic waves in the earth's crust by the explosion of dynamite, the recording of the time of initiation and arrival of such waves at a detector a known distance away, and the calculation and interpretation of results. There are two distinct schemes of survey by this method—refraction and reflection.

By the refraction method a profile is shot from a single shot point, recorders being placed at various known distances in a direct line away.

From the data so obtained, a time-distance curve is constructed, as is commonly done in the study of earthquakes, and from this curve can be determined the approximate depth of the refracting formation and its *exact velocity* of wave transmission. This geo-velocity is characteristic and, under favorable conditions, diagnostic. A profile of normal length in the Gulf coastal plain, for example, which shows a "velocity" of 16,000 ft. per second or greater, is proof positive of the existence of rock salt in the subsoil. The so-called fan-shooting in the search for salt domes on the Gulf Coast consists merely of a series of one-point profiles for comparison with normal. If a substantial "lead" indicating the presence of "high-velocity" rock is found, the area is examined in detail for the presence of a salt dome.

The most substantial successes of the refraction method have been in the finding of salt domes in the Gulf coastal plain, though its use has been extended to an examination of the structure and stratigraphy of the Permian Basin of West Texas, the determination of the Cretaceous and igneous floors beneath the Llanos in Venezuela, and the investigation of faulted areas. It is probably susceptible to great improvement both in technique of operation and interpretation. Its chief attraction lies in its usefulness for the determination of geo-velocities, which are serviceable as a means of identification of deeply buried rocks.

The reflection method consists in shooting and in recording on a detector a relatively short distance away the elastic waves reflected from suitable formations in the subsoil. From the known or determinable geo-velocity of the intervening rocks and the determinable length of the travel path, the depth to the reflecting formation can be computed accurately. The securing of reflection records requires much greater skill and care, and much more careful checking, than refraction records. Eminent physicists have even questioned the possibility of securing reflection records, but the method has already checked so well in known ground, and so satisfactorily stood the test of the drill in unknown ground, that it is definitely proved.

A variation of the reflection method is the so-called dip method, a differental method by which dip and strike may be determined from a group of records secured simultaneously from spaced detectors.

The most substantial success of the reflection method to date has been in mapping the structure of the Viola limestone in Oklahoma at depths of 4000 to 10,000 ft. below the surface. Four widely spaced wells drilled during 1930 at positions previously surveyed checked the depths to the Viola at below 4100 ft. to within an error of approximately 0.5 per cent. Reflections have been secured from the granite floor of the San Joaquin Basin, California, at depths of approximately 24,000 ft., and determinations have been made on the Cimarron limestone of Kansas, the Pecan Gap of Texas, and on various younger formations

in the Gulf coastal plain. The reflection method, also, is susceptible of considerable improvement in instrumental design, in technique and in interpretation.

The chief advantages of the sonic or seismic methods are that they give results in feet of depth to some key bed; that any practical depth can be reached; that the results admit only of unique solution and that, in refraction surveys, geo-velocities are determined which serve as an index to the type of rock encountered. Its chief disadvantages are that there are areas such as the Edwards Plateau region of Texas where it cannot be used—other methods are subject to similar limitations—its high cost as compared with other methods, and the restriction of its usefulness to regions where the rocks have sharply different physical characteristics. It has not yet demonstrated its value in California, nor in the search for deeply buried salt domes where the torsion balance has been decidedly successful.

Choice of Method

All geophysical surveys by the oil prospector are made to secure information as to the geologic structure of the area under consideration. Such information serves as a basis for the formation of an opinion as to the probability of the occurrence of oil deposits of commercial importance, and their location.

If the resulting opinion is negative, no action is required. If favorable, it will be so in degree, and the prospector is likely to translate his opinion of the degree of probability into practical terms by his decisions on a series of questions somewhat as follows:

1. Does the prospect justify taking leasehold? If so, to what extent? The decision may vary from one to take only a moderate bit of leasehold for protection on a weak prospect, to one to take a block entirely covering a strong prospect.

2. Does the prospect justify drilling? If so, what is the best site for a test well?

The net practical results of geophysical surveys, therefore, are to provide, in part at least, a basis for decisions as to whether and where to take leasehold and whether and where to drill test wells. If a test well results in the discovery of an oil pool, an accurate geophysical survey can be of great value in guiding policy for further exploratory drilling.

With this understanding of the practical uses of geophysical surveys one may proceed to a consideration of the choice of methods. There are four major factors to be considered in making a choice of geophysical survey methods, as follows:

1. Usableness of the method. Can it be expected to give its own type of data in the area under consideration?

2. Cost
    (a) On a unit-area basis
    (b) On a time-operating unit basis.
3. Time—speed of coverage of area.
4. Value of results expected to be obtained.

It is obvious that a method must be usable to entitle it to any consideration whatever beyond experimentation, with which we are not here concerned. The electrical method could not be used in water or marsh areas. The gravimetric methods would be used only with great difficulty in similar areas, and the torsion balance is not usable in mountain areas or areas of rough topography. The magnetic method would be altogether worthless in an area of igneous intrusions which were not of structural importance, such as the "Golden Lane" of Mexico. The seismic methods are not usable, in the present state of development, in areas where the surface rocks are of very high geo-velocities, such as the Edwards Plateau of western Texas.

Cost is a controlling factor. It should properly be considered in its relationship to value of results expected to be obtained; the expected result values must be great enough to justify the risk involved in estimated expenditure. Beyond this phase of the cost factor, however, are cost considerations of importance. If the investigation to be made covers a large area, and the company proposing the work is not seriously limited as to expenditure for such work, other things being equal, the cost per unit of area is the important cost. Table 1 gives comparative costs.

At the height of the intensely competitive search for salt domes in the Gulf coastal plain, the more important companies were using seismic crews and the refraction method at a cost of $18,000 to $23,000 per crew month; speed of coverage was great, and costs per unit of area were as low as could be obtained at that time. Manifestly, the budgets of most smaller companies and individual operators could not provide for so great an expenditure, though they might provide for an expenditure of $4500 per month for a two-instrument torsion balance crew, even if it operated at a competitive disadvantage because of comparative slowness of coverage, possibly higher unit costs, and, in the state of development of the geophysical technique at that time, secured less definite results than might be had from a seismic survey.

Also, certain types of geophysical survey, such as a magnetic survey, may be made, because of their cheapness, over wide areas as a speculative venture in the hope of securing information, even though it may be vague and unreliable, of contributory value great enough to justify expenditures for more reliable surveys by other methods.

Time is an essential element, especially when the purpose of the geophysical survey is to guide leasing operations in a highly competitive campaign. It may easily be of paramount importance. As an example,

TABLE 1.—*Costs of Geophysical Surveys*[a]

	Crew per Month	Costs per Station	Costs per Acre
Magnetic	$ 1,000	$ 1.30–$ 1.50	$0.0028
Torsion balance	4,500	24 – 25	0.045
Pendulum	5,000	30 – 40	0.072
Electric	6,000	25 – 30	0.055
Seismic—refraction	15,000		0.08
Seismic—reflection	12,500	80 – 100	0.18

[a] This tabulation of costs can only be approximate. Different types of survey and different purposes require different spacings. The per acre cost, except for refraction surveys which are assumed to be normal Gulf Coast fan-shooting, is based on a theoretical township with a station on each section corner. Costs are as of Jan. 1, 1931, on the basis of fees as charged by leading consulting firms plus additional expenses.

let us consider a newly discovered petroliferous province such as the Permian Basin of West Texas and New Mexico just after the discovery well had been drilled. Leasehold is cheap but on a rapidly rising market, and the indicated area in which additional pools may be found is enormous. Manifestly, any method of which the results can be of the least service in guiding the selection of acreage in a broad campaign under such conditions will be valuable almost in proportion to the speed with which results can be obtained. In the heyday of the highly competitive geophysical search for new salt domes in the Gulf Coast, when a new prospect might have a current value of one-quarter million to one-half million dollars, speed in coverage of the prospective area was imperative, and the seismic surveys almost replaced torsion balance surveys because of much greater speed of coverage.

Finally, however, after being satisfied as to the usableness or workability of any method for a given area, and accepting for any given purpose the indicated cost and time factors, one comes to the ultimate and most important factor in making a choice of method: What method is likely to give results of greatest relative value, taking time and cost into account, in advancing the purpose of the oil prospector?

In the writer's opinion and experience, and in the present state of development of the geophysical methods for areas where the occurrence of oil pools is controlled by normal folding—anticlines, faulted structures, etc.—the results obtained from seismic surveys, reflection method, are more definite and of greater value than any other type of geophysical information obtainable. He would regard a survey of this type as incomparably better than any results secured by other methods, and would regard them as essential if the purpose of the survey is to determine whether or not drilling is justified, or to make a location for a test well, or if the purpose of the survey is the purchase of high-priced leasehold.

The reflection survey secures data on the relative elevations of horizons which are competent to give reflections and under ideal conditions, such as exist in the Seminole Plateau of Oklahoma for example, gives information of the highest quality and of greater value to the prospector than can be obtained by any other method, geologic or geophysical. The obtaining of datums in Oklahoma on the Viola limestone—stratigraphically only 100 ft. or so from the producing formation (Wilcox sand) and conformable with it—at depths of 4000 ft. and more from the surface and under a deep and substantial unconformity, correct as to depth within an error of less than 0.5 per cent, represents the highest type of geophysical work that has yet been done, in the writer's opinion, and is not susceptible of improvement unless similar results can be accomplished more quickly or more cheaply or until some direct method of finding oil deposits is developed.

Results equally positive, although of somewhat different type, have been secured in prospecting for salt domes—the oil-bearing type of structure—in the Gulf coastal plain. In a search for salt domes coming to within less than 5000 ft. of the surface in this or a similar area, the writer would choose the seismic method because of rapidity of operation and definiteness of results expected. He believes that negative results in particular are more definite than those secured by gravimetric or electrical surveys. In a search for deeper domes, under similar conditions, the writer would use the torsion balance for a gravimetric survey, or the seismic method, dip shooting by reflections.

In west Texas and eastern New Mexico, the seismic method, refraction, is believed to give results more definite than those procurable by other methods. Experience in this area suggests that the refraction method has not been developed to its full possibilities. Depth calculations, only less satisfactory than those obtainable by the reflection method, can be made, and geo-velocities obtainable from the shooting of profiles are of great diagnostic value in determining the character of the stratigraphic column.

The writer's second choice for all areas, and first choice for areas in which the seismic methods are not usable, is the gravimetric method, generally a torsion balance survey. Gravimetric results are generally qualitative or indicative rather than quantitative and definite, and are subject to misinterpretation, but on their record of performance to date they are certainly entitled to second rank among geophysical methods. Positive torsion balance results indicate the existence of salt domes as definitely as seismic surveys, and are probably superior to them for deep-seated domes.

The greatest advancement in the usefulness of gravimetric surveys is likely to come from advanced ability to interpret results.

The performance of the electrical methods in the oil industry has not been impressive. Apparently they are of value in determining structure of strata at comparatively shallow depths. In the United States, at least, the problem of the prospector is with rocks at considerable depths, and is likely to be with rocks at increasingly greater depths in the future.

The results of magnetic surveys are not susceptible of interpretations sufficiently definite in terms of geologic structure to give them any considerable value to the oil prospector except in few and very special cases. They are chiefly of value, if at all, in preliminary reconnaissance, and any indications of structure deduced from them must be checked by other methods unless substantially supported by collateral geologic data.

In conclusion, as a pioneer in the application of geophysical methods to the problem of oil prospector and as promoter of their continued development and use, the writer would stress the point that the fundamental and most important part of all geophysical work is the proper interpretation in geologic terms of the physical data secured from field observations. The ingenious physicist has advanced farther in his ability to secure physical data than has the geologist in his ability to interpret it. The problem is clearly one for cooperation between the physicist and the geologist, and over a long series of efforts the highest average of success will rest with the prospector using the interpretation of the competent geologist who works with proper and critical understanding of the physical methods employed, rather than with the prospector who is imprudent enough to depend upon a poor interpretation of good physical data by the physicist or upon any sort of interpretation of poor physical data secured by any means.

DISCUSSION

D. C. Barton, Houston, Texas (written discussion).—The major factors which should be considered in the choice of a geophysical method are listed by Mr. DeGolyer as four:

1. Usableness of the methods to get their respective types of data.
2. Cost.
3. Speed of coverage of the area.
4. Value of the results to be obtained.

The writer would like to emphasize four additional major factors which must be considered:

5. Purpose of the survey: is it reconnaissance, detailed reconnaissance or detailed in character; are the results to be used for checkerboarding, blocking, or the location of tests?
6. State of knowledge of the general area available to the company which is meditating in regard to the use of geophysics.
7. Appropriation available.
8. Type of structures to be sought or mapped.

In choice of a geophysical method, as in choice of a geologic method in mapping for oil structures, a less accurate but less costly reconnaissance method which will give the general features of the area and the larger and more easily evident structures

is more efficient than a far more accurate but more costly detail method in the early work in an area of which practically nothing is known of the subsurface structure. On the other hand, in an area in which the major features of the structure are known and in which all the larger and more clearly evident structures have been discovered, the less accurate reconnaissance methods are useless.

The torsion balance and magnetometer methods to the writer are primarily reconnaissance methods. The torsion balance registers the effects of structure on both sides of a profile line as well as to great depths under the line, and a reconnaissance net of lines in general will give a fair picture of the general structural framework of the area, of much of the larger structures, and of some of the lesser structures, at moderate cost. The actual cost of most torsion balance work runs considerably lower than the figures given by Mr. DeGolyer. Most seismic and electrical work has been done by consulting geophysical companies, and a prudent oil company will do well to have its work done by one of the leading seismic and/or electrical consulting companies. Most of the torsion balance work, however, has been done by company troops. The costs run on an average about $12 per station, not over $15, and with some sacrifice of accuracy, as low as $8 per station.

The seismic method is a reconnaissance method only where refraction fans may be used in search of such exaggerated structures as salt domes; elsewhere the seismic method can do reconnaissance only by detailing; it gives far more accurate information down to the deepest key horizon used and immediately along the line of profile, but does not give information about what lies to the side of the line of profile or what lies below that deepest key horizon.

The magnetic method is the fastest and cheapest, and although it is by far the most unreliable method, a complete magnetic picture of an area does give a certain amount of valuable information about a structure of which little is known.

For the earlier reconnaissance and detailed reconnaissance surveys of areas in which practically nothing is known of the subsurface structure, the writer's choice of a single method would fall upon the torsion balance, but a slightly more detailed magnetic survey may supplement the torsion balance data with valuable information and add only slightly to the cost. But the maximum of information detailed and general about the area and the larger structures can be obtained by a combination of the methods: reconnaissance by the torsion balance and magnetic methods, and then detailing by the seismic method of special areas and structures which were indicated by the torsion balance and magnetic surveys.

In areas in which much is known about the geology and subsurface structure, reconnaissance torsion balance and magnetic surveys may give supplementary information which will be worth the cost of surveys.

For detail work in search of structures of small relief, for detailed final evaluation of the structure of a particular area (except for deep salt domes), etc., the choice, of course, would fall upon the more accurate seismic methods rather than upon the less accurate torsion balance and magnetic methods.

For $30,000 per year, a small company can keep a torsion balance party with two or three instruments in the field year in and year out but can do only two to two and one-half months of seismic work. Most small oil companies tend to spread their geologic activities over a somewhat wider area in reference to their geologic personnel than would a large company, and to be satisfied with work of reconnaissance grade where a large company would do work of detailed accuracy. If the torsion balance gives fair results in areas in which the small company is interested, the torsion balance method may be in line with the general grade of the company's geologic work and be within the limits of its financial resources. The small company may be able to attain greater efficiency in the handling and direction of the torsion balance work spread out over a year than of the seismic work concentrated in two to two and one-

half months. If a company has a yet smaller sum of money to spend on geophysical work, in some areas it may make the best use of its money by using the magnetic method.

A torsion balance picture of an area is the sum of the effects of the differences of density to enormous depths. If the station is properly taken, the value obtained for the gradient and the differential curvature is good forever; the values of new and old stations are equally usable; if the density of stations is sufficient, old surveys are as usable as new surveys; there has been no increase in the accuracy of station observation during the past 30 years. With the passage of time and with the consequent increase in our knowledge of the area, and with increase of the depth to which the oil man is interested, a given torsion balance survey may be reinterpreted over and over again and in regard to deeper and deeper structure. The depth to which the results of a given seismic survey are effective depends to a considerable extent upon how the observations are made; also upon the depth at which convenient seismic markers are found. The cost of a seismic survey increases with depth to which it is made effective. Most seismic surveys are run to be effective to some moderate depth (rather commonly 4000 to 5000 ft.). The observations of such a survey cannot be worked over at a later date if information is desired in regard to greater depths. Absolescence will probably not affect present-day and future seismic observations as much as it has those of the past. The seismic observations of 1924–1925 are practically totally obsolete; those of 1927–1928 are regarded as usable to a certain extent, but as not nearly so accurate as those of 1930–1931. Such factors again color the choice of the method to be used.

As examples of the effect of the type of structure on the choice of methods: the torsion balance is the best method with which to search for very deep salt domes; the reflection seismic method is the only one that will map small structures in the "Wilcox" sand in central Oklahoma; the magnetic method is preferable for determining whether a core of certain structure may be laccolith or volcanic plug.

Certain minor factors under special conditions may affect the choice of methods. The use of dynamite is prohibited in Angola and in other areas is hedged with onerous restrictions and regulations. In thickly settled areas, it may be difficult to get permission or permits for the refraction method and it may be difficult or impossible to convince officials and individuals that the small charges in the reflection method will do no damage. The service of the supply of dynamite in areas without railroads, good roads, or water transportation may preclude the use of the refraction method.

The electric methods would be of great value if the seismic and torsion balance methods did not exist, but unless they can be perfected directly to detect the presence of oil down to moderate depth, they will not have much of a place in geology.

The pendulum surveys may be of service in supplementing torsion balance surveys or in substitution for torsion balance in areas in which the topography is too rugged or the subsoil too heterogeneous for the torsion balance. A pendulum survey gives only ΔG (*i.e.*, the variation of the intensity of gravity). The torsion balance gives the gradient of ΔG, the differential curvature, and ΔG. The curve of the gradient depicts the variation of ΔG more sharply than does the curve of variation of ΔG itself, and therefore gives information about depths and form of the structure which cannot be obtained from the curve of ΔG itself. The differential curvature also in many cases gives additional information not given by either the gradient or ΔG. Pendulum surveys, then, cannot replace torsion balance surveys in areas in which reasonably good observations can be made with the torsion balance.

Theoretically, Mr. DeGolyer's statement that seismic observations permit only unique solution is true; practically, indefiniteness and uncertainty are not uncommonly met with in the interpretation of seismic data, particularly of refraction data.

E. DeGolyer (written discussion).—The paper under discussion will be more clearly understood, perhaps, if it is realized that it was not prepared as a contribution to geophysics, but as a simple discussion for the young engineer.

The differences between Dr. Barton and myself are largely of viewpoint—the differences between the geophysicist and the prospector. The geophysicist inquires what appropriation is available; the prospector asks what service the geophysicist can render. It is the problem of the prospector to weigh cost against value of results expected to be obtained, and to decide, as a practical matter, what use can be made of the results after they have been obtained. The ultimate purpose of all geophysical surveying in the oil industry is oil finding on a more profitable basis than can otherwise be accomplished, and it is from this viewpoint that geophysics should be considered. It has even approached the degree of usefulness in certain favorable areas where it might be considered from the standpoint of rendering an actual saving, as against the more expensive option of drilling needless wildcat wells, rather than as an expensive form of geology.

The writer's preference for the type of data secured by the seismic method—reflection—over that secured from gravity or magnetic surveys, is based chiefly upon the uniqueness of solution of the reflection data as compared with results of surveys by other methods. It is subject to personal and instrumental error, but is not more vulnerable in this respect than are the other methods. The gap which must be bridged between the observed and computed physical measurements and their proper geologic interpretation is infinitely smaller than that existing with the other methods. Feet in depth to a key horizon—absolute or relative elevation—is geologic data of the highest and most usable quality, and is greatly superior to physical data which cannot be directly resolved into definite structural geology.

Dr. Barton will recall that a torsion balance high in the Seminole plateau might be interpreted as a structural high, but might also mean, and in one notable instance did mean, a marked structural trough containing an erosional remnant of Hunton limestone; that the great Crosby County, Texas, torsion balance high might have meant a great structural dome in the sedimentary rocks, but apparently was only the expression of some marked difference in specific gravity of components of the deeply buried basement complex; and that the marked structural high near Gainesville, Texas, was not coincident with its gravity expression. Even a salt dome may express itself as a gravity high or low, according to whether it is shallow enough for the relatively small mass of the higher specific gravity cap rock to register its plus effect, or so deep that the normal plus effect of the cap rock is of little consequence as compared with the minus effect of the greater volume of the relatively light salt mass. A torsion balance high may mean a structural high in Kansas; the marked Lost Hills anticline in the San Joaquin Valley of California is a decided gravity low. I cite these examples in support of my argument of difficulty in the geologic interpretation of the qualitative methods—the gravity and magnetic surveys.

The cost discussion of Dr. Barton is hardly fair to the present paper. The cost table presented is too high all around. It is as of Jan. 1, 1931. Costs have been reduced substantially since that time. The torsion balance costs used were those furnished by Dr. Barton.

Summaries of Results from Geophysical Surveys at Various Properties*

(New York Meeting, February, 1930)

Introduction

By Donald H. McLaughlin,* Cambridge, Mass.

In our sessions devoted to geophysical prospecting, the greater part of the time heretofore has been given to the presentation of theoretical subjects or highly technical details by specialists directly engaged in the development and perfection of the various methods. This was necessarily so, for we wanted such information available for the benefit of other workers and for our members who were anxious to gain a reliable understanding of the principles involved, to enable them to form an opinion of the adaptability of this new technique to their special problems.

The methods have now been actively applied by many reliable organizations for several years and in this period there has been opportunity for operating mining men, engineers and geologists to appraise them. We therefore felt that it would be desirable at this meeting to give some time to interchange of opinion concerning results from actual cases. Consequently, the Committee on Geophysical Methods of Prospecting sent out letters of inquiry to numerous mining people, asking for frank statements of opinion based on results from geophysical surveys at their properties. We have received many replies, among which 30 or more are of specific interest. On the whole, the point of view is remarkably uniform. Except for one or two who were, perhaps, a bit irritated, the operating men who have replied consider the methods to be capable of yielding valuable information when suitable conditions exist. Their attitude is definitely one of tolerance and of confidence in the integrity and sincerity of the outstanding workers. There is, however, a very general feeling that the chances of attaining positive and trustworthy results are small unless the work is directed by, or at least carried on with the close cooperation of, a geologist or mining man who is thoroughly familiar with the local geologic and mineralogic conditions and with the relations of the orebodies to rocks and structure.

From certain comments, I am inclined to think that some operators are not greatly impressed with the cheapness of the work. Probably they are not attaching much weight to cost per acre, but are thinking primarily of total cost in terms of significant results obtained. It is

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recognized, at any rate, that geophysical surveys are far too expensive for indiscriminate wildcat exploration.

Letters

Before calling on some of the members who are present, I shall read a few paragraphs from certain letters that are of interest. In order to save time, I shall paraphrase certain letters somewhat.

Mr. Allen W. Pinger, of the New Jersey Zinc Co., writes: "After application of the methods at a number of our properties, we have come to the conclusion that the ores with which we are dealing are not favorable to direct exploration by geophysical methods in their present development. Our zinc ores do not offer sufficient contrast, in physical properties, to the enclosing rocks. Or where such contrast does exist, topographic or soil and ground-water conditions are unsuitable. Indirectly, we have found interesting relationships between structure associated with ore and the geophysical 'indications.' Unfortunately, the relationships have been developed following intensive exploration work based on highly quantitative structural mapping; the 'indications' have not been of sufficient positive character to assist this mapping or to eliminate nonproductive ground, nor to lessen the amount of quantitative mapping necessary in a particular area.

"We consider the technical quality of the work done for us to be of a high order; we believe the character of our ores to be responsible for the negative results obtained."

Success with Molybdenite Deposits

Mr. J. W. Weitzenkorn, president of the Molybdenum Corpn. of America, very kindly furnished a very interesting summary of the work done at their property at Questa, N. M. The details concerning this survey have already been published.[1] The ore deposits in this district consist of veins of molybdenite in alaskite granite porphyry. The orebodies have both horizontal and vertical dips and are generally lenticular. They range from 6 to 8 ft. wide down to practically nothing. Their average width is about 18 in. The slopes of the region are steep and a considerable part of the area is covered with shallow overburden consisting of slide rock and soil. The purpose of the electrical survey was to locate new veins of molybdenite and extensions of known veins, chiefly in the area covered with overburden. Alternating current was introduced to the ground through linear electrodes. The following statement is made concerning results obtained by subsequent exploration at the places indicated to be most favorable for ore by the electrical survey:

"Since January, 1928, the most accessible of these mineral indications were opened up. To date, six groups have been investigated; two have proved extremely valuable in that they have revealed large quantities of ore, two have not shown any indications and two are now in the course of development.

"The indications tend to follow the general strike of our mineralization; that is, in an east-west direction, and we have accordingly grouped them for our own convenience.

"Should the other groups prove to be as much an aid as to finding new ore as the six groups now under investigation—in other words, if but a third of the remaining numerous indications prove to be commercial mineralization, the survey may be considered to be a wonderful success. In fact, we are completely satisfied with the results as of today and we contemplate in the future to have a survey made covering the remainder of our property."

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[1] K. Lundberg and A. Nordstrom: Electrical Prospecting for Molybdenite at Questa, New Mexico. Geophysical Prospecting, A. I. M. E. (1929) 125.

Indications in Sericite Schists

Through the courtesy of Mr. J. Gordon Hardy, vice-president of Ventures, Ltd., we are given information from Mr. L. G. Morrell concerning a geological and electrical survey in a region of sericite schists impregnated with disseminated sulfides. Unimportant siliceous sulfide-bearing veins occur in hospitable planes in the schist. The dimensions of the veins are small, but one outcrop 15 ft. wide and 30 ft. long was known with pyrite and chalcopyrite of possible commercial grade.

A high-frequency electrical method was employed over an area of ½ mile square. Several lines of conductor axes were plotted and in the end indicator lines were fairly evenly distributed over the entire area. Some of the lines were reported as being stronger than others. There was no outstanding strong indication.

Twenty-three shallow, oblique diamond-drill holes were subsequently put down. It is stated that in more than one-half of the instances in which an indicator line crossed above a drill hole the indication on the surface was directly above the best mineralized section of the hole. The mineralization appears to consist chiefly of zones of fine-grained, disseminated sulfides. The small veinlets containing massive sulfides appeared to have no appreciable effect. No strong conductors were indicated by the geophysical survey and none were found by the diamond-drilling, but the diamond-drilling revealed a heavier mineralization under several weak indications than in adjacent material.

Reports from Canada

Mr. George Bury furnished information concerning electrical surveys using a high-frequency method on two properties in Canada, the Abbey Mines, Ltd. and the St. Lawrence Metals, Ltd. The results at the latter are of possible significance. The ground adjoins the Tetreault property in southeastern Quebec, where sphalerite-galena ores occur as replacements in a band of limestone largely altered to tremolite, in an area of gneisses and quartzites. Previous exploration had revealed some small lenses of ore in the property. An electrical survey indicated several conducting zones. The principal one was subsequently drilled and is now being explored by underground work. Mineralized limestone is said to have been encountered, with sufficient local concentrations of ore to encourage further work. The results increase the confidence the writer has in geophysical prospecting as a means of helping the geologist and engineer in working out specific problems. He concludes, from the results at the two properties, that the practical value of information from electrical surveys increases with the effort given in correlating it with the geology.

Mr. Joseph Errington, of Toronto, wrote that in two instances favorable results have been obtained by geophysical surveys on properties with which he is associated, in the Sudbury Basin, and stated that in the Vermilion Lake section the electrical surveys outlined an ore zone which was subsequently confirmed by drilling.

At the adjacent Treadwell-Yukon properties, however, results were unsatisfactory, according to a letter from Mr. C. V. Clausen, the general superintendent. Experimental surveys were made by several organizations. The failure to secure definite results was attributed to the large percentage of graphite in the country rock, to pyrrhotite abundantly disseminated and in seams in the wall rocks without relation to the orebodies, and to basic dikes in addition to complex structural conditions.

Mr. W. H. Emens, manager of the Mining Corpn. of Canada, sent an interesting letter giving valuable information concerning results at various properties of his company. He says that three years ago his company had a survey made by the induction method over a group of claims in Quebec. At that time it was felt that with negative results the options could be safely dropped. Later it developed that

elsewhere in the region there were certain types of ore occurrences which did not give much, if any, reaction by the induction method, but which could be traced by the equipotential method.

A little more than a year ago, surveys were made for them over narrow veins in the Cobalt area by an organization employing a high-frequency method. Mr. Emens describes the work as follows:

"They were first put over the tailings filling part of Cobalt Lake. However, there was so much interference from the railroad, power lines and pipe lines, all of which were 'conductors,' that the underground conductors, which at the most were 25 to 30 ft. of backs of stopes, were difficult to follow.

"We then put them on the Trout Lake property, where we had developed a network of narrow calcite veins with small pockets of arsenical ore on the 300-ft. level. The overburden was 30 ft. or more of clay. The topography gave no clue to the trend of the veins. They traced out the veins surprisingly well, considering the fact that these veins contain very little of the metallic minerals.

"They were then put over the outcrop of a narrow cobalt vein on an adjoining property and to our surprise they got no 'kick.' However, we felt that the results justified our putting them on some drift covered with virgin ground. In due course of time we received the reports with many 'indicators' and 'traces' plotted.

"We trenched across the strongest indicator and located a pronounced shearing or fault with 4 ft. of crushed material and gouge between the two walls.

"Since then a shaft has been sunk 400 ft. and exploratory work has been done. We have found that the fault, though it seemed to correspond at the outcrop with the 'indicator,' does not have the same strike at lower levels, the two forming a wide V. Another 'indicator' proved, as we suspected it would, to be the contact between the Nipissing diabase and the Cobalt series.

"While this work has been entirely unproductive and, incidentally, very expensive, as the indicators appear to be water-filled fractures which gave heavy flows, we still feel they serve a useful purpose in areas where there are no outcrops, if for no other reason than to trace out the directions of differences of conductivity. These differences of relative conductivity probably in only rare instances represent ore.

"We would certainly not use this method in virgin territory; that is, not near known ore. They will always get reactions from contacts, faults, etc. But to pick up extensions of ore, or to locate orebodies in favorable ground where they can standardize their reactions near known ore, we feel they have a useful field."

Limitations of Geophysical Work

Mr. Robert E. Tally, general manager of United Verde Copper Co., wrote as follows: "It is our opinion that geophysical methods of prospecting have considerable merits under favorable conditions, when recommended by a geologist who appreciates the limitations of geophysical work and who has given consideration to structural and other geological conditions.

"In other words, geophysical work is in the nature of a specialty for certain well-known conditions, and has much merit if guided by experienced geologists.

"However, there has been some inclination toward wildcatting in this branch of work."

Mr. Fred Searls, of the Newmont Mining Corpn., advised us that his organization had electrical surveys made by three different groups at properties in South Africa, in South America and in Newfoundland, respectively. At O'Okiep, Namaqualand, they endeavored to locate heavy sulfide orebodies or disseminated orebodies contained in noritic rock, which in turn was enclosed in gneiss. The self-potential method was used for the most part, but induction methods were used to a limited extent. The

efforts were unsuccessful. In general, the work led to indications of ore where none existed and to indications that no ore existed where ore was subsequently found.

In Newfoundland, electrical surveys were made for the Newmont company in the Millertown Basin. Results obtained were conflicting, and the general outcome of the investigation was not such as to encourage further efforts by this company on the ground being examined. Indications reported as favorable were disproved by drilling and trenching.

In South America, the electrical surveys were used to outline massive sulfide orebodies enclosed in limestone under glacial drift. These efforts were successful, perhaps, in that they indicated that the sulfide was confined to the area where it was known to exist, except that one important tongue or projection, then unknown, was not located by the geophysical work but was subsequently discovered by drilling. Inasmuch as the area where negative results were obtained has not been drilled, there has been no corroboration of the negative results, but they are believed to be sufficiently sound to obviate the additional drilling, since the indications about the known ore were unmistakable.

Mr. J. M. Snow, of the Tintic Standard Mining Co., writes that electrical surveys using equipotential methods and high-frequency methods were made at different times at the properties of that company, but that neither proved of value, due, it was believed, to numerous local interferences that made it impossible to interpret results. In his opinion, however, both processes have merit when used within the narrow limits necessary for accurate results.

Kennecott Mines, Alaska

By Alan M. Bateman,* New Haven, Conn.

Before speaking of geophysical work at Kennecott, I should like to discuss two things.

1. I have felt for some time that discussions on geophysical methods need to include the viewpoint of the user of the methods as well as that of the supplier. It is, of course, natural that the geophysicists are for the most part proponents of their particular methods, so their viewpoints unfortunately are somewhat biased at times, inasmuch as they wish to see their particular methods employed. When one who is not a geophysicist—and I am not, I am just a plain economic geologist who has occasion to use geophysical methods—wishes to determine which methods or commercial organizations he shall employ, he faces a problem.

The literature is full of the application of given methods as used in certain deposits, and these articles are practically all prepared by persons interested in their own methods. The successes only are written up and the literature is surprisingly free from discussion of the failures. But the one who is not a geophysicist and who wishes to decide which method, or group of methods, he shall employ for the search for ore in a given property in which he is interested, finds little to guide him in the literature. The failures as well as the successes should be presented. Several

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methods apparently are adapted to his type of deposit and the various solicitors of his business will likewise claim superiority for their methods. But, his judgment tells him that one or two are probably better than the others. He cannot employ them all; consequently he has to decide on one organization. Therefore, if this series of discussions will bring out the viewpoints of the employers of geophysical methods (and I hope it will), as well as those of the proponents, it will serve an extremely useful purpose. It should enable the user to determine whether he is justified in the expenditure involved and, if so, which method or organization he shall employ. Today, the millman, for example, can turn to reliable information from users regarding processes and machines that might be adapted to his particular problem. So it should be regarding geophysical methods.

In this connection, I am much pleased to note the excellent experimental work under natural conditions that the U. S. Bureau of Mines is now carrying on, to enable the mine manager, the engineer and the geologist to obtain independent information regarding the adaptability of various methods to particular types of deposits. I hope funds may be forthcoming so that they may continue such work.

2. A second point is the possibility of adaptation of geophysical methods to underground mine workings. I believe, if suitable methods can be perfected, this offers one of the greatest fields in which geophysical prospecting can aid mining exploration. It is a thought I have had in my mind for many years. There are many mines that are said to be worked out, or are approaching that stage, but before they are abandoned, the best known geological methods are commonly utilized to make sure that favorable ground is tested and ore possibilities are not passed over. If, however, more precise geophysical methods can be employed in shafts, long crosscuts and deep levels, the life of such mines may be prolonged, or if they are abandoned they can be given up with the assurance that no orebodies are being overlooked. In other words, there may be a rich opportunity to prove in underground workings, far below the reach of surface methods, the existence of orebodies that were unsuspected in previous exploration. Here lies a golden opportunity for geophysical organizations to carry on experimentation and perfect methods adaptable to the exigencies of underground workings and mining operations.

Work at Kennecott

I had occasion to recommend some geophysical work at the Kennecott mines in Alaska, and had to make a layman's decision as to which methods and company should be employed. It seemed, from the information available, that equipotential or electromagnetic methods would serve best, and the work was carried on in a highly satisfactory manner by

the Swedish-American Prospecting Co., which cooperated excellently in the many difficult problems that arose.

The first work was at the Beatson copper mine, in Prince William Sound. The deposit is a fat lens of closely disseminated sulfide ore several hundred feet long by 100 to 300 ft. wide. The ore replaces quartzite and slate along part of a large shear zone. Sulfides constitute 12 to 15 per cent. of the rock, in places reaching 25 to 50 per cent. It was desired to determine whether other lenses might be found along the continuation of the shear zone of which the position was approximately known. The region is topographically relatively flattish. The surface is tundra covered and in part lightly forested. The water table lies practically at the surface and bed rock is a few feet to a few tens of feet in depth.

Equipotential methods were employed first. Rather amazing results developed. Equipotential lines in the absence of a conductor in the ground normally will be relatively straight (with line-potentials) and trend at right angles to the flow of the current. But at this place they did not follow such a course; they trended almost parallel to the flow of the current. They indicated what appeared to be a fine looking conductor approximately 900 ft. long by 300 ft. wide. There were also some anomalous features not understood.

Then an electromagnetic survey was run over the same area as a check. This more precise method indicated a conducting body some 600 or 700 ft. long and 200 ft. wide with relatively sharp boundaries. It looked so good we thought there was going to be another Beatson mine.

We lost little time in putting down some diamond-drill holes to test it. Alas, our hopes were rudely dashed. We did not find the mine we thought should be there. The first drill hole was put down under the part of the conductor that by both methods gave the strongest indication. It yielded about 12 ft. of mineralized slate peppered with a small amount of pyrite and chalcopyrite, which together indicated about 3 or 4 per cent. of the rock. In the copper assay, (this was a copper pyrite metallization), the pyrite ran less than 1 per cent. The remainder of the hole was blank except for two small spots. However, the indications were so strong, I felt that as long as we had expended so much money in the survey we should gamble some more holes. Three more were put down, with somewhat similar results. In this particular case the results were frankly disappointing.

The peculiar equipotential lines are rather difficult to explain. The formations dip steeply and some of the beds are better water carriers than others. Possibly this may have been a factor. The "conductor" was aligned along the shear zone. It is possible there have been enough scattered specks of pyrite in the rock to give this effect.

At Kennecott different methods were needed. But one must understand first the geologic setting of the ores. There is a series of over 3000

ft. of limestones lying conformably on top of greenstone, the whole dipping 20° to 35°. The orebodies lie in the limestones. They consist of replacement veins and irregular replacement masses up to about 125 ft. in width. The veins dip rather steeply, and they strike at right angles to the limestone beds. The orebodies range from a few tens of hundreds up to nearly 3000 ft. along their stope length. The ore consists chiefly of chalcocite and covellite with considerable carbonate. Pyrite is absent. There may be bodies of pure chalcocite as large as 6000 cu. ft. without a speck of gangue in them. In the orebodies the sulfide particles are connecting; therefore they are ideal conductors. The topography is rugged and largely inaccessible above the mines. The surface in part is covered by talus. Vegetation is absent, as it is above timber line. The surface is dry; ground water is largely absent, since the ground is deeply frozen.

The equipotential method was precluded on account of the surface and the topography. The rugged topography precluded the possibility of surface work save for limited sections such as steep talus slopes and the tops of glaciers. Consequently underground methods were tried, in part experimental. We used two methods—the electromagnetic method and a variation of it. In the first, a regular loop was placed on the surface, carrying the high-frequency current in insulated cables, and the observations were carried on underground within the influence of this surface loop. The results from this loop proved negative. I watched the work from day to day, and feel that the reception was in general satisfactory, although some doubt might be entertained as to its perfection on the lower levels. I have no reason, therefore, to think that within the radius of recording (which is small) any unsuspected orebodies were passed by. A great deal of prospecting has been done in that locality and no body of appreciable size has been found outside of the regular veins.

Then another method was tried for places where we could not put a loop on the surface; namely, the intensity method. A single insulated cable was carefully grounded at both ends and supplied with high-frequency current. Thus both induced and direct currents were set up. This cable was stretched along long crosscuts or down an incline shaft. Then level readings were taken above and below it. Several small conductors and two of fair strength were indicated by this method.

Many difficulties had to be contended with in the underground work. For example, all wires had to be cut and rails and pipe lines disconnected at intervals. It is expensive work and handicaps mining operations. The work was in large part experimental, and the Swedish-American Prospecting Co. rose to the occasion in helping to solve its difficulties.

As to the results of this type of survey, subsequent work showed that one of the indications was a little mineralization in a fault. another proved

to be a wet fault gage without mineralization, and two failed to disclose anything to cause the indications.

Conclusions

As to my conclusions—and I speak not as a trained geophysicist but as a plain geologist, with the wish to avoid becoming entangled in any technical arguments—I think (1) that under ordinary conditions geophysical methods are an extremely valuable adjunct to geological work in ore finding. The two, however, cannot be separated. It is essential that geological interpretation go hand in hand with the deciphering of the geophysical results, and often will play as great a part as that of the geophysical interpretations themselves. Without it, erroneous conclusions may readily be drawn, as is shown by the citations I have made.

2. I think that geophysical work in mining regions has its value chiefly in surveying specific areas in which mineralization is strongly suspected, as in the vicinity of known orebodies or in potential ore areas. I am not convinced that the expense and slowness of the work justify it for blind prospecting in areas that might be favorable but in which no ore is known.

3. I do not think, from my own brief experience with it, that in all cases geophysical work is sufficiently standardized to convince the mine operator that negative areas contain nothing and that indicated conductors mean ore. I hope that may yet be achieved.

4. I think that possibly for certain types of work some of the companies should do more experimentation of their own in the field in order to know whether their methods are adaptable to certain types of deposits, before they accept such work under fee. More than once, in discussing with mining companies the possibility of having geophysical work done, I have met this retort: "We have to pay a high fee for work of which we do not feel entirely confident about the results. Often we have to pay for experimentation when the workers do not know whether their method will give dependable results on our type of deposits. We have to allow them to experiment at our expense."

I have confined my remarks largely to the work done at Kennecott, but I have also observed it at other places. In some it has been highly successful, both from a positive and a negative standpoint. Incidentally, I also observed some interesting work in the Belgian Congo where the self-potential method was used, not for finding ore, for it was unsuccessful for that purpose, but for tracing out a horizon marker of graphitic shale which formed a key bed by means of which the structure could be deciphered and borehole locations determined.

Southern British Columbia

By J. J. O'Neill,* Montreal, Que.

In Southern British Columbia we explored a series of five limestones, separated by shales or schists; the limestone bands ranged from 300 or 400 to 900 ft. in width. The average strike is N. 70° E. and the dip 60° SE. Along the belt there were places where the limestone pinched and offset towards the west very abruptly, and in at least three such places orebodies were found. In one case the orebody was developed to at least 2,000,000 tons of commercial ore, and a considerable amount of lower grade material.

The ore is predominantly zincblende, but there was a fair amount of galena and there were lenses of solid pyrite in association with the ore, which was selective replacement of limestone bands.

Two of the geophysical prospecting companies, partly at their own suggestion, were invited to demonstrate whether they could indicate such orebodies. First the Radiore Co. went over the larger of the known orebodies and indicated a weak conductor along the top of the orebody near the footwall, perhaps 30 ft. from the footwall; the end of the conductor terminated where we knew that the orebody terminated on the surface. Unfortunately, they stipulated that the depth of the conductor they were indicating on the maps was such that it threw the conductor entirely out of the orebody; that is to say, with an orebody dipping about 60° the conductor along the footwall, if projected to the depth which they indicated as being the location of the ore, would be thrown entirely out of the orebody into the footwall, and into the quartzites below the limestone.

They also indicated a weak conductor in the hanging wall of that particular orebody. It also terminated where the orebody terminated, but the depth did not correspond with the depth of the ore and the conductor followed a basic dike. It checked almost exactly with the location of the dike. If they had said nothing at all about the depth the results would have been more nearly correct.

They did indicate the twists in the formation, however. Where their indicators came towards the bend in the limestone it was shown very clearly on their map. Their indicators did cease at the ends of the orebody, although the points indicating the depth of ore were incorrect. They were engaged to carry the survey further but no more beneficial results were obtained.

The other company, Mason-Slichter, first went over the orebody, using the coil method right across the center of this large orebody, but showed no indication whatever. They continued along the line, which

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was at right angles to the strike of the formation, crossed over an overlying schist into a second belt of limestone, and there they obtained a very remarkable curve. They compared it to the curve which they got over the Frood orebody at Sudbury; it was just as strong and just as typical a curve, and they stated that it was a curve that indicated a solid sulfide orebody, a perfect conductor.

Then they took the pots. They ran their pots across the same line and picked up the first orebody (the only method that had really indicated it) and, continuing along the line across the orebody indicated by the coils, they got twice the strength of the curve that they did in the first instance. That confirmed them in the belief that there was a solid body of sulfides there. It was pointed out to them that the limestone was slightly carbonaceous and that there were little lenses of graphite through it, and they were shown an outcrop across the formation. They took specimens of this and tested it with no result, until several days later they happened to put the contacts, both contacts, on one of the little lenses of graphite, and then got a remarkable indication. Taking it in mass, they got no indication in the small specimen, so they still thought they had a solid sulfide body. The drilling showed there was no sulfide there at all; it was merely a carbonaceous limestone.

On the strength of the indication over the known orebody—the indications by the pot method—we asked them to use that method on the limestone in which we knew there was ore, to follow along that and see if they could duplicate the result they got over the known orebody. We cut about 28 miles of line for them through the bush. They did not get any indication along a considerable strip in which ore has been found recently by underground working and on a second orebody and a third one, which we already knew, they got no indication whatever. The latter type of orebody was exactly the same as the one indicated in the first instance.

The results to my mind merely indicate that there has to be a great deal more standardization in interpretation of results. If properly applied, a great deal of assistance can be obtained from the method, but so far at least results have not been interpreted in such a way as to gain very much confidence.

DISCUSSION

L. B. Slichter, Madison, Wis.—Dr. O'Neill points out that the self-potential method, although responsive to the known orebody at one part, failed to pick it up farther along the strike. This disappointing result has been observed at other properties under quite different geological conditions. For example, Dr. Mason, in his 1927 paper before this society,[2] illustrates the inconsistent results obtained by this method at the Falconbridge nickel body near Sudbury. Here encouraging results

[2] M. Mason: Geophysical Exploration for Ores. Geophysical Prospecting, A. I. M. E. (1929) 9.

were obtained in the Edison Kettle Hole, but at a point several thousand feet to the east no response could be obtained over any of the traverses. Similar inconsistencies may be pointed out at other properties, and we have always cautioned against the difficulties of this method. One should not, however, utterly condemn this method because of these serious failings. The method has yielded beautiful results in some cases, with as clean-cut a discovery of ore as one could wish. Also, as an auxiliary to other, more reliable geophysical methods, it yields independent data which sometimes contribute greatly to successful interpretation.

It is often difficult to trace the specific causes of the inconsistent results referred to. In the case of the Reeves-MacDonald body, success was had at one end of a sulfide body, which, of course, is the natural place to expect strong polarization effects. Farther along the strike, much difficulty was encountered in getting electrical contact with the ground because of scanty soil and the unusually dry season. These two circumstances may well account for the different results. At the Falconbridge body, however, such explanations cannot be assigned, and the cause of the discrepancies has not been suggested. Clearly, however, the causes of the local variations in the natural ground potentials are complex and sometimes quite variable locally. For example, one must expect that such a thing as the intensity of chemical reaction will vary considerably along the strike of an orebody, due to innumerable possible changes in composition of the ore, its surroundings and the ground water. Drainage and joining and minor faulting doubtless have influence.

A. S. Eve, Montreal Que.—Some eminent geophysicists assure me they can tell the difference between graphite and pyrite. Is that simply a question of the positive and negative sign, or is there any other difference in the character of the curve?

L. B. Slichter.—I can answer Dr. Eve's question by explaining in greater detail the situation encountered at the conductor discovered. This conductor, when first crossed, showed the negative self-potentials ordinarily associated with sulfides. On further exploring it along the strike, a blank interval several hundred feet long was obtained, followed by a separate distinct zone at which the sign of the response changed to positive, which is usually associated with graphite. A search of the ground in the vicinity of the positive peaks fortunately revealed abundant outcrops of graphite, which, then, was a satisfactory explanation, not only of the self-potential results but also of the inductive ones. The negative self-potential peaks in alignment down the strike, however, were not so clearly explained.

Under such conflicting findings, opinions differed as to whether or not the conductor was a sulfide, and as the mining company had a drill at hand and desired to test this ground anyway, a hole was drilled to settle the nature of the discovery. It passed through an 80-ft. zone of banded limestone and graphite, with veins of massive pyrite up to ½ in. wide, and with disseminated pyrite in between. These bands of pyrite undoubtedly explain the negative peak obtained. However, I am not personally familiar with the details of this survey, as I was not on the property. As I said before, one cannot always explain the vagaries of the self-potential method.

If the area of positive peaks, with the exposed graphite, had been discovered first, there would have been greater delay and skepticism in drilling, and possibly the hole would have been saved.

In summary, the sulfides normally react with a negative peak and graphite with a positive one. There are special conditions that reverse the rule, as has been pointed out above. The shape of the curve is related to the geometrical configuration of the causal structures rather than to the composition of the substance.

Bushveld, Transvaal, South Africa

By R. D. Hoffman,* New York, N. Y.

In the Bushveld area, Transvaal, South Africa, there occurs a series of pyrrhotite lenses that carry nickel. A huge basic laccolith extends some 300 miles east and west and 100 miles north and south. This intrusive is called the Great Bushveld Complex. The nickel is found over a wide area, but no deposit is found closer than three miles to the lower norite-diabase contact. I shall refer to four of the deposits, which are roughly 1000 ft. apart and occur in the norite. These lenses are vertical and vary from 10 to 50 ft. in diameter.

Before I examined these deposits, a German company made an electrical survey and indicated several large conductors. When geological work was conducted underground it was found that in three cases the pyrrhotite orebodies were cutting through flat-lying lenses of a very basic differentiate, of high magnetite content, in the norite. Clearly, the conductors were these beds of basic rocks. At some distance from the known orebodies, also, outcropped a number of these basic differentiates. Consequently, since this electrical survey had not been correlated with surface and underground geology, it was felt that the electrical work was a waste of time and money.

Briefly summarized, my ideas concerning electrical surveying are as follows:

1. Unless there is a detailed geological surface map and, if possible, one of the underlying rock structure, plus a competent engineer who is qualified to interpret results, in all likelihood the work will prove ineffective.
2. Unless the character of the ore and the size of the deposit or deposits lend themselves to electrical prospecting, the survey is of little use.

In a country like northern Canada, where there is heavy overburden, high cost of labor and a water level close to the surface—factors which make for expensive trenching—an electric survey to extend the boundaries of a known deposit or deposits may well be conducted. In South Africa, however, where trenching costs are very low, because there is little water trouble and cheap negro labor, such a survey, in most cases, would be unwise.

Gull Lake, North Central Newfoundland

By E. Y. Dougherty,† Toronto, Ont.

Gull Lake mainly occupies a northeast elongated belt of andesitic lavas and interlayered cherts and siliceous tuffs. Granite and diorite

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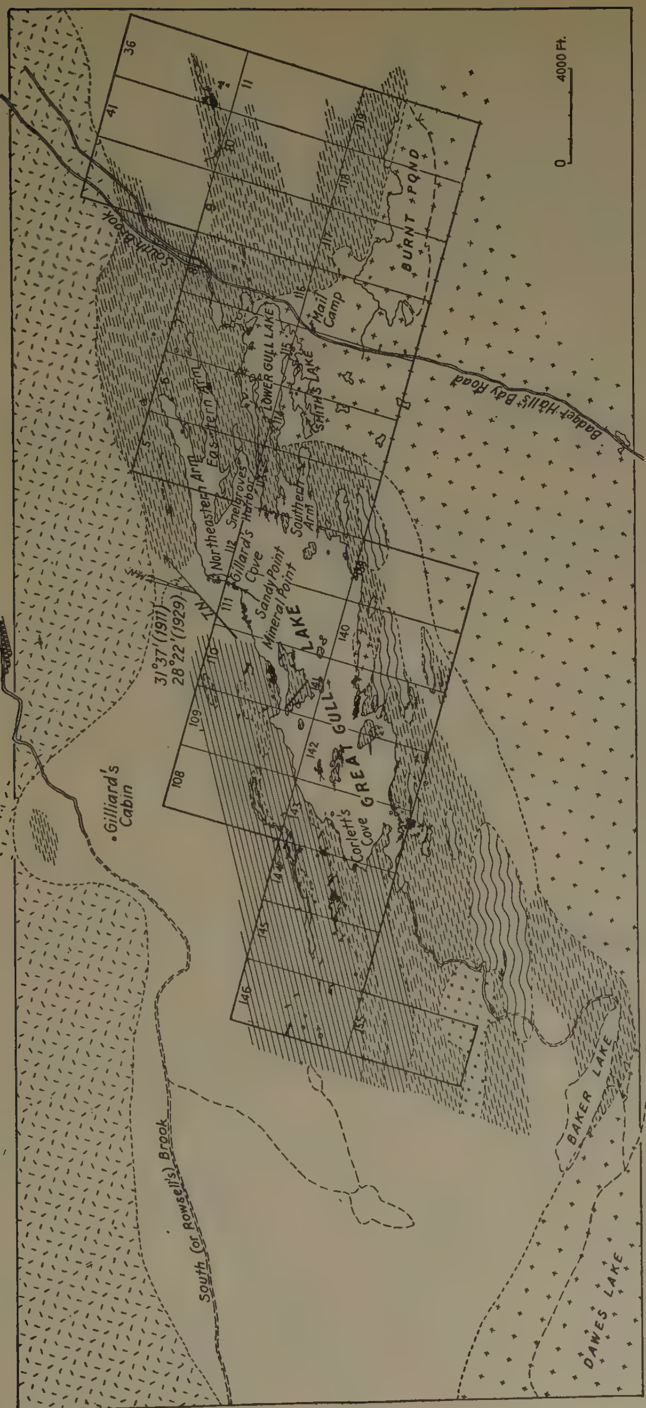

FIG. 1.—CAPTION ON OPPOSITE PAGE.

rocks, which are part of batholithic or sill-like intrusives of central Newfoundland, border and intrude and probably underlie these andesitic lavas, tuffs and cherts. Swarms of offshoots from the granite-diorite rocks penetrate the andesitic lavas, tuffs and cherts. Contact metamorphic effects on the intruded rocks are intense in places. Both basic (mainly diabase) dikes and acid (mainly feldspar porphyry) dikes intrude the lavas, tuffs and cherts. Many basic dikes also intrude the granite-diorite masses, and it is inferred that the acid dikes also intrude the granite-diorite masses. The geological conditions of the area are shown in Fig. 1.

Ore Deposits

At Mineral Point, on the northwest shore of Gull Lake, a copper orebody, averaging about 50 ft. wide and about 1150 ft. long at shallow horizons, has been indicated by diamond drilling and some underground

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Fig. 1.—Generalized geological plan of Great Gull Lake and vicinity.

- Andesitic lavas, in part pillow lavas and with subordinate interbedded tuffs and cherts
- Diorite with subordinate granite
- Dikes of quartz porphyry
- Red conglomerate, Carboniferous?
- Tuff and chert, with subordinate interbedded andesitic lavas
- Very basic diorite
- Granite with subordinate diorite
- Dikes and sills of dolerite
- Paragneiss, predominantly metamorphosed tuffs but in part altered andesitic lavas and possibly igneous gneiss
- Dikes and sills of feldspar porphyry
- Oxidized mineralized outcrops
- Contacts (largely inferred)
- Ore boulders
- Oxidized boulders
- Mine or shaft
- Magnetic attraction
- Glacial striae
- Roche moutonée, etc.

development and surface trenching. A picture of this orebody and the sulfide zone in which it lies, as deduced from diamond-drill hole intersections, is shown in Fig. 2. The metallic sulfide minerals are chalcopyrite, pyrrhotite and pyrite. Small quantities of magnetite are distributed in the metallized zone. Cordierite, secondary amphibole which is probably actinolite, biotite and quartz are the characteristic gangue minerals. These gangue minerals have replaced andesitic lava, cherts and tuffs. Cordierite is especially characteristic and abundant. Disseminations, clusters, veinlets and streaks of the sulfides penetrate

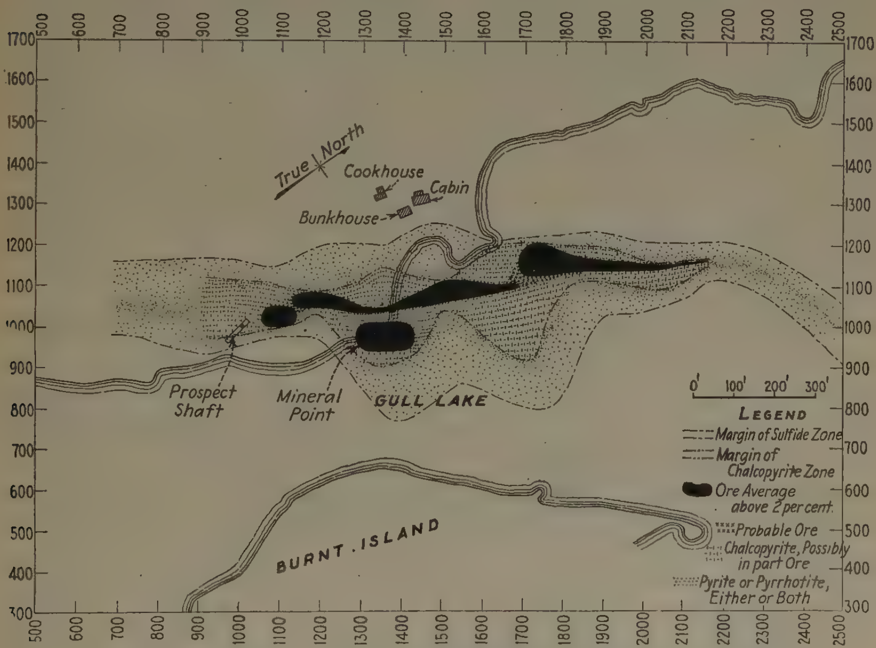

FIG. 2.—COPPER OREBODY IN SULFIDE ZONE, DEDUCED FROM DIAMOND-DRILL HOLE INTERSECTIONS.

and replace these gangue minerals. Pyrrhotite is almost invariably intergrown with or contiguous to chalcopyrite but the relative proportion of the two minerals is extremely variable. Pyrite is nearly ubiquitous in the sulfide zone both in the ore and as an envelope around the ore. It usually occurs as disseminations but locally as concentrated masses.

About 1¼ miles southwest of the Mineral Point orebody, a small copper deposit, known as the southwest shaft orebody, has been partly exposed by surface trenching and underground development. Its mineralogy is similar to that of the Mineral Point orebody. Various oxidized pyrite outcrops are disposed along a zone embracing Mineral Point and the southwest shaft area (Fig. 1) but chalcopyrite is scant in these.

Factors Determining Decision to Prospect Geophysically

Prior to our investigation, the Mineral Point orebody had been partly indicated by diamond drilling. Study of the diamond-drill results and of the geology as disclosed from logs of the holes, ore dump, surrounding rock exposures and surface trenches on the orebody outcrop led to the conclusion that a good chance existed for extensions of this orebody northeast under the lake and southwest under glacial overburden. The geological structure of the Gull Lake depression and the contact metasomatic origin of the ore deposits due to the bordering granite-diorite masses, were envisaged, and it was concluded that the Gull Lake zone presented favorable geological conditions for ore deposits like that at Mineral Point. Furthermore, nests of chalcopyrite ore boulders and pyritic boulders, of glacial origin, are distributed northeast of Gull Lake (Fig. 1). Geological reconnaissance showed that Gull Lake occupies a glacial carved depression. Glacial striae, *roche moutonée* and stoss and lee structure of elongated rock ridges indicated glacial movement from southwest to northeast. This suggested a source for the ore boulders in the Gull Lake depression, or between its northeast end and the site of the ore boulders.

Prominent rock ridges parallel the long axis of Gull Lake but much of the area is covered by lake water, woods and bogs, all of which in turn usually mantle glacial drift. It was obvious that geophysical methods must be employed in order to prospect the area at all thoroughly. Possible extensions of the Mineral Point orebody could be tested readily by diamond drilling but, on the other hand, this could be more intelligently directed if zones of maximum sulfide concentration could be outlined first by electrical survey. It was concluded that the highly electrically conductive ore mineral chalcopyrite, associated with the magnetic mineral pyrrhotite, presented favorable conditions for the effective joint use of electrical and magnetic surveys. Two main objectives were in mind: (1) to locate and outline possible extensions of the known orebodies; (2) to locate new orebodies, particularly orebodies other than the known orebodies that might be the source of the ore boulders.

History of Geophysical Prospecting at Gull Lake

During the summer of 1928 a Swedish-American Prospecting Co. field party employed the Lundberg equipotential electrical method, supplemented by electromagnetic and dip-needle surveys. Surveys over the known Mineral Point orebody, and its possible extension southwest, gave good electrical and magnetic indications. Within a broad zone of electrical disturbance about 200 to 300 ft. wide, Mr. Lundberg, of the Swedish-American Prospecting Co., outlined certain lenticular areas as the strongest indications. Two diamond-drill holes have been put down in two of these outlined areas southwest of the known orebody. These

corroborated the electrical indications by penetrating zones of abundant pyrite and pyrrhotite mineralization but unfortunately the chalcopyrite content is too low to constitute ore. The general picture of a wide conductive zone within which more conductive zones lie, as presented by the equipotential survey map, agrees with the conditions of sulfide mineralization shown in Fig. 2, except, perhaps, that lenticularity of the orebody is probably overemphasized if Mr. Lundberg's lenticular areas of maximum conductivity are visualized as orebodies.

An equipotential electrical survey on the southwest shaft area suggested narrow zones of metallization. This agrees with known conditions.

An equipotential electrical survey was also started near the northeast end of Gull Lake and carried on northeast towards the ore boulders. About 600 acres were surveyed but only weak disturbances were found.

During the winter of 1928–1929 a Gurley dip-needle survey was made on the ice by the Newfoundland Mining Corpn. organization, covering the entire length of Gull Lake along the suspected mineralized zone. This was followed in the summer of 1929 by detailed geological mapping and study. An electromagnetic survey, using a large vertical loop, was then conducted by a Swedish-American Prospecting Co. field party. Boats and islands were utilized to take profiles across zones in Gull Lake which the dip-needle survey and geological mapping suggested as the most favorable. Profiles were also taken across the land area between the northeast end of Gull Lake and the ore boulders, over part of the ground covered by the equipotential survey in the summer of 1928, in order to check the equipotential method in an area where the overburden was probably thick. No indications meriting further exploration were discovered by the electromagnetic survey. Indications were obtained, however, across the known orebody at Mineral Point on both land and water.

Results and Conclusions

The net result of geophysical prospecting on the Gull Lake property, costing about $25,000, was disappointing. The two drill holes put down on the Mineral Point southwest extension to verify the electrical indications demonstrated the efficacy of the equipotential method to locate concealed zones of conductivity in this area, but in spite of the joint use of electrical and magnetic surveys, under favorable conditions, it was impossible to predict whether certain indications resulted from a concentration of chalcopyrite sufficient to make ore or merely from noncommercial sulfides. Magnetic reactions were due in part to pyrrhotite and in part to magnetite and did not necessarily indicate important concentrations of associated chalcopyrite. Electrical indications were pronounced over uncommercial sulfides as well as over ore. The fundamental limitations of geophysical surveys were evident, therefore.

The geophysical survey having for its object the location of unknown orebodies gave no certain evidence of the efficacy of the methods used. Negative results might be attributed to the inability of the methods to locate conductive zones under a considerable covering of lake water or glacial overburden, but a fairer conclusion is that no strong conductive zones exist in the areas prospected.

General Discussion

(Donald H. McLaughlin presiding)

A. H. Rogers, New York, N. Y.—The matter of expense has been brought out today. It was spoken of particularly by Dr. Bateman. It does seem expensive to spend $4 or $5 an acre to prospect a piece of mineral ground. When you consider, however, what it costs to prospect it by any other method, it is not so expensive. The reason for that cost is largely in the expense of doing the geophysical business. I do not believe that mining people generally consider what is going on in the field of geophysics. I can speak particularly for the electrical field, because I am somewhat concerned with that. My company spends anywhere from $30,000 to $50,000 a year solely on experimental work, and I have no doubt the other electrical companies are spending sums more or less similar. The other geophysical companies probably are not spending so much, but the cost of management, the cost of keeping the engineers under salaries when they have no engagements, counts up. Geophysical work is not so tremendously profitable as some people seem to think it is.

D. C. Barton, Houston, Tex.—There are certain difficulties actually in checking up on the methods. We have had experience with that on the Gulf Coast. For example, in the few years the leading consulting company in the seismic method had four troops in the field, one troop leader found six salt domes, as I remember. That figure is not accurate. The other three troop leaders did not find any salt domes. I have not got the exact data as to where the troops worked, but in general they worked over much the same area. We felt at that time that one troop leader had something that the other three leaders did not have. They all had the same equipment. They were supposed to be doing the same thing. Yet one troop leader could find salt domes and the others could not do so. If the method had been tried out on the Gulf Coast only with those three troops, we would probably have turned down the seismic method for use on the Gulf Coast. One of my friends later had one of those three troops run a line across a known salt dome that rises within 800 ft. of the surface but failed to inform the troop of the presence of the salt dome. The troop leader reported that there was no dome present. The troop afterwards went back and finally succeeded in picking up the dome.

We also have found, on the Gulf Coast, that if you take what I call a seismic month—an average month with an average crew—and calculate the number of seismic months worked by the two main consulting companies, they compare their success in discovering domes with the success of all the other companies, made up mainly of individual oil companies trying to run their own seismographs, you will find that those two main consulting companies have a much greater efficiency in the discovery of the domes. If you had tested out the seismic method only by the "other" troops, you would get a much more unfavorable opinion of the capacity of the method than the seismic method now has, largely as the result of the work of the two main consulting companies.

So in evaluating the success of a method, the personal equation and the equipment equation both come in.

I was interested in another experience of one of my friends. They used two companies. I will not say which method was used. One was a very expensive company and the other company cost only half as much. The less expensive did not pick up the structure; the much more expensive company did.

L. B. Slichter.—Just a word about the subject of interpreting geophysical data and the relative role of the geologist and the physicist. This has often been a point of discussion at these meetings. Now the interpretation and analysis of electrical and electromagnetic data in the three-dimensional case, as in a geophysical problem, is certainly a specialized province of mathematical physics. Many of the failures in method and interpretation may be attributed, I believe, to a complete lack of training in the fundamentals of electromagnetic theory on the part of the personnel in charge, and to the undue optimism and consequent energetic exploitation which sometimes follows when ignorance is bliss. Competent physicists are essential in the present stage of development of electromagnetic prospecting.

In this connection, it is useful to stress the essential independence of the electrical and geological problems. In the electrical problem, the objective is the utilization of the observed electrical data for discovering distributions of electrical material which are competent to explain these data. In this search, the observed electrical data may be analyzed, scientifically speaking, only in terms of the special rules and laws which apply—that is, in terms of electromagnetic theory. The geological data, similarly, are independently mapped and analyzed in accordance with geological theory. The two are obviously quite independent studies, governed by laws which are about as distinct as the rules of bridge and poker, and although there is a certain temptation to mix the two, and gain an electrical interpretation directly from geological information, such a mixture is dangerous, and as impossible, logically, as the attempt to play, at one time, partly bridge and partly poker. The science of the matter should be kept straight, and if the independent findings of the two processes need to be reconciled, the compromises necessary should be recognized as such. Clearly, the more independent are the conclusions, the greater will be the value of consistent findings; and if divergencies exist, the more productive will be efforts focused upon the discrepancies.

D. C. Barton.—I should like to emphasize one more point, which is my favorite point in geophysics. The first step in geophysics is mapping a physical effect at the surface. That is purely a physical problem. You do not worry about geology at all. You map the variation of some physical effect at the surface. Then, in general, the next thing to do is to take the map of the variation of the particular physical effects at the surface and from it interpret the distribution of some physical property in the subsurface. That again, as Dr. Slichter says, is a physical problem. You have to know the physical laws governing that physical property and how that physical effect can be produced at the surface. That is the second step in the physical survey. The third is to take the distribution of that physical property in the subsurface and try to interpret what it means geologically. That is where you must know geological relations.

In connection with the electrical geophysical work, I think that at the present time it is in a state of flux and therefore belongs mostly in the physicist's hands rather than in the geologist's. In the seismic methods the theory is not changing very rapidly and the field men therefore should primarily be geologists. In both these methods, however, we should have one physicist-geophysicist studying new instruments and another physicist-geophysicist, or mathematical geophysicist, studying improvements in the mathematical and physical end of the interpretation. The physicist type of research man should be in the background working on his end of the problem, and the geologist-geophysicist type of man should be in the field.

I like to compare the geophysicist in geophysical prospecting to the petrographer. The petrographer applies a physical instrument and method to the study of small sections of the earth's crust in a way that is very similar to the way in which the modern methods of geophysical prospecting are applied to the study of large sections of the earth's crust. But the petrographer has been in geology so long that everyone has forgotten the highly mathematical-physical character of his method. I think that the geophysicist, like the petrographer, must have a strong training in mathematics and physics and in the special theory of his science superimposed on the fundamental training of a geologist.

D. H. McLaughlin, Cambridge, Mass.—In my opinion, geophysical prospecting will not gain an assured place in the mining field until the basic physical problems have been so well worked out and standardized that the methods can be used by geologists as part of their regular technique. In their present stage of development, the skill of the physicists is of first importance, in devising methods for measuring the physical quantities and for stating the results in significant terms. This was equally true in the perfection of the X-ray technique, but it would now hardly seem fair to expect the physicist to make a diagnosis from an X-ray negative in a clinic. Likewise, it is unreasonable to ask the physicist for the final answer with respect to the subtleties of geology from the data he obtains from the measurements of physical fields in the various geophysical surveys. It is clearly the geologist's responsibility to make effective use of the valuable new data that the geophysical methods will provide.

I believe, however, that the physicists need to give more serious attention to the problem of expressing their results in the most serviceable way. It is their responsibility to interpret the data in accordance with physical theory and to state their results in terms that are significant to the geologist but which leave the final answer, in terms of rocks, structures and ores to him. Lucky guesses are possible from geophysical surveys, by physicists as well as by geologists, but unless thorough cooperation exists between them misses are more probable, and, if too numerous, are likely to obscure the true value of the information that can, without any question, be gained by geophysical surveys.

E. E. Ellis, New York, N. Y.—Our company has made several investigations with geophysical methods with the hope of obtaining clues which might supplement geological studies and assist in finding orebodies.

At our Tennessee operations the zinc ores occur in dolomitized phases of a limestone series at shallow depths and the resistivity method was employed in the hope that there might be a difference in the resistant properties of limestone and dolomite sufficient to differentiate the areas, and that if this feature did not develop, the presence of solution cavities might be more prevalent in one type than the other and be indicated. Marked differentiation in resistivity was found, which made an attractive map, but our judgment was that the mapping was essentially that of the residual soil covering.

An unexpected result was a fair map of an oxidized orebody lying at depths of 10 to 70 ft. below the surface. It was presumed that this was due to the fact that the oxidized ore carries high moisture content and therefore furnished a better conductor.

The electrical method has not been continued for this class of ore, partly because it was felt that similar and confusing results might arise from other causes and mainly because other methods are cheaper.

From this work we deduced that this method might have decided value where the contour of the rock surface below residual soil or other covering is of interest.

This year several methods have been employed in the northern New York zinc district. It was, of course, recognized that as zincblende is a nonconductor, electrical

methods could be employed only for associated minerals or indirectly and the purpose was to use the geophysical methods only as an adjunct to geology. In all cases some indications were checked, but only at points where the geological work indicated a promising drilling location where indications were found.

The first method used was the high-frequency inductive method and indications of moderate strength were found, but drill checks gave no results and a known orebody with associated pyrite at a depth of over 300 ft. gave no response. It is our belief that this method is well adapted to shallow sulfide bodies, but does not have penetrating power.

The next method tried was the magnetometric, and very interesting maps of magnetic attraction resulted. It was first tried on a previously drilled orebody with curving outlines and the magnetometer check was very exact. The magnetometer was tested on various fragments of ores and rocks and it was found that some of the zincblende had definite polarity and exerted a strong influence on the magnetometer, while most of the blende caused no response. It might be stated that outside of the known checked area no zinc ore was found under magnetometric highs.

A group of marked magnetometric highs were drilled and in four instances were found to be pre-Cambrian channels filled with Potsdam sandstone. It is presumed but not known that this sandstone carries some finely divided magnetite. If there were any commercial object in outlining these channels the method would be effective. One small but strong high was drilled and nothing found to explain the anomaly.

In another area a marked magnetic high was drilled and a substantial thickness of massive pyrite and pyrrhotite was found—an adequate explanation. In this same field, however, within a few hundred yards, and in a meadow, an area of magnetic highs was found corresponding in intensity to that of the pyrrhotite area, but the slope of the high gave reason to believe that it represented a valley filled with sands carrying magnetite. The one drill hole put down found 130 ft. of sand and gravel wash and magnetite was found by panning.

The self-potential or measurement of natural earth currents produced by oxidizing sulfides was given a rapid test. This method checked the pyrrhotite area but gave negative results for the meadow.

A fourth method, the electrical inductive with low frequency, has given undoubted indications of pyrite zones but the application to ore finding is still in doubt. For our particular purpose, among those referred to, it is decidedly of the greatest promise.

It is the writer's opinion that each of the methods mentioned above will definitely operate under certain conditions and for certain materials, but the findings must be scrutinized with the utmost care by men familiar with the geology and if possible with any peculiar geological features of the area under observation.

In our own case we have expended several thousands of dollars and no monetary return is in sight, and it cannot be said that the results have any future value even as negative findings, yet there is no regret over the expenditure because the work has indicated clearly that commercial applications are possible under conditions within the limits of the process employed.

It is improbable that any of these methods will ever be developed to a foolproof system and any concern using them must have a competent interpreter of its own on the meaning of the results and must be very cautious in the expenditure of money to check the results. Furthermore, it is essential that the daily results of the geophysical prospecting be available at all times to this representative and not withheld for a final or even weekly report.

For a going concern, with a known orebody of unknown extension, there is a good possibility of satisfactory return from expenditure in the class of geophysical prospecting to which its type of orebody will respond. In an area of known geological conditions and ore types, the right method under the right conditions should find

unknown orebodies. In an area of obscure geology, the odds are heavily against commercial success.

It is my opinion that too much of the geophysical work for the purpose of ore finding has been done without a proper interpreter for the party most interested; *i. e.*, the party who pays not only for the geophysical work but for the much greater expenditures necessary to check the meaning of "indications."

A New Development in Electrical Prospecting

By Hans Lundberg* and Theodor Zuschlag,† New York, N. Y.

(New York Meeting, February, 1931)

Based upon an instrumental improvement, a new development has taken place in the art of electrical prospecting, and some remarkable results have already been obtained with regard to potential exploration.

In order to visualize the importance of this development, it is deemed advisable to review briefly the fundamental facts about potential surveys.

Potential exploration may be carried out qualitatively or quantitatively. Qualitative investigations are made by determining and plotting on a map points of equal potential, whereby any anomalies are revealed, without, however, much clue to their magnitude. Quantitative investigations determine the variations of potential or potential drop, and thus show not only the location of the anomalies, but also their magnitude. Figs. 1 to 6 illustrate the influence of simple types of ground inhomogeneities, assuming one effective exciter potential only. Such a condition can be closely realized by connecting an electric current source to two electrodes, one of which is located at a great distance. Assuming uniform ground resistivity, the resultant potential distribution may be represented by concentric spheres. Any change from the normal condition causes a characteristic distortion of the spheres, which becomes different for different types of resistivity variation.

The series of Figs. 1*a* to 6*a* illustrates schematically results of qualitative investigations and series 1*b* to 6*b* results of quantitative investigations. Figs. 1 and 2 picture the well-known case of local resistivity variations normally encountered in ore prospecting. Figs. 3 and 4 show the reactions caused by more or less vertical resistivity contacts and Figs. 5 and 6 refer to the problem of ground-water and bedrock investigations where the resistivity contact is horizontal or nearly so.

It is evident from Figs. 5*a* and 6*a* that qualitative measurements do not permit an identification of such general geological features as widespread horizontal strata, because the resultant contraction or expansion of the equipotential circles at the surface cannot be recognized without quantitative determinations. On the other hand, qualitative measurements are well suited for the exploration of local or lateral resistivity

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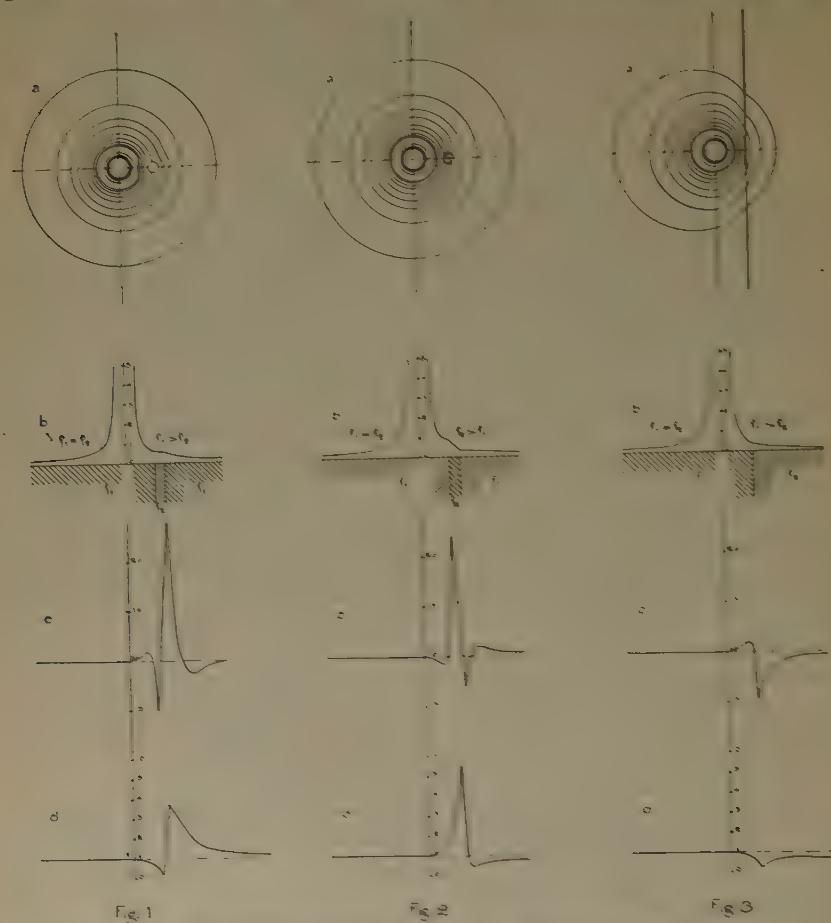
† Research Engineer, Swedish American Prospecting Corporation.

variations, particularly if one considers that instrumental requirements are extremely simple and economical. Quantitative measurements (Figs. 1 to 6, *c* and *d*), on the other hand, may give results under all



FIGS. 1–6.—INFLUENCE OF SIMPLE TYPES OF GROUND INHOMOGENEITIES.

Figs. 1 and 2. Local resistivity variations normally encountered in ore prospecting.

Figs. 3 and 4. Reactions from vertical resistivity contacts.

conditions, yet their execution requires a higher class of equipment than is necessary for qualitative measurements. This fact is responsible for the relatively slow development of the quantitative methods, because adequate equipment first had to be developed and even now may be capable of further improvement.

Instruments for this purpose measure either potential drop or potential drop ratio values. Potential drop determinations may be carried out

with the help of direct reading instrument combinations[1] or by means of comparative measuring arrangements.[2]

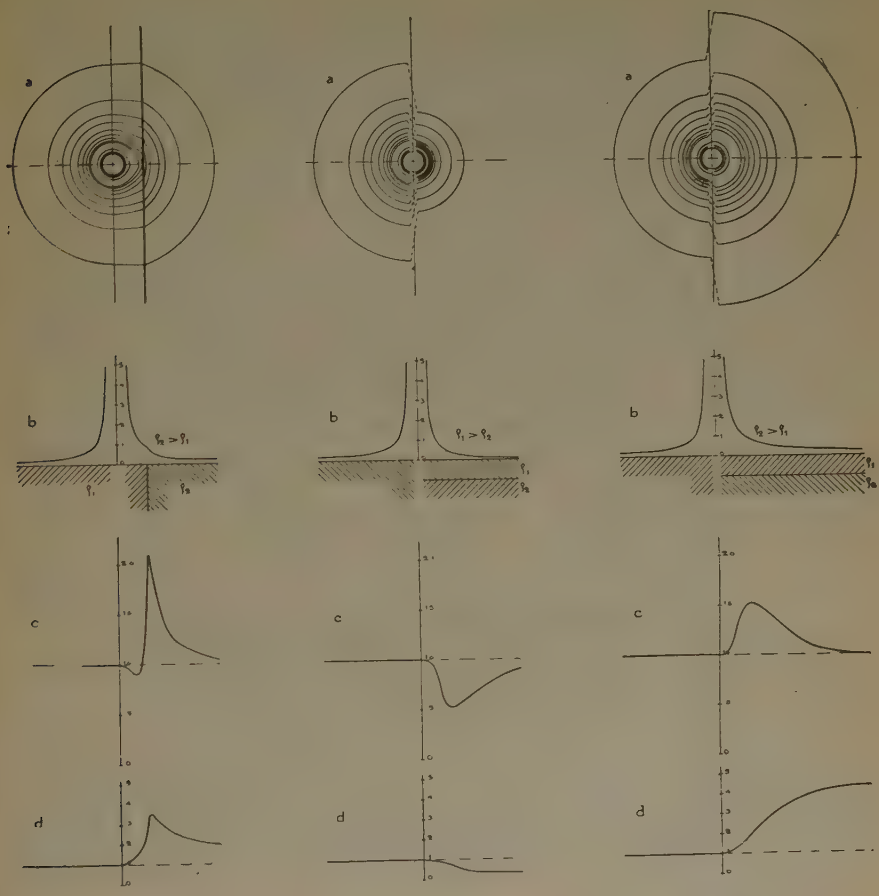


Fig. 4 Fig. 5 Fig. 6.

FIG. 1–6.—CAPTION ON OPPOSITE PAGE.

Figs. 5 and 6. Reactions from horizontal resistivity in ground-water and bedrock investigations. Series *a* shows results of qualitative investigations; series *b*, *c* and *d* results of quantitative.

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[1] F. Wenner: A Method of Measuring Earth Resistivity. U. S. Bur. Stds. *Sci. Paper* 258 (1915).

F. W. Lee: Measuring the Variation of Ground Resistivity with a Megger. U. S. Bur. Mines *Tech. Paper* 440 (1928).

[2] C. Schlumberger: U. S. Patent 1719786 (1926).

O. M. Gish and W. T. Rooney: Measurement of Resistivity of Large Masses of Undisturbed Earth. *Terrestrial Magnetism and Atmospheric Electricity* (1925) **30**, 161.

Potential drop ratio determinations are based upon some kind of bridge arrangement, as proposed by C. Schlumberger,[3] Hans Lundberg[4] and J. G. Königsberger.[5]

Direct reading instrument combinations are easy to operate, yet possess a relatively low sensitivity, which depreciates their reliability as well as range of application. Comparative measuring arrangements are less simple, but more sensitive in operation, thereby insuring a more satisfactory as well as more extended range of application.

All potential drop measuring arrangements require a permanent or temporary connection between power source and measuring device. This condition, however, becomes a source of complication, as well as error, if the exploration is carried to great depth or distance, because it forces the operator either to employ long leads to or from his measuring device or to stay close to his power supply line. Long leads are cumbersome to handle and, if current-carrying, are subjected to thermal resistivity variations. Furthermore, in employing alternating current, even of a frequency as low as 50 cycles, long leads are liable to cause distorted

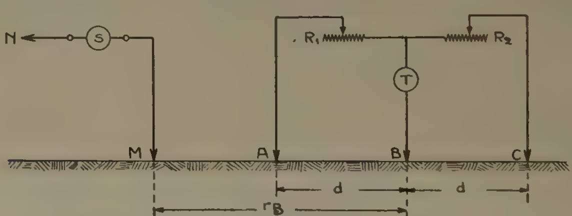

FIG. 7.—DETERMINATION OF POTENTIAL DROP RATIO.

readings through their tendency to pick up electromagnetic effects. This condition is aggravated if potential drop determinations are taken near the alternating current supply line, because in this case the strong electromagnetic field of the line affects not only the leads but also the resultant ground potential.

Bridge arrangements for potential drop ratio determinations do not require a connection between current source and measuring device, but this advantage is completely offset by the relatively low sensitivity, lack of accuracy and cumbersome mode of operation of the aforementioned systems of potential drop ratio investigation. The common cause of these three deficiencies is the existence of unknown and variable contact resistances. Their elimination or determination would undoubtedly

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[3] C. Schlumberger: U. S. Patent 1163468 (1913).

[4] H. Lundberg: Potential Metod for Elektrisk Malmletning. *Jernkontorets Ann.* (1919).

[5] J. G. Königsberger: *Ztsch. f. Geophysik* (1930) **6**.

improve sensitivity and accuracy, as well as speed of operation. A physical elimination evidently is impossible, while a numerical determination of their value is a rather complicated affair. However, the problem has now been solved by means of a mathematical procedure which has already been used for similar measuring problems and which, in this case, permits the elimination of the contact resistances. This procedure was made the basis for the design of a potential drop ratio compensator called the "Racom," developed by Theodor Zuschlag.

The Racom

Generally speaking, the Racom is a sensitive bridge arrangement for the accurate and speedy execution of potential drop ratio determinations, and is thus adapted to geoelectrical exploration. In order to explain the principle of its operation, reference is made to Fig. 7. A power source S is connected to the exciter electrodes M and N, the electrode N being located at such a distance that for practical purposes the potential distribution due to electrode M only need be considered. This potential distribution at the surface of the ground is tapped by means of suitable rods at points A, B and C. Rods A and C are connected in series with variable resistances R_1 and R_2, while rod B is connected in series with an indicator T to the common end of resistances R_1 and R_2.

The potential drop P_{AB} from A to B and P_{BC} from B to C causes a current flow through R_1 and R_2 and normally also through T. Any current flow through T deflects the indicator, which enables the operator to recognize and neutralize it by adjusting either resistance R_1 or R_2. The remaining current flow i through R_1 and R_2 is then determined by the potential drop P_{AB} and P_{BC}, the potential rod contact resistances R_A and R_C and the instrument resistances R_1 and R_2 in accordance with the following equations:

$$P_{AB} = i(R_A + R_1)$$
$$P_{BC} = i(R_C + R_2)$$

or

$$\frac{P_{AB}}{P_{BC}} = \frac{R_A + R_1}{R_C + R_2} \quad [1]$$

This equation contains the unknown resistances R_A and R_C, which makes it impossible to determine the potential drop ratio accurately.

Assuming now that the value of resistance R_1 is arbitrarily changed to R_1', we destroy, of course, the neutralization and cause a deflection of T_1 which may again be compensated by a readjustment of resistances R_2 to R_2' in accordance with the new equation

$$P_{AB} = i'(R_A + R_1')$$
$$P_{BC} = i'(R_C + R_2')$$

or

$$\frac{P_{AB}}{P_{BC}} = \frac{R_A + R_1'}{R_C + R_2'} \qquad [2]$$

Equations 1 and 2 may be transformed into expressions giving the value of contact resistance R_A.

$$R_A = \frac{P_{AB}}{P_{BC}}(R_C + R_2) - R_1; \qquad \text{and} \quad R_A = \frac{P_{AB}}{P_{BC}}(R_C + R_2') - R_1'$$

The new equations may now be combined and developed in regard to the potential drop ratio

$$\frac{P_{AB}}{P_{BC}} = \frac{R_1' - R_1}{R_2' - R_2} \qquad [3]$$

It is seen that the resultant equation 3 is free from the contact resistances R_A and R_C and permits an accurate computation of the potential drop ratio P_{AB}/P_{BC} by means of a simple procedure and formula.

FIG. 8.—RACOM IN USE.

The determination of the potential drop ratio in this manner requires that a certain amount of ground current must be shunted through the instrument. Theoretically, such an arrangement is bound to distort the resultant ground potential distribution, the distortion increasing with decreasing shunt resistance. Extensive tests, however, have shown that the shunt resistance, comprising contact plus instrument resistance, is almost always of such a magnitude that the resultant distortion is negligible for all practical purposes.

The elementary Racom combination described above is not suitable for the exploration of alternating fields, because no provision is made for the compensation of phase displacements, and it is also rather limited with regard to the range of contact resistances, which can be eliminated. These limitations, however, are not found in the field type of Racom shown in Fig. 8, which permits large phase adjustments and covers a wide range of contact resistivities. The readings are usually taken with a telephone, the impulses being stepped up by means of a high gain amplifier, which acts at the same time as a filter and, when using frequencies above or below the audible range, as a frequency converter. A detailed description of the complete arrangement, however, has to be postponed on account of pending patent applications.

Exploration with the Racom

Potential exploration by means of the Racom is based upon potential drop ratio determination.

Ratio profiles, representing a systematic succession of potential drop ratio determinations, are radiated from a common exciter electrode or are laid out as parallel lines from a line electrode or a number of different exciter electrodes in such a manner that the area to be investigated is systematically covered (Fig. 9).

The individual ratio determinations may be carried to any distance from the exciter electrode, using any suitable spacing between the three

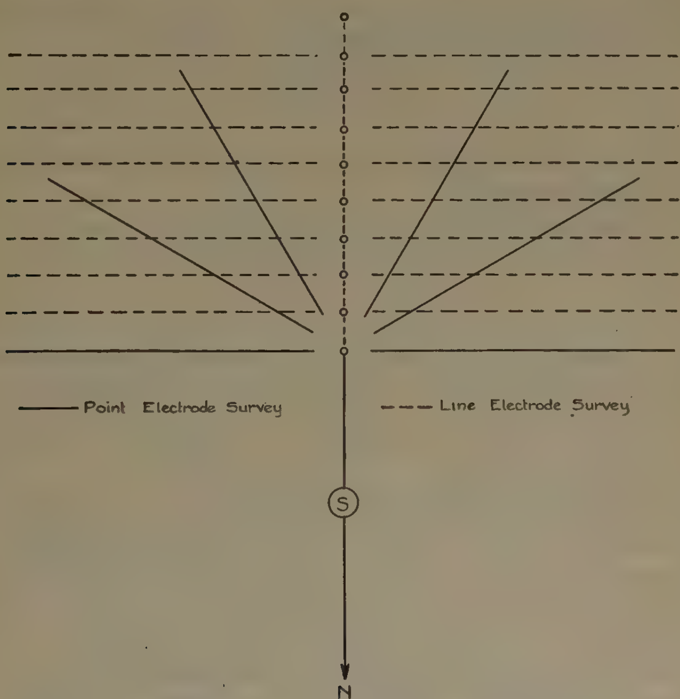


Fig. 9.—Method of radiating ratio profiles.

potential rods. The spacing is preferably kept equal and constant, but may also be changed in accordance with any arbitrary distance ratio.

The measured potential drop ratios are either referred to a standard ratio by multiplication with the distance ratio $(r_B - d)/(r_B + d)$ (See Fig. 7) and plotted against the distance r_B to the center electrode, or they are transformed into individual ground resistivity ratios and plotted against the corresponding potential rod locations.

The apparent ground resistivity ρ_x of a point x in r_x meters distance from the exciter electrode is given by the equation

$$\rho_x = 2\pi r_x \frac{P_x}{I} \text{ ohms per cu. m.} \tag{4}$$

Where I is the strength of the exciter current and $P_x/I = (P_x)$ the semi-absolute potential in point x.

The ground resistivity in short distance from the exciter electrode may be considered identical with the surface resistivity ρ_o, while in greater distance from the exciter electrode it represents the combined effect of the different ground resistivity variations and may be expressed as a function of the surface resistivity

$$\rho_x = K_x \cdot \rho_o \tag{5}$$

where K_x is called the ground resistivity ratio. Substituting equation 4, the ground resistivity ratio may evidently be written

$$K_x = \frac{r_x(P_x)}{r_o(P_o)} \tag{6}$$

where (P_o) is the semiabsolute potential in point O located in distance r_o from the exciter electrode.

If the ground resistivity is uniform, we have $K = 1$ and

$$P_x:P_o = r_o:r_x \tag{7}$$

Arranging the points A, B, C of Fig. 7 in such a manner that relation 7 applies to point A and $B(P_A = P_o)$, it is possible to develop equation 3 in regard to the semiabsolute potential ratio P_c/P_o and hence to compute the ground resistivity ratio K_c. In the same or similar manner, the ground resistivity ratio for any point located upon a ratio profile may be easily determined and plotted.

The interpretation of the Racom results presumes a profound knowledge of potential theory, as well as extensive practical experience. The limited scope of this paper makes it necessary to refer in this regard to special publications, as, for instance, T. V. Hummel: The Apparent Specific Resistance. [*Ztsch. f. Geophysik* (1929) Nos. 3 to 6.]

Generally speaking, any variation in the continuity of the actual ground resistance influences the resultant potential distribution and thereby causes varying potential drop, as well as potential drop ratio values. Assuming the conditions of Figs. 1 to 6, the resultant potential drop ratios are schematically illustrated in the series 1*c* to 6*c*. The reactions are pronounced and differ considerably for the various assumptions, thereby permitting a definite characterization of the different cases.

It may be noted that the boundaries of the local and lateral variations of Figs. 1 to 4 are distinctly indicated and thereby easily located. Referring to Figs. 5*c* and 6*c*, it is seen that a better conductive, as well as a poorer conductive, bottom layer—for instance, water table or bedrock—causes a characteristic peak of the ratio curve. The distance to the

peak is a function of the thickness of the top layer and may be used to determine the depth to the bottom layer. The numerical value, however, of the ratio "thickness of top layer over distance of ratio peak" is not a constant, but varies with different resistivities of the two layers. In Figs. 5 and 6 it has been assumed as 0.66, a value that represents a fair practical average. Greater accuracy may be obtained by basing the

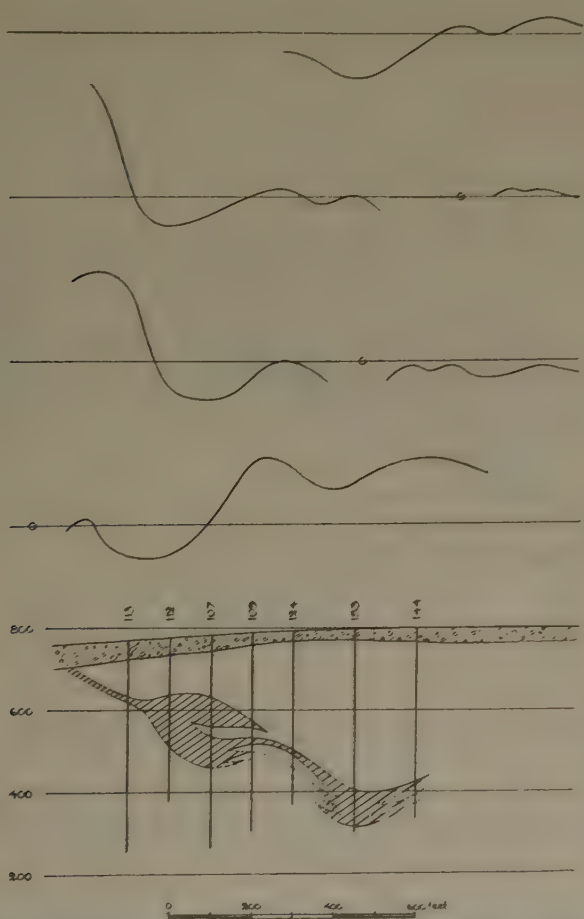


FIG. 10.—RATIO PROFILES FOR DIFFERENT ELECTRODE POSITIONS OVER A GEOLOGICAL SECTION ACROSS ORIENTAL OREBODY AT BUCHANS, NEWFOUNDLAND.

interpretations upon theoretical ratio curves, which may be computed for different two-layer conditions.

The analysis of the potential drop ratios alone, however, is not always sufficient to permit an accurate interpretation of any exploration problem. This is particularly true of cases where it is necessary to apply corrections;

for instance, in order to compensate for certain topographical, stratigraphical or lithological changes. *Potential drop ratios* are composed of three potential values referring to different locations, which makes it rather difficult to apply any correction. On the other hand, *ground resistivity ratios* as defined above refer to only one location and therefore are well suited to amend and improve the potential drop ratio analysis in this respect.

Referring again to Figs. 1 to 6, the series 1*d* to 6*d* illustrates schematically the resultant ground resistivity ratio. The reactions are some-

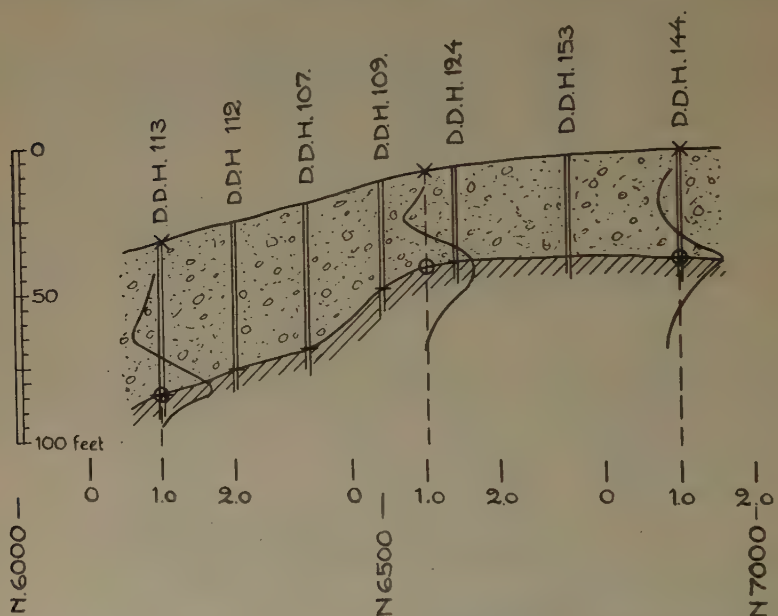


Fig. 11.—Determination of depth to bedrock, Buchans, Newfoundland.

what less pronounced and characteristic than for the potential drop ratio curves. It will be noted that the peak of curves 5*c* and 6*c* is replaced by an inflection point in curves 5*d* and 6*d*.

Potential Drop Survey during Development of Orebody

A potential drop survey may render useful information during the development of an orebody, as shown by the following example:

Fig. 10 is a section across the Oriental orebody at Buchans, Newfoundland. This orebody was known from several drill holes (Nos. 107, 109, 112, 113) but in holes 124 and 144 very little ore and only scattered mineralization had been encountered. From the electrical profiles it could be noticed, however, that besides indications from the orebody, a

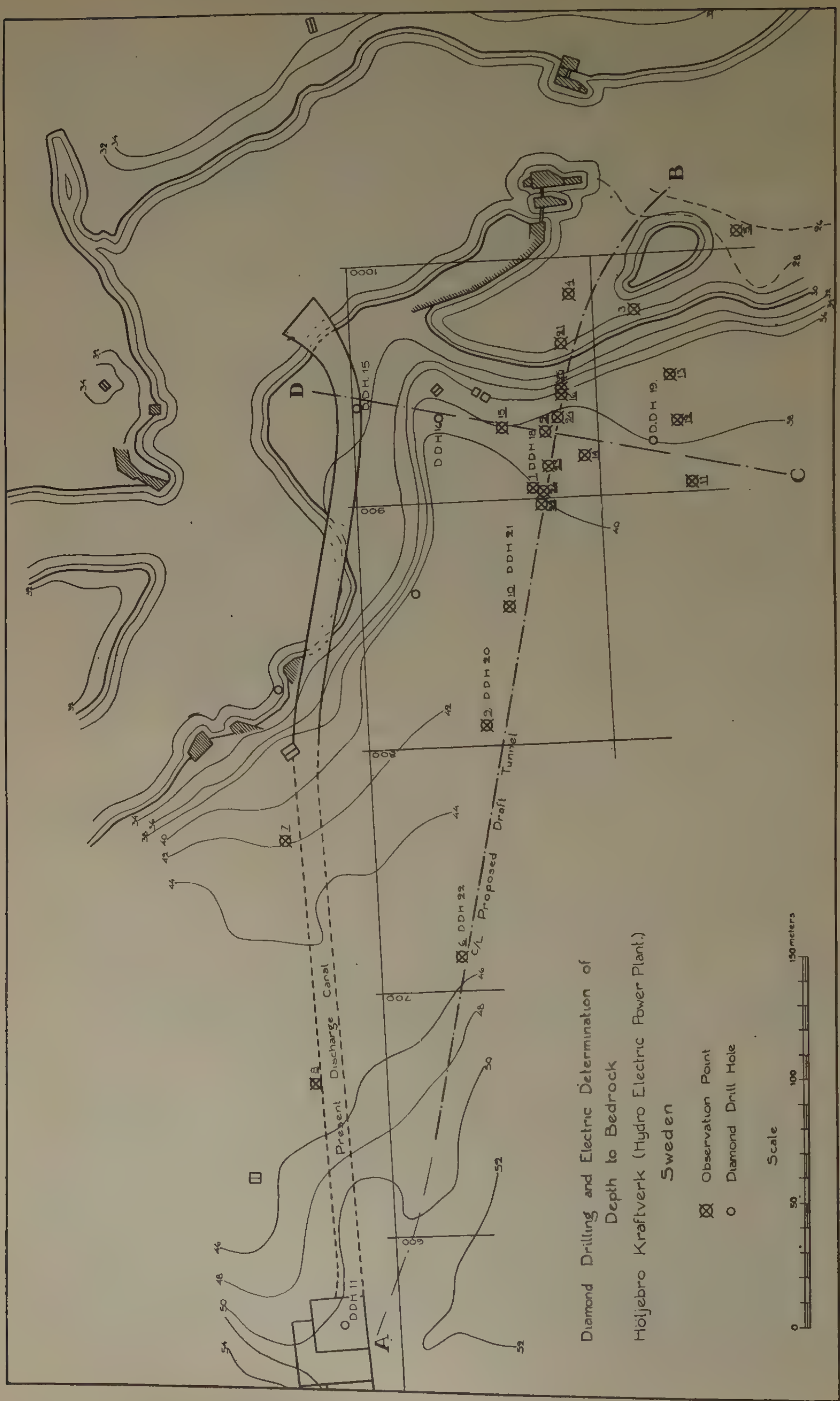
distinct disturbance existed between holes 124 and 144. This latter disturbance was interpreted as being caused by a conductor at a depth considerably greater than the known orebody. Hole 153 was then drilled and from 400 to 460 ft. depth, heavy sulfide mineralization was encountered, which thus well confirmed the electrical prediction.

The original application of electrical prospecting was principally the locating and outlining of ore deposits, whereas modern geoelectrical exploration now may be applied to solve many other problems. This refers especially to irrigation and foundation problems, as well as to stratigraphic and structural exploration.

In many irrigation and foundation projects, the question frequently is asked: What is the depth to the ground-water table, or to the bedrock? Both problems may be solved electrically with the help of the generally existing resistivity differences between top soil and water table or top soil and bedrock, as schematically indicated in Figs. 5 and 6. The actual results of a depth to bedrock determination by means of potential drop ratio measurements are given in Fig. 11. The investigated points lie upon the geological section of Fig. 10 and were analyzed with smaller electrode spacing than used for the ore exploration work, in order to obtain clear indications despite the relatively shallow depth of the bedrock. The resultant potential drop ratio values are plotted vertically and in corrected and reduced scale against the location of their exciter electrodes. Two different peaks may be noticed on every move, the upper one indicating the upper boundary of a zone of better conductivity in the lower part of the glacial drift, probably caused by higher water saturation; and the lower one indicating the boundary plane between drift and arkose. The electrical depth determinations check with the logs of the diamond-drill holes 113, 107, 109, and 144. It is interesting to note that the time required for the electrical investigation was less than four hours.

A more elaborate investigation of this type, requiring about eight days, is shown in Fig. 12. The purpose of this work was to outline the course of a new draft tunnel for the Holjebiro Power Station of the Bergvik Ala Nya Aktiebolag, Askesta, Sweden. The tunnel was to be blasted through a plateau of gray gneiss covered completely with glacial drift of varying thicknesses. The electrical investigation revealed that the bedrock along the eastern shore of the River Ljusnan, into which the water was to be discharged, is sharply sloping to the north (profile *CD*), while in an easterly direction (profile *AB*) only a slight dip is encountered. Guided by this result, the most suitable location for the outlet of the tunnel could be selected, the situation of which was at a more southerly point than originally planned.

One of the most important applications of geoelectrical exploration is the electrical analysis of the sedimentary column in regard to structural configuration.


Diamond Drilling and Electric Determination of
Depth to Bedrock
Höljebro Kraftverk (Hydro Electric Power Plant.)
Sweden
Observation Point
Diamond Drill Hole
Scale
150 meters
Proposed Draft Tunnel
Present Discharge Canal
A
B
C
D

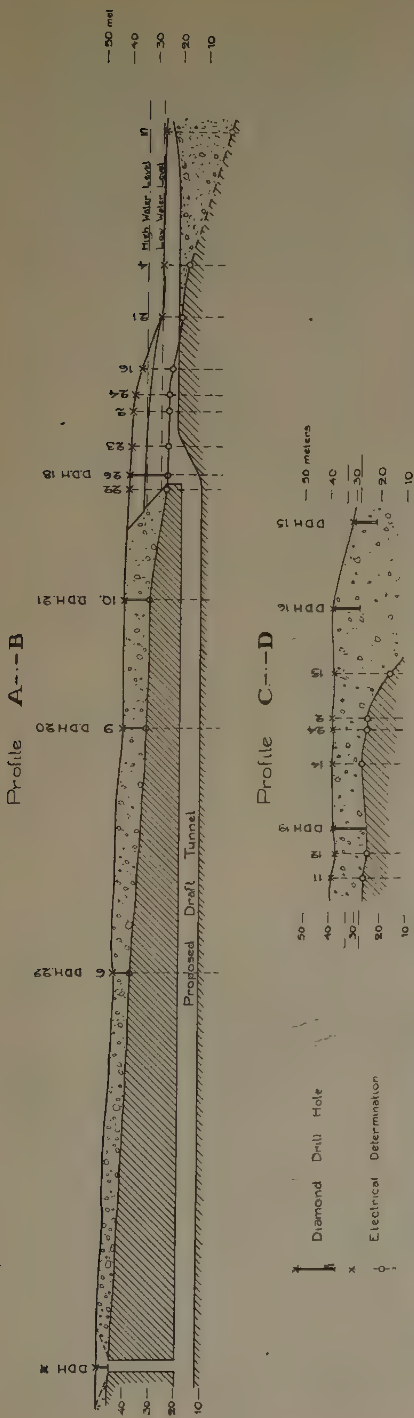


FIG. 12.—DIAMOND DRILLING AND ELECTRIC DETERMINATION OF DEPTH TO BEDROCK, HYDROELECTRIC POWER PLANT, SWEDEN.

Almost any lithological change is accompanied by more or less distinct changes in electrical resistivity (of the different rocks). The resistivity of a rock is chiefly determined by its water content and by the degree of salinity of the water. The water content of sedimentary rocks is practically identical with the pore volume and, in the case of rocks of fine grain, the salinity generally depends upon the kind of water in which the sediments settled. Such a bed has the tendency to retain the connate water, whereas a coarse-grained bed permits a freer flow of water and thus the original water is often washed out and replaced by purer water of secondary origin.

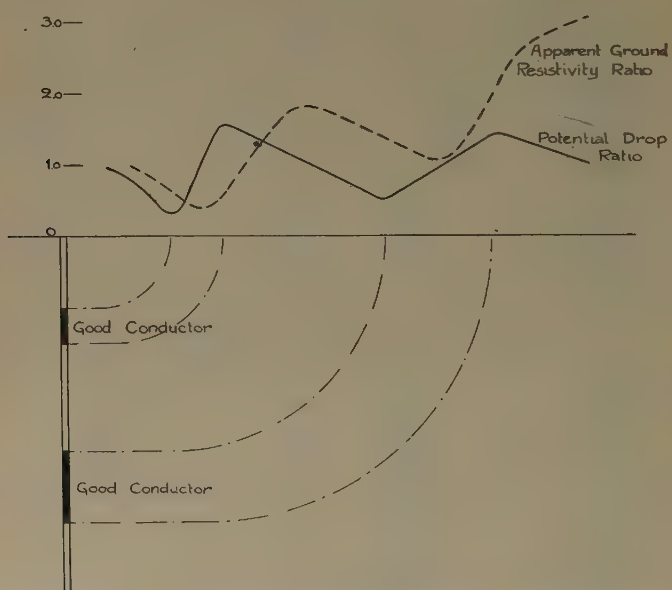


FIG. 13.—POTENTIAL DROP PROFILES FROM DEEP-PROBING POTENTIAL INVESTIGATIONS.

The best conductors of the column are the marine clay and shale beds, while fresh-water gravel beds have usually a high resistivity.

Assuming a sedimentary column with a number of pronounced conductivity changes it can be proved theoretically, as well as experimentally, that these changes influence the surface potential distribution in the same succession as they actually occur in vertical succession. Deep-probing potential investigations mean, therefore, far-reaching potential drop ratio profiles, as illustrated in Fig. 13. Any contact between formations of different resistivity causes a characteristic peak of the potential drop ratio curve and an inflection of the apparent ground resistivity ratio curve.

It may be noted in this connection that the ratio "depth to contact over distance to corresponding peak or inflection point" varies for differ-

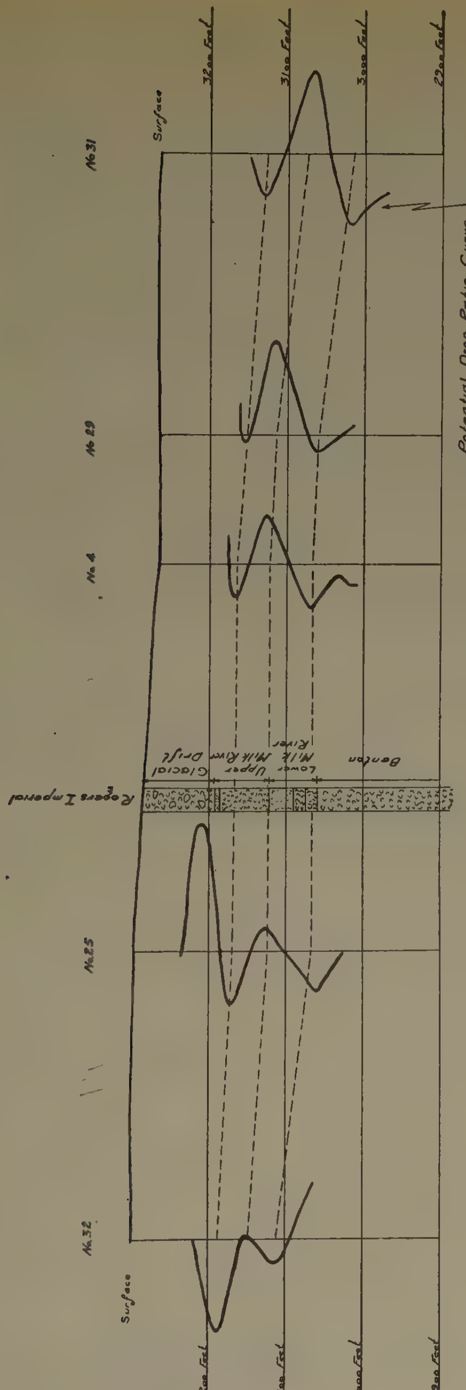


FIG. 14.—ELECTRICAL LOGS AT ROGERS IMPERIAL WELL.

ent contacts with any factor influencing the resultant potential distribution, thereby making a numerical interpretation a rather complicated affair. Experience, however, has shown that such variation is relatively small when the area under investigation is fairly free from abrupt topographical and sudden lithological changes. In this case, satisfactory structural interpretation may generally be obtained by the following simplified procedure: The distances to the measured potential drop ratio values are reduced by the same correction factor; the resultant curves are vertically plotted against their exciter electrode locations, and correlated peaks are connected with each other.

Fig. 14 represents an actual example of this type of interpretation, showing a number of Racom logs taken in the vicinity of the Rogers Imperial Well, at Range, Alberta.

The logs are taken in geological section composed of glacial drift, Upper Milk River, Lower Milk River and Benton shale. The contact between the good conductive Upper Milk River, with its finely grained clayey shales, and the poorly conductive Lower Milk River sandstone, as well as the top of the good conductive Benton shale, produces outstanding indications which may be followed over the whole profile.

Summary

A new measuring procedure permitting potential drop ratio determinations is described. Following a brief nontechnical discussion of some fundamental applications, the results of four actual investigations of different types are discussed.

Geophysical Examination of Meteor Crater, Arizona

By J. J. Jakosky,* C. H. Wilson† and J. W. Daly,‡ Culver City, Calif.

(New York Meeting, February, 1931)

Meteor Crater, Arizona, is a natural wonder which for years has been the subject of considerable discussion and study as to its origin and age (Fig. 1). Of the various theories advanced regarding the origin, two have been considered as most probable by a majority of investigators. The first theory is that the crater was formed by a meteorite or a swarm of meteoric material striking the earth at a high velocity and burying itself. The second theory is that the crater was formed by a "steam explosion" from hot solutions or gases coming from underneath the present sedimentary beds which overlie the area. In order to obtain more detailed information regarding the conditions existing at the crater, the present management decided to have a detailed geophysical examination made.

The origin of the crater is of considerable scientific importance and interest. More or less active mining operations have been carried out during the past 20 years by various groups who believed that the crater was of meteoric origin, and who hoped that sufficient quantities of such material had been buried to be commercially valuable. Fragments of meteoric material found on the surface of the ground in the debris around the rim gave analysis of approximately 92 per cent iron and 8 per cent nickel, with some platinum and iridium. The material having a gross value of approximately $50 per ton, mining would be commercially profitable if sufficient quantities of the material existed in condition for mining. In the early work numerous drill holes and shallow shafts were put down, mainly in the northern portion of the crater. Pulverized material and debris caused considerable difficulty, and in an effort to overcome this the present management started a 1500-ft., two-compartment shaft on the south rim of the crater and outside of the fill zone. The intention was to sink the shaft in the solid rock outside of the crater, and then to crosscut toward the crater at a depth of 1500 ft. The work was finally stopped when a depth of 650 ft. was reached, because water poured into the shaft through large cracks and fissures in the rock. The enormous force responsible for the crater had also broken

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† Mining Engineer, International Geophysics.

‡ Geologist, International Geophysics.

Fig. 1.—Aerial photograph of Meteor Crater, Arizona.

and shattered, in a radial fashion, the sedimentary rocks of the area for considerable distances from the crater.

No definite figures are available as to the total cost of the exploratory work at the crater to date, but it is estimated to be well over $500,000. The general purpose of the geophysical work described in this paper was to obtain information as to the advisability of continuing with the exploration and the direction such exploratory efforts should take.

During this geophysical investigation, detailed geological studies were made over an area of approximately 1000 acres; magnetic studies over an area of approximately 5000 acres, and detailed electrical studies over an area of about 700 acres. The entire work in the field occupied approximately five weeks. The complete report covering this work cannot be presented here, because the space is too limited, but an attempt is made to give sufficient data and details to show the general nature of the investigations, and the general method of interpretation and conclusions.

General Procedure

This work was conducted by our usual procedure of applying various independent studies to an area and then working out the final conclusions by correlations of the data from the various studies. After inspection of the area and considering the time available for the work, three main independent studies finally were decided upon: (1) geological, (2) electrical and (3) magnetic. Each study was in charge of an independent worker or group. These various studies are presented individually in this report, together with the general conclusions possible from results of each study. Upon completion of the field work, the necessary laboratory studies were made to allow final calculations. All data were then correlated and the following interpretation derived.

Brief Summary of Results

The results of the work may be summarized as follows:

1. From a geological examination, we believe the crater was formed by impact of a meteorite or a swarm of meteoric material. The explanation previously advanced by some investigators, that a steam explosion is responsible for the crater, does not seem probable. The age of the crater is considerable, possibly 50,000 years. Fig. 2 shows the chief geological conditions, together with important faults, and the dips and strikes of outcropping strata. The geological study indicates the presence of a foreign material in the southern portion of the crater. This material lies within the crater, as shown by the maps, and does not lie underneath the rim, as originally supposed from early studies conducted by the crater exploration company. The indicated position of this material corresponds with the electrically conductive zone and the magnetic anomalies.

2. The electrical survey gives indications of the presence of an area of higher conductivity in the southwest quadrant of the crater, between the center and the rim, the main mass of which lies at an effective depth of approximately 750 ft. A careful study of the original and altered

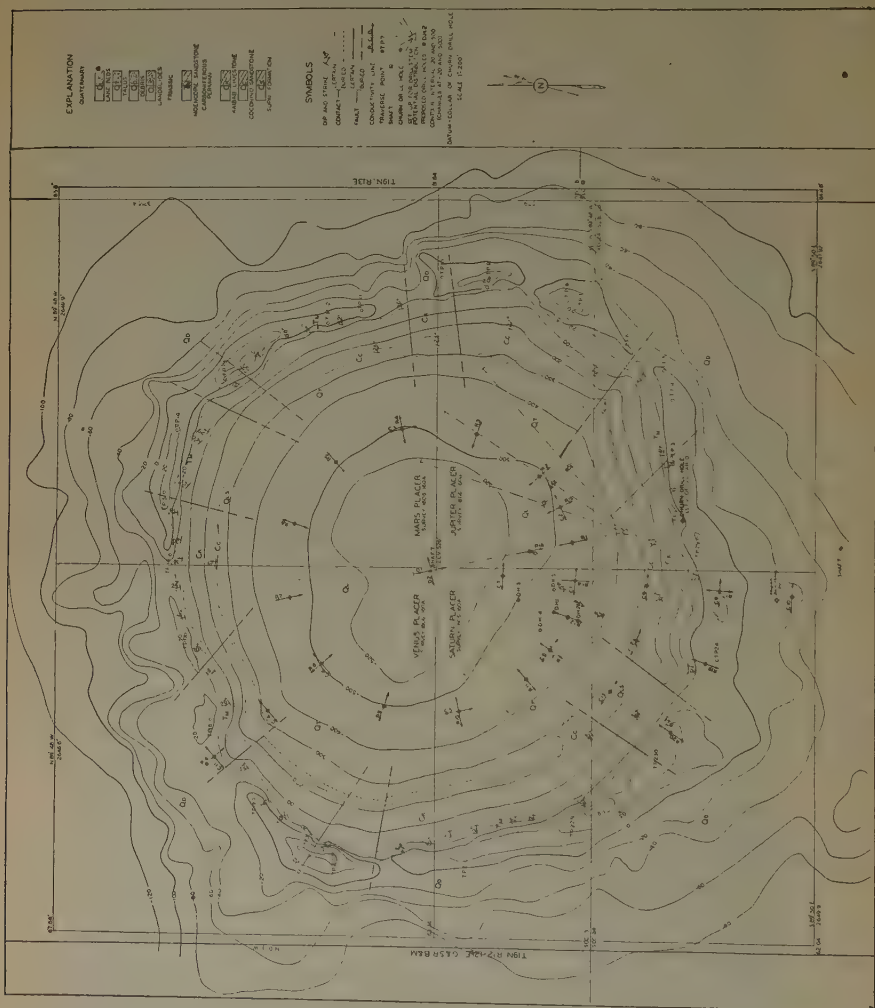

FIG. 2.—Geology and electrical stations, Meteor Crater.

materials found in the area indicates that this zone of higher conductivity is not due entirely to fill material or structural conditions. The conclusions are that this area contains material of metallic character. The chief electrical relationships are shown graphically in Figs. 3 and 4. The material is not in the form of a sphere, but probably a fragmental zone having its greatest length in a general southwest direction, underneath station R12.

3. The magnetic studies indicate the presence of an area containing magnetic material in the southwest portion of the crater. This material starts at depths of approximately 200 ft. and continues downward, probably concentrating with depth. The magnetic anomalies, after correcting for regional gradient, are shown on Fig. 5.

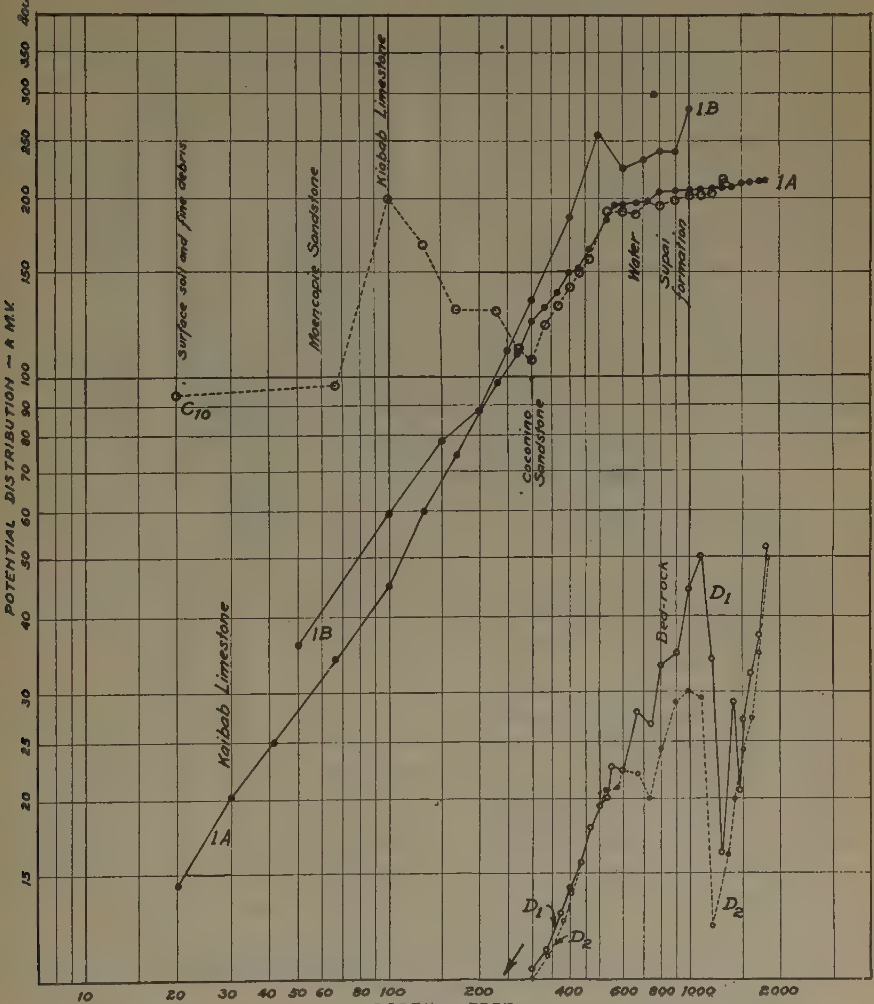

FIG. 3.—POTENTIAL SECTIONS, STRUCTURAL EFFECTS, METEOR CRATER.

4. The geological evidence and the electrical and magnetic indications individually would be classed as fair or moderate effects of a buried mass. The general agreement as regards plan location, depth and other factors gives sufficient added strength to the results to warrant further explorations to the extent of churn-drill holes. The recommended locations for

such drill holes are shown in Fig. 2, and are numbered in importance for proving-up the presence and extent of the fragmental meteoric zone.

5. Results of the work are not considered sufficiently definite, in view of complicating factors encountered, to warrant calculations or

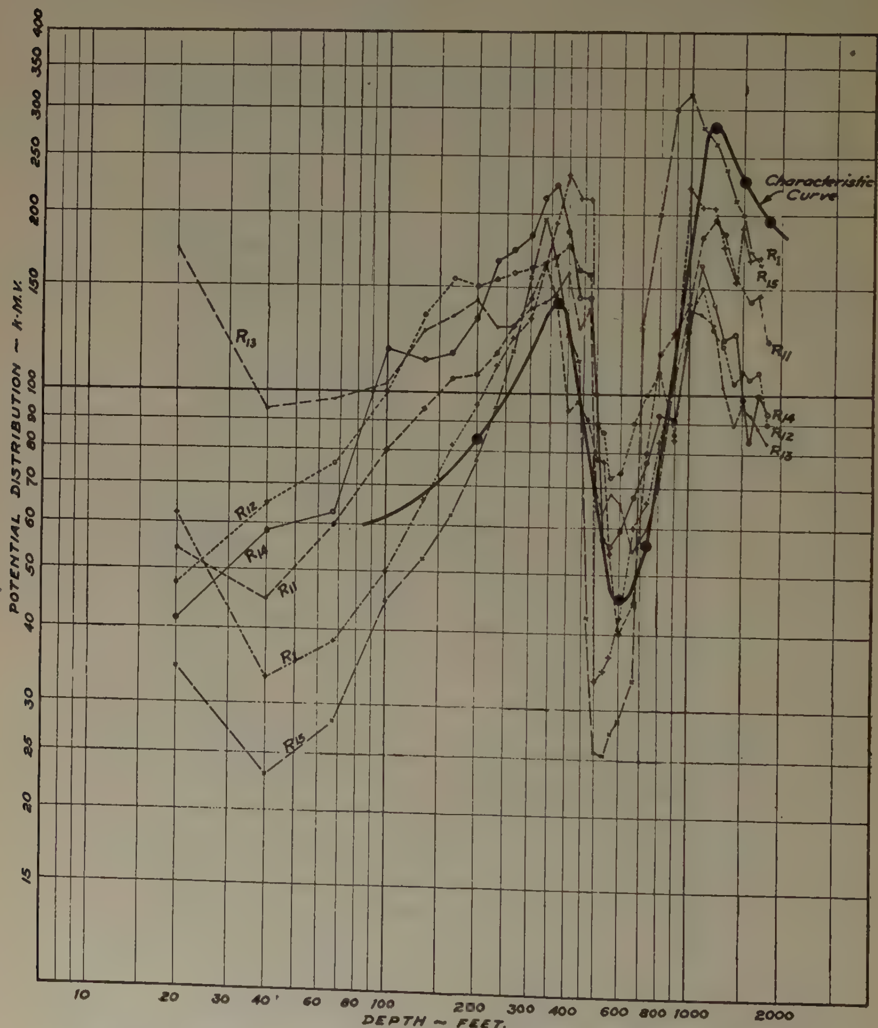

FIG. 4.—POTENTIAL SECTIONS, CRATER CONDITIONS, METEOR CRATER.

predictions regarding the tonnage or mass of material which may be present. The chief result of the work has been to delineate clearly the area wherein future development work is to be concentrated.

6. The water level in the crater is comparable with that outside of the crater.

Geology of Meteor Crater

Location and Topography

Meteor Crater lies in the high plateau of northern Arizona, about 7 miles south of U. S. Highway 66, approximately 35 miles from Flagstaff

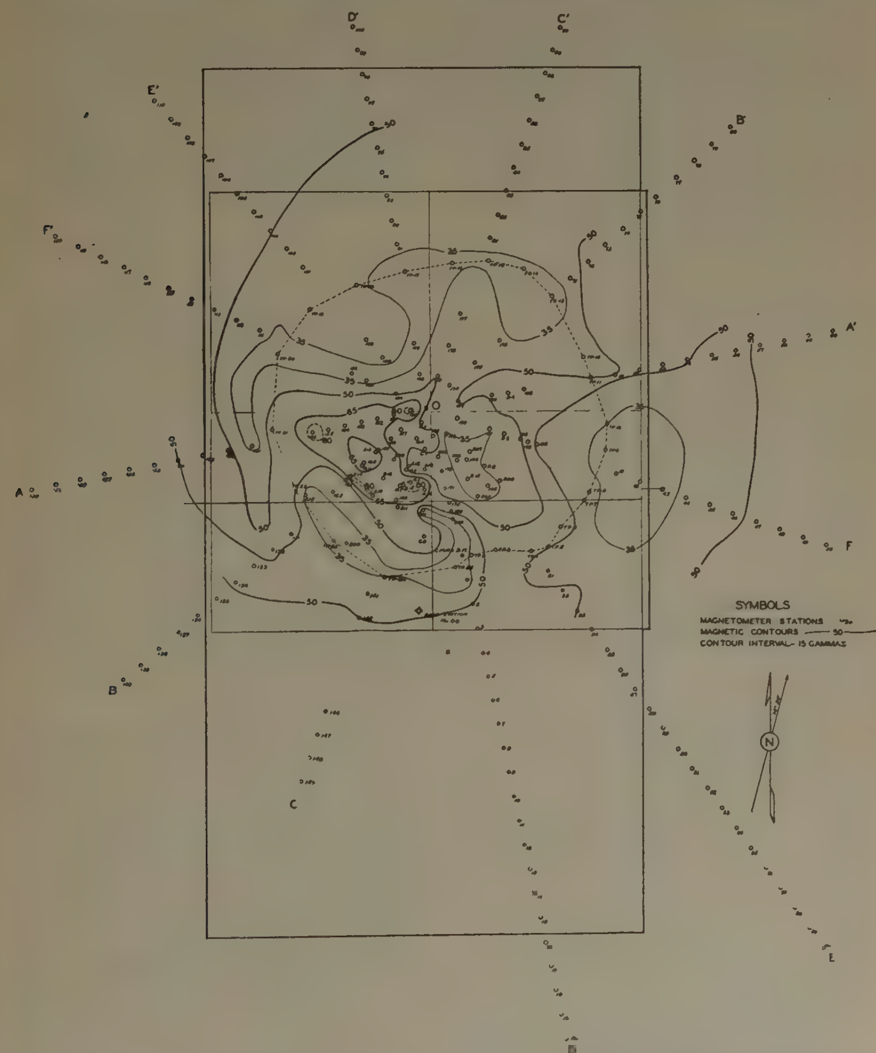

Fig. 5.—Magnetometer stations and magnetic contours, Meteor Crater.

and 20 miles from Winslow. The regional topography is that characteristic of plateau country, with a gentle slope to the north and northwest, which is broken by small buttes rising abruptly above the surrounding country.

The crater rim stands approximately 160 ft. above the general level of the plateau and surrounds a bowl-shaped area, 570 ft. deep, having a maximum diameter of 4250 ft. and a minimum diameter of 4000 ft. (See Figs. 1 and 2.) The elongation is in a northwest-southeast direction, while the lowest part of the rim is in the octant between a north line and a northwest line measured from the center of the crater. A playa lake bed occupies the floor of the crater and the walls become steep toward the rim until, in some places, they are almost vertical.

Outside of the crater, the regional water run-off drains toward the north and northwest, but this is altered slightly in the vicinity of the crater. Here the drainage pattern is roughly radial from the crest of the rim outward to the water courses of the plateau and inward to the playa lake bed at the bottom.

Stratigraphy

The formations which have been affected in the cataclysmic formation of Meteor Crater are the same as those which outcrop or underlie the plateau region of northern Arizona. Their designations, divisions, geologic age and correlation, in this report, are according to Darton.[1] The lithologic units of Quaternary age have been divided into debris, talus, landslides and lake beds.

Supai Formation (*Permian*).—This formation is not exposed in the walls of the crater, but it is included here because it is the oldest rock (as drill records and electrical data show) to have been disturbed by the forces responsible for the crater. As described by Darton,[2] the Supai consists of shales, interbedded with massive, coarse and fine-bedded red sandstones, which have a thickness of roughly 1000 ft. in this vicinity. The Supai formation rests on the Redwall limestone.

Coconino Sandstone (*Permian*).—About 150 ft. of this sandstone can be seen in the crater walls, but most of it has been covered with talus and debris. Where exposed, the Coconino is seen to consist of thick- and thin-bedded, white or gray, finely laminated, friable, cross-bedded sandstone. The chief constituent mineral is quartz in rounded, fine to medium sized grains which have been cemented into a rock of considerable hardness but still extremely porous. A few scattered limonitic concretions are seen in the sandstone. Some of the beds weather to a characteristic light brown or faun color. The formation is of Permian age and lies conformably on the Supai beds.

Since the base of the section is not exposed, the thickness could not be measured. A boring at Winslow penetrated the Coconino from 100 to 965 ft.[3] The log of the churn-drill hole drilled by the Standard Iron

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[1] N. H. Darton: A Résumé of Arizona Geology. Ariz. Bur. Mines *Bull.* 119 (1925).

[2] N. H. Darton: *Op. cit.*

[3] N. H. Darton: *Op. cit.*

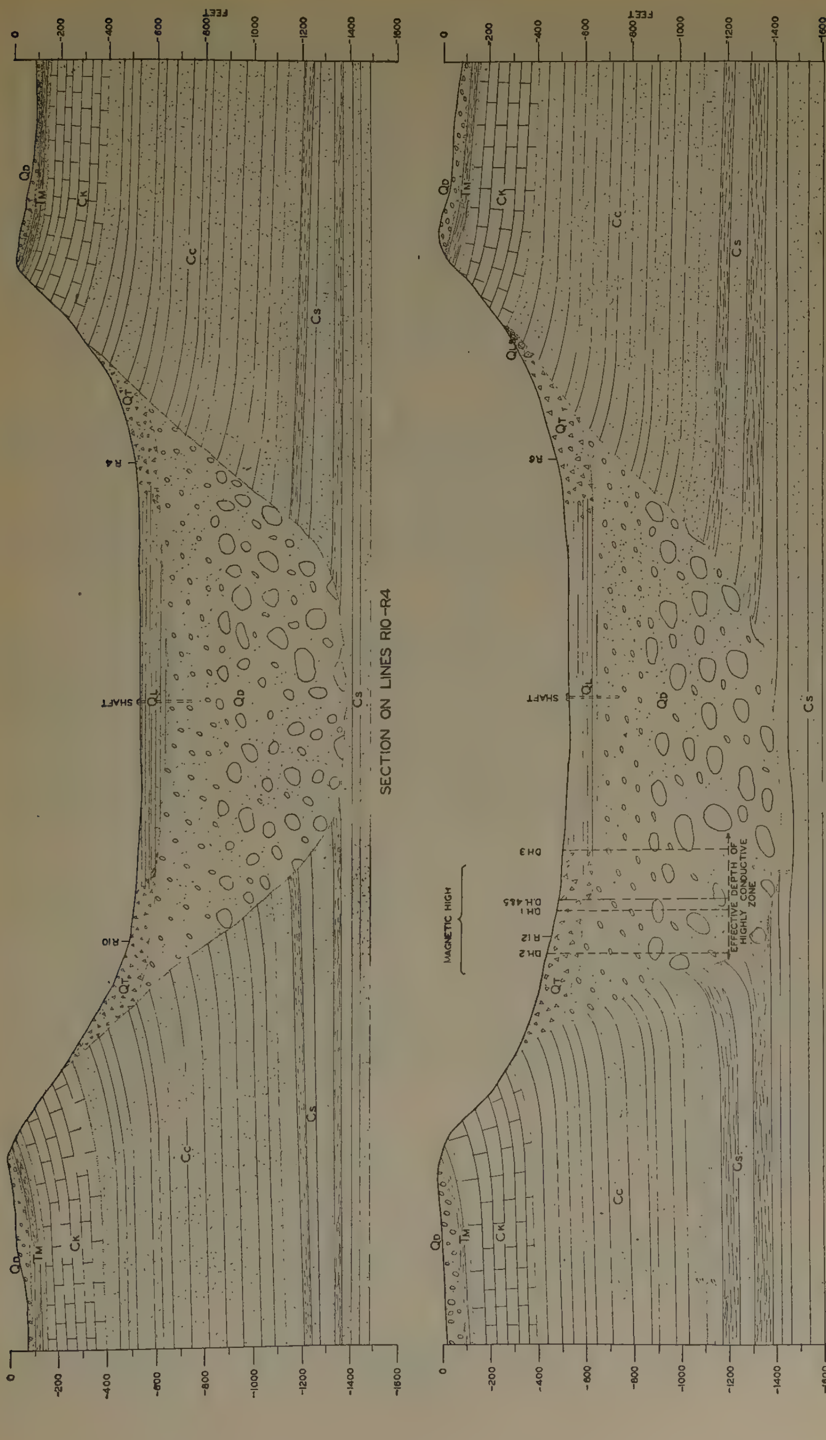

FIG. 6.—PROBABLE GEOLOGIC SECTIONS, METEOR CRATER.

Co. on the south rim shows that Coconino was penetrated from about 325 to 1300 ft., or for about 975 ft. This cannot be ascertained accurately because the log has been poorly kept and no corrections have been made for the dip of the strata. An electrical section on the plateau one mile west of the crater (Fig. 2) indicates that the Coconino sandstone is 660 ft. thick. This figure has been used in the construction of the cross-section of the crater (Fig. 6).

Kaibab Limestone (*Permian*).—The contact of the Kaibab limestone and the underlying Coconino is not sharp but is gradational over about 8 ft. stratigraphically, this 8-ft. formation consisting of alternating thin beds of white quartzitic sandstone and buff colored sandy limestone. In mapping, this transitional series has been included in the Coconino.

The beds mapped as Kaibab are buff colored sandy limestone with a variability of bedding thickness from 10 ft. to a few inches. In general, the beds are thinner in the upper portion than in the lower part; soft, thin-bedded gray sandstone forms the upper 2 ft. of the series. This formation, like the Coconino sandstone, has a few limonitic concretions which are not confined to any definite beds but which are scattered throughout the whole strata.

According to the log of the shaft sunk by the Meteor Crater Mining and Exploration Co., 250 ft. of Kaibab limestone was encountered in sinking. The logs of the diamond-drill holes of the same company show a thickness of 220 ft. Computations made from the areal geologic data give a figure of 320 ft. in the crater. Data obtained from the electrical section one mile west of the crater (Fig. 2) show a thickness of 250 ft. This figure is comparable with that obtained from the log of the shaft.

Numerous fossil beds occur throughout the section but as no paleontological studies were deemed necessary collections were not made.

Moencopie Formation (*Triassic*).—In Meteor Crater the Permian-Triassic unconformity, which is noted elsewhere in the plateau country, is marked only by a distinct change in lithology from the upper sands of the Kaibab to the cross-bedded, pink, thick-bedded, basal sandstone of the Moencopie. As closely as it could be determined, the two formations are angularly concordant.

The thickness varies around the rim of the crater from 60 ft. on the northern half to zero on the southwest and southeast sides, where it has been faulted up and covered with debris. It is entirely in keeping with the facts to be observed at the present to suppose that the Moencopie had a variable thickness, due to erosion, at the time of the formation of the crater.

The exposed section can be divided into three distinct members: (1) the basal sandstone with a thickness of 17 ft., (2) 34 ft. of thin-

bedded, red shales, and (3) 9 ft. of red, cross-bedded sandstone. Although the basal sandstone and red shales have been recognized in the outcrop outside of the crater, the upper sandstone member was not seen.

Debris (*Quaternary*).—The outer slopes of the rim consist of the ejectamenta from the crater. This mantle extends continuously for 1000 ft. from the rim, and there are some isolated patches and boulders more than a mile away. This debris is a heterogeneous mixture of shale, meteorites, rock fragments of all the original formations, the metamorphosed Coconino sandstone described by Barringer,[4] and rock flour.

Barringer described two varieties of the metamorphosed sandstone, A and B. It has since been shown by Rogers[5] to consist of Le Chatelierite, the amorphous, high-temperature variety of quartz, which forms at a temperature of 1625° C. The varietal difference is based only on texture; variety A is massive, variety B, pumaceous.

The rock flour consists of extremely fine sand which the microscope shows to be angular shreds of quartz.

The distribution of the rock fragments about the rim is noteworthy. Huge sandstone blocks cover the area close to the rim on the south side. To the east and west sides the slopes are covered principally by huge blocks of limestone and smaller fragments of the same material. On the north rim, the cover of the debris thins, either as a result of erosion or because less of this material was deposited.

Regarding the debris which fills the center of the crater, Tilgham[6] states that below the lake beds are found about 200 ft. of rock flour and fused silica and below that are the fragments of the Coconino, Kaibab, and Moencopie formations in the order named; *i. e.*, in the reverse spatial relation of the undisturbed rocks. The electrical data show that the boundary between the fine and coarse debris in the crater is 310 ft. below the present crater floor. This checks the figures given by Tilgham.

Lake Beds.—The bottom of the crater has been occupied by a lake, probably of the playa or intermittent type. The old shaft in the bottom of the crater was sunk through these beds but was unsafe for examination. However, it was possible to descend a shaft 425 ft. north, 11.5° west, of the center shaft, which section is shown on page 12. According to Barringer,[7] Dr. Dall examined these fossils and reported them to be "all recent species local to the region of the southwestern United States."

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[4] D. M. Barringer: Meteor Crater in Northern Central Arizona Natl. Acad. of Sci. (1909).

[5] A. F. Rogers: The Natural History of the Silicate Minerals. *Amer. Miner* (1928) **13**.

[6] B. C. Tilgham: Acad. of Natural Science (1905).

[7] D. M. Barringer: *Op. cit.*

	Feet	Inches
Fine sand	1	
Fossil bed		6
Fine-bedded marl and clay	1	6
Conglomerate	6	
Fine-bedded sand	3	
Fossil bed		1
Fine-grained, fine-bedded sand	11	
Fossil bed		2
Medium-grained, fine-bedded sands	6	
Clay		2
Fossil bed		3
Coarse sand with some fossils	6	
Bottom of shaft—Total	35	8

Tilgham notes these beds as being from 60 to 100 ft. thick, while Fairchild[8] notes the beds but fails to state the depth of the occurrence of a little volcanic lapilli and ash in them, and the depth of the occurrence or the chemical character of the fragments. According to G. M. Colvocoresses, the lowest fossil horizon is at 70 ft. Electrical data show 110 to 120 ft. of lake beds.

From the nature of the sediments, it seems that these beds represent an extremely long period of sedimentation under arid conditions, and include a record of at least the last effusions of the San Francisco volcano group.

Landslides (*Quaternary*).—These slides are composed of the Kaibab limestone and Coconino sandstone which have been crushed in sliding.

Talus (*Quaternary*).—The material mapped as talus consists of angular rock fragments which have broken from the steep walls of the crater and have fallen until they repose at their angle of rest.

Structure

Regionally the structure consists of a thick series of concordant strata having a dip of a few feet per mile in a northwest direction. This series is broken at Sunshine Mountain, 12 miles southeast, at Black Mesa, 20 miles southwest, and at San Francisco Mountain, 45 miles to the northwest, by the feeder vents of the extrusive igneous rocks which occur at these places.

Locally the structure is somewhat more complicated. Here the rocks which lie nearly horizontal on the plateau are uptilted along the margin of the crater. The angles vary from 11° to 80°, but average about 30° and dip away from the center of the crater in a roughly radial manner.

[8] H. L. Fairchild: Origin of Meteor Crater, Arizona. *Bull.* Geol. Soc. Amer. (1907) **18**.

Dips taken in the upper portion of the section around the rim of the crater bring to light the fact that, in general, the beds on the north and south rim dip less steeply (by 10° or 15°) than do those on the east and west sides. This accounts for the more perpendicular cliffs on the north and south sides.

A slight dome can be seen on the south side, but this is more a visual effect than real, as a study of the plotted dips shows. The manner in which the contour of the crater cuts this structure, as well as a slight slumping of the limestone beds on either end, causes the structure to seem greater than it is.

The length of unbroken strata on the west side of the crater is bowed down between the two limiting faults and dips away from the crater in a manner which suggests a structural basin. The dips taken in this portion (Fig. 2) indicate that the strata are flat. This, however, is slightly misleading because some of the outcrops are slumped. There is reason to believe that the strata probably dip at about 30° instead of 12°, as shown on the map.

Thirteen faults are seen in the face of the crater walls but cannot be traced beneath the debris on either the outside or inside of the crater. If these faults are projected toward the center, most of them meet roughly in an area about 300 ft. dia. and south of the center of the crater. The faults are more numerous on the east portion of the crater than on the west portion. Of the 13 faults, only 3 have registered a stratigraphic displacement of 50 to 100 ft. These are between TP 5, TP 6, and TP 18, TP 19, TP 23 and TP 24 (Fig. 2).

Origin of Meteor Crater

The origin of Meteor Crater has long been a moot question, which has attracted the attention of numerous scientists. Several hypotheses have been advanced but of these we need consider seriously only two, the volcanic or steam-explosion theory, first proposed by G. K. Gilbert and elaborated by W. Johnston, N. H. Darton, and others, and the meteoric origin for which D. M. Barringer has been given the credit of discovery and which has been the theory held by Merrill, Thomson, Fairchild, Tilgham, Magie, Day, Wright, numerous astronomers, and others.

The following factors have been considered in the probable causes for the formation of Meteor Crater:

1. From drill-hole and geoelectrical data, it is known that the underlying Supai strata are undisturbed at a distance of 850 ft. below the present crater bed. Thus the effect of a steam explosion would have to be concentrated in the upper Supai beds and in the Coconino sandstone. If such is the case, it is necessary to account for the source of heat. Since the source of heat could not have been vertically below the present crater bed, the hot solutions must have been accumulated by lateral

filtration from some adjacent buried mass. The magnetometric work does not show a change of regional magnetic gradient within distances of over one mile from the crater. This indicates the absence of such a buried igneous mass, hence this lateral filtration must have occurred over a distance greater than one mile. It is inconceivable that the hot waters would be concentrated by such a lateral seepage in a rock as porous as the Coconino sandstone when the overlying rocks are so porous (Table 1) that they would allow easy escape of the waters in springs and fumaroles.

2. If the steam explosion were the cause, one would expect the accumulation of hot waters to extend over a certain period of time and, from the nature of the rocks affected, to have some egress through numerous fissures and joints. Under such circumstances one would expect to find some evidence of solfataric or hydrothermal action, but such evidence is lacking.

3. The metamorphosed sandstone, or Le Chatelierite, is a mineral of which the temperature of formation (1625° C.) is higher than the temperatures accredited to igneous activity. The steam-explosion theory as stated by Darton[9] would produce forces which would be competent to explain the formation of the rock flour but are incompetent to produce the Le Chatelierite.

4. The intimate association of fragments that are of undoubted meteoric origin with the debris of both the rim and central portions of the crater can have but one interpretation—that their deposition was simultaneous and must be attributed to the same cause. It is conceivable that a shower of meteoric material might have fallen on an area at which a steam explosion had occurred but to suppose that the two effects were simultaneous would multiply the ratios of probabilities so as to make them infinitesimal and practically impossible of occurrence.

5. The chief contention of those favoring the steam-explosion theory was that a meteoric mass was not located by a previous magnetic survey. The equipment used in this early survey was a magnetic needle, which probably did not possess sufficient sensitivity to measure accurately the small magnetic anomalies present in the area. The anomalies are of the order of 50 to 80 gammas, while the regional gradient from one side of the crater to the other, in a northeast-southwest direction, is a maximum of 100 gammas. At best, the early needles were not accurate for measuring anomalies of less than 100 gammas.

From the preceding facts, it is concluded that Meteor Crater is of meteoric origin, or has resulted from the impinging of an extra-terrestrial body with a tremendous force, and which was accompanied by an explosion of compressed gases and water vapor.

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[9] N. H. Darton: A Reconnaissance of Parts of Northwestern New Mexico and Northern Arizona. U. S. Geol. Sur. *Bull.* 435 (1910).

Geologic Age of Meteor Crater

Many investigators believe that Meteor Crater has been formed within the last 5000 years. Merrill, Wright and Day have estimated a much earlier date, in the neighborhood of 20,000 to 50,000 years. The following estimate is based on a study of the physiographic and stratigraphic evidence listed below. Before an analysis of this evidence is made, it should be pointed out that Meteor Crater lies in a region of arid climate where the average rainfall seldom exceeds 10 in. and strong prevailing winds blow from the west and southwest.

1. About 120 ft. of lake beds occupy the present floor of Meteor Crater, and at least the upper 35 ft. of these are principally thin-bedded sands, clays and marls, with numerous fossil beds. Near the top are about 6 ft. of conglomerates, which probably represent cloud-burst deposition. Fairchild[10] has noted the occurrence of some volcanic lapilli and ash in these beds, thus suggesting that Meteor Crater is previous to the last eruption of the San Francisco volcanoes. The eruptions of the San Francisco volcanoes have been divided into three periods. Robinson[11] writes:

> The general conclusion is reached that the eruptions of the earlier stage of the third period of activity covered a considerable interval, beginning not long after the close of the second period and extending into comparatively recent time. That is, they certainly occurred during the Quaternary period and presumably during the latter part of that period.
>
> But of more recent date are the small number of cones and flows of the later stage of the third period, which are situated in the eastern part of the field. They are probably not less than 300 years old as judged from the pine trees that grow along the border of Bonito flow and they may be (in round numbers) 1000 years old.

Robinson further states that Sunset Mountain belongs to the earlier part of the third period. It seems more probable that the volcanic ejecta found in the lake beds would be derived from the nearest source of such material—Sunset Mountain—rather than from the San Francisco mountains proper. The fact that the material is lapilli and ash and not volcanic dust is also significant in this respect. Thus the time of the deposition of these beds would be placed well within the Quaternary period.

2. It has been noted above that limonitic concretions occur sparsely in the Coconino sandstone and Kaibab limestone. On the ridges and saddles along the rim, these have been concentrated to such a degree that the surface is thickly covered with them. It is thought that this concentration is the residue remaining after the erosion of a vast amount of the Kaibab and Coconino.

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[10] H. L. Fairchild: *Op. cit.*

[11] H. H. Robinson: The San Francisco Volcanic Field, Arizona. U. S. Geol. Sur. *Prof. Paper* 76 (1913).

3. When the detritus was ejected from the crater, probably it was not distributed in a uniform slope, allowing a radial drainage toward the plateau, but was hummocky and enclosed some small basins which caught part of the drainage. There are what appear to be remnants of these basins, which have long since been sapped by the headward erosion of the streams farther down the slope. These streams have incised themselves into the debris and have cut canyons of considerable size.

4. The thickness of the Moencopie on the present surface cannot be more than 25 ft., except in the small buttes to the north, while in the crater, where it has been protected by the debris, as much as 60 ft. is exposed. This would indicate that about 35 ft. has been eroded. However, there exists a possibility that the present site of Meteor Crater marks the former location of one of the small buttes in the Moencopie similar to those about one mile north of the crater rim. In this respect it should be pointed out that the upper sandstone member that still remains in the inner part of the crater where it has been protected from erosion by the debris was not seen in the adjacent plateau.

5. It also appears that the amount of erosion within the rim has been greatly underestimated and that the fresh looking cliffs are the products of centuries of erosion.

To summarize all the evidence which presents itself, we have, first, a series of sediments deposited in arid climatic conditions under which conditions sedimentation is recognized as an extremely slow process: second, the record in these beds of a volcanic disturbance in this vicinity, probably at the Sunshine Mountains: third, the proof of a very considerable period of erosion. For these reasons, the age is estimated to be measurable in terms of tens of thousands of years and probably to be in the neighborhood of 50,000 years. To this estimate there are only two possible objections:

(1) an Indian legend having to do with some flaming god which fell from the sky; (2) the occurrence of recent fossils in the lake beds.

At best, the Indian legend is probably only an attempt of people to explain a natural phenomenon and should not be given preference to facts which can be obtained by scientific observations. As for the occurrence of recent fossils, one can only ask, "How long is the recent period?" The separation of the Pleistocene period by Sir Charles Lyell was based on the occurrence of 90 to 95 per cent living species in the fauna of those beds. So, from fossil evidence, all we can say is that the lake beds are of recent origin, with no means of evaluating the age in terms of years. It should also be pointed out that the lake need not, and probably did not, occupy the floor of the crater immediately after its formation but was a factor which depended on regional climatic variations. In addition, the lowest fossil bed occurs 70 ft. below the top of the present lake bottom,

which leaves 40 to 50 ft. below this bed of sediments, in which no shells are found.

Probable Location of Meteoric Mass Indicated by Geologic Data

From the data cited by Tilgham and obtained from holes drilled in the bottom of the crater, as well as from the electrical data, it is known that the undisturbed Supai beds are reached at a depth of about 850 ft. below the present crater bed. These, of course, would have the flat dip which is characteristic of the formations throughout the plateau. Thus one would expect a gradual diminution of the angle of dip from the steeper beds at the top to the flat-lying Supai at the bottom. This is exactly what one finds in all parts of the crater except in the center of the sector included between a line S.10° E. and a line S.40° W. drawn from the center of the crater. In this sector the dips on top are in the magnitude of 20° but increase as one goes down the slope of the crater walls to as much as 35° in the Coconino sandstone, or an increase of 15°. If one tries to draw a section through this sector, he sees that there is a void between these steeper Coconino beds and the flat underlying Supai. This void must be filled with some foreign material—meteoric, debris or altered material.

The limestone beds east of resistivity line R_1 and those west of resistivity line R_{12} are slumped as if they were insufficiently supported by the underlying sandstone, thus indicating slight overhang, due to the excavation of the lower material by explosion. Barringer[12] has noted the slightly domed areas of the south rim of Meteor Crater and has pointed out the fact that this shows a vertical stratigraphical uplift of some 105 ft. above the corresponding strata of the level plain but neglected to mention or did not observe that it is stratigraphically 85 to 100 ft. below the adjacent fault block on either side. He attributes the cause of the uplift of this mass to the wedging beneath it of a huge mass of meteoric material. An explanation should be made for the even greater uplift of the strata to either side. Let us arbitrarily assume a huge body striking the earth at an angle of approximately 70° or more from the horizontal and accompanied by a large volume of hot, highly compressed gases, and take for granted that the Coconino sandstone contained some water. Penetrating these strata, the speed will decrease with approximately a deceleration proportional to the square of the speed, and the force, a product of mass times acceleration, will diminish proportionally. There would be two factors operating to excavate the crater: (1) the rock splash at impact, and (2) an explosion following impact due to (*a*) the expansion of the highly compressed gases and (*b*) the water vapor caused by an increase of temperature due to both the heat of impact and the heat of

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[12] D. M. Barringer: *Op. cit.*

compression. The water permeating the pores of the sandstone, on sudden rise in temperature and consequent vaporization due to the heat introduced by the meteorite, would exert a tremendous force and rend the sandstone particles into rock flour, as has already been suggested by Thomson.[13]

After the projectile came to rest, the escape of the compressed gases would follow the general line of least resistance; *viz.*, the path of the projectile. At the same time, there would be an immense component of force in a lateral and vertical direction, which would account for the up-arching of the strata and the roughly circular form of the crater. After the explosion, a portion of the large fragments dropped over the rim, part fell back into the crater and part of the rock flour was dissipated into the atmosphere.

From one viewpoint, it seems probable that the force of the impact of the meteorite would be so greatly diminished that it could not raise the materials, under which the meteorite came to rest, stratigraphically higher than the adjacent blocks where the vertical component of the explosive forces would be greater. Nor could the meteorite be buried wholly beneath the rim, for the material immediately above it would be ejected by the subsequent explosion.

An opposite viewpoint is held, however, by others, and as wholly justified assumptions show that the meteorite would be expelled from the crater. Dr. F. R. Moulton concludes,[14] "If the meteorite entered at a speed of 10 miles per second and only 1 per cent of its kinetic energy were used in expelling it, then it would be thrown out with a speed of one mile per second."

The many unknown factors which enter into the mechanics of a celestial body striking the earth negate the definite prediction as to the resting place or fate of such a body from theoretical considerations alone.

From the geological evidence presented by the doming in the south rim, the configuration of the faulting system and the increase in the dip of the strata as one descends the crater walls in that sector, it is concluded that the projectile which caused Meteor Crater would lie under the present crater floor, near the south and southwest walls.

Conditions Affecting Application of Geophysical Methods

A number of factors had to be considered in making a choice as to the geophysical studies to be conducted. As is well known, there is no one geophysical method or field procedure that is suitable and useful for all

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[13] E. A. Thomson: The Fall of a Meteor. Amer. Acad. of Arts and Sci. (1912) 42.

[14] Letter from F. R. Moulton to Q. A. Shaw.

purposes. Each geophysical process can supply one general type of information under suitable geological conditions. The physical properties of meteoric fragments found in the vicinity of the crater gave two effects which allowed geophysical examination for a mass lying at greater depth: (1) The electrical conductivity of the unoxidized material was over one million times greater than the surrounding country rock (Table 1), therefore if sufficient quantities of this material existed underground, the electrical studies would indicate its presence (assuming no other electrically complicating factors) by a decrease in the effective resistivity; (2) the magnetic permeability of the unoxidized meteoric material was also over one million times greater than the surrounding sedimentary series, and should cause a magnetic anomaly if the material existed in sufficient amount at depth, and again assuming no other complicating magnetic factors. Because of the limited time available it was decided to conduct preliminary electrical and magnetic studies over the entire area, and then such detailed and check studies as might be indicated over localities that gave favorable indications during the preliminary work.

Factors Governing Electrical Methods

The detectability and differentiation of meteoric material and the surrounding country rock depends upon the physical and chemical composition of the meteoric material, its configuration, mass, and unit conductivity. The conductivity in the country rock is electrolytic and depends almost entirely on the amount and character of moisture contained and on dissolved materials. For the meteoric material, the conductivity may be both metallic and electrolytic, depending upon the degree of oxidation and the continuity of unoxidized portions. For the meteorite there are two possible extremes in composition: (1) unoxidized metallic iron, and (2) completely oxidized material. In the first case, the absolute conductivity would be many millions of times greater than that for the country rock; in the latter, the difference might be so slight as to make detection difficult. See Table 1 for relative conductivities.

In the same way, there are several possible conditions of physical aggregation in the meteorite material: (1) one large mass or a small number of large masses, and (2) a great number of smaller masses. In either case, the material might be partly oxidized; perhaps entirely so in the second instance. From the standpoint of oxidation, as well as the fact that in the second case introduction of fragments of rock between the smaller meteoric masses would serve to lower the composite conductivity, the ratio of conductivity in case 1, above, would, in all probability, be much higher than in case 2.

However, the presence of many scattered smaller pieces of meteoric material, if only partly oxidized, certainly would produce a detectable ratio of conductivity when embedded in country rock of the highly

compression. The water permeating the pores of the sandstone, on sudden rise in temperature and consequent vaporization due to the heat introduced by the meteorite, would exert a tremendous force and rend the sandstone particles into rock flour, as has already been suggested by Thomson.[13]

After the projectile came to rest, the escape of the compressed gases would follow the general line of least resistance; *viz.*, the path of the projectile. At the same time, there would be an immense component of force in a lateral and vertical direction, which would account for the up-arching of the strata and the roughly circular form of the crater. After the explosion, a portion of the large fragments dropped over the rim, part fell back into the crater and part of the rock flour was dissipated into the atmosphere.

From one viewpoint, it seems probable that the force of the impact of the meteorite would be so greatly diminished that it could not raise the materials, under which the meteorite came to rest, stratigraphically higher than the adjacent blocks where the vertical component of the explosive forces would be greater. Nor could the meteorite be buried wholly beneath the rim, for the material immediately above it would be ejected by the subsequent explosion.

An opposite viewpoint is held, however, by others, and as wholly justified assumptions show that the meteorite would be expelled from the crater. Dr. F. R. Moulton concludes,[14] "If the meteorite entered at a speed of 10 miles per second and only 1 per cent of its kinetic energy were used in expelling it, then it would be thrown out with a speed of one mile per second."

The many unknown factors which enter into the mechanics of a celestial body striking the earth negate the definite prediction as to the resting place or fate of such a body from theoretical considerations alone.

From the geological evidence presented by the doming in the south rim, the configuration of the faulting system and the increase in the dip of the strata as one descends the crater walls in that sector, it is concluded that the projectile which caused Meteor Crater would lie under the present crater floor, near the south and southwest walls.

Conditions Affecting Application of Geophysical Methods

A number of factors had to be considered in making a choice as to the geophysical studies to be conducted. As is well known, there is no one geophysical method or field procedure that is suitable and useful for all

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[13] E. A. Thomson: The Fall of a Meteor. Amer. Acad. of Arts and Sci. (1912) 42.

[14] Letter from F. R. Moulton to Q. A. Shaw.

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1. Moencopie sandstone } undisturbed
2. Kaibab limestone } undisturbed
3. Coconino sandstone } undisturbed
4. Supai formation
5. Rim debris
6. Fractured and broken rim rocks
7. Talus material
8. Landslide debris
9. Coarse and fine fill material in the floor of the crater.
10. Lake beds in crater bottom

Knowledge of the relative conductivities is essential to a complete interpretation of the potential curves. The values given herein are approximate and average, and are obtained from laboratory and field measurements, as well as from data built up from field experience with similar rock type. Main emphasis has been put on the two latter, because laboratory measurements on small samples may lead to a large error, since it is practically impossible to reproduce actual field conditions in the laboratory. Particularly is this true in the present instance because of the influential effect of ground water, shattering and pulverizing and the unknown factors of pressure and gradation of particle size at varying depths below the surface.

Plan of the Survey

The electrical survey consisted of three separate but dependent parts: (1) a preliminary reconnaissance survey to delineate favorable and unfavorable areas, and to obtain data as to the electrical properties (conductivities) of the various rocks, of the crater structure as a whole, other structural features of importance, and of water levels; (2) detailed potential studies in areas selected after study of results of the preliminary survey, and (3) studies of potential distribution around the drill stem on the south rim of the crater, utilizing the drill-hole casing as an electrode through which current was applied to the ground.

A topographic survey was conducted and a map prepared to give data from which the plat of the electrical survey could be planned and from which the corrections necessary for topography could be obtained. The results of this work are shown in Fig. 2.

The general plan of the electric survey, after checking by small-scale models, has been as follows. The almost circular symmetrical shape of the crater rim afforded a convenient and desirable means of obtaining comparative studies at various points in the area. A center point selected within the crater was used as a hub or reference point. Potential studies were made at stations distributed circumferentially around the crater, as follows: (1) 1000 ft. from the center point and entirely within the crater proper; (2) 1500 ft. from the center on the inside slopes of the crater rim; (3) 2000 ft. from the center point and on the crater rim;

(4) additional studies at points within and outside of the crater at several stations selected after the first section had been completed and a general idea obtained as to depth of bed rock in the crater. Spacing between stations was selected to give sufficient overlap to allow reasonable assurance of detecting any underlying mass between the stations. The direction in which these studies have been made are: (*a*) along lines radially from the center; (*b*) along lines at right angles to *a* above, and hereafter designated as chord lines. On both radial and chord lines studies were made to effective depths of 1800 feet.

TABLE 1.—*Relative Conductivities of Rocks at Meteor Crater*

Lithologic Unit	Specific Resistance, Ohms per Cu. Cm.		Maximum Water, Per Cent by Volume
	Field Data[a]	Laboratory Studies (Maximum Water Content)	
Moencopie shale[b]		21,000 to 25,000	3.6
Moencopie sandstone[b]	150,000 to 200,000	20,000 to 25,000	6.7
Kaibab limestone[b]	100,000 to 150,000	15,000 to 20,000	18.4
Coconino sandstone[b]	150,000 to 200,000	25,000 to 28,000	15.6
Supai formation[b]	100,000 to 125,000	7,000 to 10,000	16.3
Rim debris (fragmentary limestone and sandstone	200,000 to 800,000	[c]	[c]
Fractured and broken rim rocks (limestone and sandstone)	400,000 to 500,000	[c]	[c]
Talus material	50,000 to 75,000	[c]	[c]
Landslide debris	100,000 to 150,000	[c]	[c]
Fine crater material	10,000 to 20,000	[c]	[c]
Deeper, coarser fill material	50,000 to 100,000	[c]	[c]
Crater lake beds	4,000 to 6,000	[c]	[c]
Meteoric iron (metallic)		2.4 to 3.2×10^{-6}	none
Meteoric iron (oxidized)		13,000 to 15,000	

[a] Effective values, down to and including each formation listed.
[b] Undisturbed rocks, in place.
[c] Values vary widely with pressure, particle size, etc.

The preliminary survey consisted of: (1) studies at 12 stations all 1000 ft. from the center point, spaced at 30° intervals around the center of the crater, made along radial lines at stations R1, R2 . . . R12; (2) three additional radial lines in the crater, D1, D2 and R16, to give data as to character of fill material, topographic effects, etc. All other radial and chord lines are part of the detailed survey. Most of these are in the area selected as favorable from the results of the preliminary survey but some also are in other parts of the area and were utilized to obtain comparative data. All other chord lines are designated by the prefix C, as C1, C2, etc.

Plotting Results of Electrical Work

The data from these studies are plotted on logarithmic paper as curves showing the variation of potential for constant current input, with depth. In making final interpretations, detailed corrections were made for topography. Owing to the symmetrical conditions existing around the crater, these corrected curves were not more comparable than the original field curves. In order to simplify this paper, the original curves are presented and are used for final comparison and interpretation.

Potential Studies around the Drill-hole Casing

These measurements were undertaken to endeavor to check the results of the previous preliminary and detailed survey. For this purpose, current was applied through the drill stem casing as one electrode, and the other power electrode was placed at a distance of 7500 ft. diametrically across the crater. The potential field set up around the drill stem was studied by measuring potential drops along radial lines laid off from a center point on the line of the power electrodes and 400 ft. north of the drill stem. Readings were made along seven lines, 1500 ft. long, using a constant separation of 600 ft. between the pick-up electrodes.

Apparatus Used and General Theory

The work was conducted with the International Geophysics five-electrode, low-frequency alternating-current potential apparatus. Using a certain configuration of electrodes making contact with the surface of the earth, and a separation depending upon depth being worked to, it is possible, by passing a current of equivalent value into the ground through two sets of the electrodes, to measure the potentials across the remaining electrodes. From the data obtained in such a series of measurements, calculations can be made to give the effective conductivity of the subsurface material at various depths.

Analysis of the Electrical Data, Five-electrode System

Table 2 gives the probable sections as constructed from the electrical data illustrated in Fig. 3. To facilitate conclusions as to the condition of ground water occurrence, Table 3 was prepared, showing elevations of the stations 1000 ft. from the center, depth to water, and absolute elevations of water table with drill-stem collar as datum. The distribution and elevation of ground water is vital to future mining operations. The present two-compartment shaft was sunk to a depth of 650 ft., at a cost of over $250,000, and stopped at that point because over 1000 gal. per minute of water came in through the fractures or cracks in the supposedly solid rock.

TABLE 2.—*Geologic Sections Constructed from Electrical Data of Fig. 2*

Material	Depth Interval, Ft.	Thickness, Ft.
SECTION IN VALLEY ONE MILE WEST OF CRATER Water Level at 580 Ft. More Impervious Strata at 750 and 1000 Ft.		
Moencopie sandstone		20
	(Boundary indefinite)	
Kaibab limestone	20–280	260
Coconino sandstone	280–940	660
Supai formation	at 940	
SECTION AT CRATER CENTER. ELEVATION, −526. (VALUES AVERAGED) Water Level at 210 Ft.		
Lake beds	0 to 120	120
Fine fill material (sand and silt)	120 to 310	190
Coarse fill material (sand and large rock fragments)	310 to 850	540
SECTION APPROXIMATELY 500 FT. NORTH OF MAIN SHAFT. ELEVATION, −40 (ESTIMATED) Water Level at 650 Ft., or Elevation −690		
Debris	0 to 55	55
Moencopie sandstone	55 to 100	45
Kaibab limestone	100 to 320	220
Coconino sandstone	320 to 900	580
Supai formation	at 900	

The information contained in the tables and curves can be summarized as follows:

1. Geologic sections at representative points have been given.

2. The crater structure is shown to be a bowl-shaped depression, containing a central pluglike and very conductive fill, delineated into lake beds, fine fill and coarse fill. Surrounding this is a rim of dry, highly fractured and hence highly resistant rock (sandstone and limestone) covered on top by coarse and fine debris, and on the inside slopes by coarse talus debris. Still farther from the rim, the debris thins and the rock is less disturbed. Here, the rock conductivity approaches normal values. The lithologic units have been shown to have characteristic conductivity values. From these considerations, it will be seen that the radial lines to be discussed presently will show average values and a form of curve governed by the composite effect of the petrologic differences plus the effect of any anomalous structural condition not discussed (occurrence of meteoric material).

3. Correlation was made between water levels inside and outside of the crater. At the crater center, the water level is shown to be lower than on the rim or at the point near the shaft. The level of the water under

TABLE 3.—*Water-level Data*

Average Elevation of Water under Crater Rim Is −690 Ft.

Station	Elevation	Depth to Water, Ft.	Elevation of Water, Ft.
R1	−470	240	710
R15	−490		
R2	−510	190	700
R3	−470	250	720
R4	−500	200	700
R5	−490	200	690
R6	−470		
R7	−460	250	710
R8	−500	240	740
R9	−500	190	690
R10	−470	190	660
R11	−430	190	620
R13	−435	230	665
R12	−440	200	640
R14	−450	250	700
D1D2[a]	−526	210	736
Valley[a]	−140 to −150[b]	580	720 to 730
		650	690
C10	−40		

[a] Elevations of water at crater center and at station in valley outside crater.
[b] Estimated.

the rim is 10 to 20 ft. higher than in the valley. The values given by Table 3 can be considered only approximate in each case, but a general average of the values shows a definite higher level under the rim. It thus appears that the water level is bulged slightly under the rim following topographic effects, but that there is close correlation between the water level in the center of the crater and the point in the valley to the west of the crater.

Brief Description of Potential Curves Showing Geological Conditions (Fig. 3)

Curves 1*A and* 1*B—Section in Valley* 1 *Mile West of Crater.*—The curves as a whole show a rather uniform variation of potential. On curve 1*A* the points of inflection due to structural change are somewhat masked by the influence of surface water from recent rainfalls. The depths to the various structural changes are indicated on the curves and in Table 2. Due to the low resistance of the material near the surface and the probable shallow extent of Moencopie sandstone at this point, the boundary between it and the Kaibab limestone does not show up. To show this would have required detailed measurements at the shallow depths. The work described in this paper was conducted during the fall

rainy season. Showers to heavy rains were encountered almost daily while the work was in progress. The effects of the rains in masking the deeper effects may be seen by comparison of the two curves on Fig. 3. Curve $1B$ was obtained during a preliminary survey in June, preceding the rainy season.

Station D1, in Center of Crater.—The absolute values shown by this curve are the lowest found in any part of the crater area. The large drop from the surface to a depth of 130 ft. delineates the very conductive lake beds. The other structural changes are indicated. The depth to water is also indicated at approximately 200 ft. below the surface.

Station D2 in Center of Crater.—This curve is very similar to $D1$. The sudden breaks at the ends of the two curves are topographic and contact effects. Average values from curves $D1$ and $D2$ were used to construct Table 2.

Curve C10.—This curve illustrates the structure of the rim at a point approximately halfway between the main shaft and the edge of the rim. The various structural changes are indicated and tabulated. Comparison of this curve to that of Curve IA and IB, which show the structure in the valley to the west of the crater, proves very interesting. It shows that curve $C10$, from a depth of 300 to 1800 ft., coincides almost exactly both in form and absolute values with the corresponding section of the valley curve. This shows that between those depth limits the structure at the two points is almost identical. However, comparison of the two curves shows that the Coconino is thicker in the valley and that correspondingly the depth to the Coconino Supai contact is shallower near the shaft. This may be only apparent and due to an indefinite boundary horizon between them at one point or the other. The high values occurring on curve $C10$ for the shallower depths are due to the presence of debris and highly fractured rock near the surface.

Radial Lines, Stations Placed 1000 Ft. from Crater Center

In the preliminary survey, studies were made at 12 stations each 1000 ft. from the center of the crater. The curves from these 12 stations were practically identical, with the exception of curves $R1, R11, R12$. To facilitate comparison of the various radial curves within the crater, a characteristic curve was constructed. The shallower section has been omitted, since variations in the surface material are unimportant for the present consideration. The curve was constructed for the portion of the crater that was indicated to be barren of material of better conducting power. The curve is an average curve, inasmuch as points on it represent both average depths to corresponding points of inflection and average absolute values at those points. The curve represents the average condition over approximately three-fourths of the crater rim ($R10$ to $R15$ clockwise). As has been mentioned previously, the form of the radial curves and the

values on them are governed by the composite effect of several different lithologic units and the structure and configuration of the crater. The rapid variation in the middle portion of the curve is a characteristic function of topography and the location and angle of the boundaries of the rim rocks and the central conductive plug of fill material. In the following general descriptions, each curve will be compared to this characteristic curve. Due consideration is given both to the form of the curve and the actual values of potential. The characteristic curve is shown in Fig. 4, together with the following curves which differ from it.

*Curve R*1.—Curve *R*1 is similar to the characteristic curve. The main points of difference are as follows:

1. The maximum value reached on the first peak near 400 ft. is considerably higher than in the characteristic curve. This is due to the very steep slope of the rim at this point.

2. The drop in potential at 467 ft. is abrupt and the minimum value reached at 500 ft. is considerably lower than on the characteristic curve. This feature is due to the effect of the vertical cliffs at this point and in part also to the effect of the drill stem in concentrating the current and sending it to somewhat greater depths than would ordinarily be the case, thus lowering the potential drop or increasing the apparent conductivity. The average slopes of corresponding portions of the two curves are very similar.

*Curve R*15.—This is very similar to the characteristic curve. Essential points of difference are the same as for *R*1.

*Curve R*11.—This differs from the characteristic curve in the following particulars: (1) the potential value for the shallower depths and to a depth of 350 ft. are higher, because of the greater extent of the coarse boulder talus at this point; (2) the slopes of various portions of the curves are closely similar. At a depth of approximately 1000 ft., curve *R*11 shows a tendency to hold down in value, indicating the inclusion of better conducting material beyond that depth.

*Curve R*13.—This is markedly different from the characteristic curve in the following respects: (1) The potential values to 350 ft. are considerably higher than is characteristic, because not only is the talus more extensive at this point, introducing more resistant material, but there is also considerable coarse landslide debris; (2) between depths of 200 and 450 ft. the curve holds down and approaches the characteristic curve, and follows it until a depth of about 900 ft. is reached; (3) from a depth of 900 ft. to a depth of 1800 ft., the curve again holds down, showing pronounced low values at those depths, and indicates the presence of much more conductive material below 900 feet.

*Curve R*12.—This curve is similar to *R*13 but shows even more strongly the effects of a deep-lying conductive zone. As was true of *R*13, the absolute values on the curve between the depths of 150 and 400

ft. are higher than characteristic, because of the inclusion of a greater amount of more resistant talus. At a depth of about 800 ft., the slope of *R*12 departs from that of the characteristic curve. The decreased slope and low values of potential (about 50 per cent of normal) indicate the presence of deep-lying conductive material.

*Curve R*14.—Curve *R*14 is similar to curves *R*12 and *R*13, and quite different in several respects from the characteristic. To a depth of 400 ft., the inclusion of talus and a structural effect result in abnormally high values. Beyond a depth of 800 ft., the curve holds down again, indicating the deeper conducting material shown by *R*12 and *R*13; the effects, however, being less pronounced.

Summary.—The discussion of the radial lines 1000 ft. from the crater center has shown:

1. A well defined conductive zone occurring in the crater, in the sector bounded by radial lines *R*14 and *R*11.
2. This zone consists of two portions; *i. e.*, a comparatively shallow zone extending practically to the surface and a deeper-lying, much more pronounced zone, occurring at a depth of 750 ft. near *R*12.
3. The first zone is shown by all four curves; the second, to some extent, by *R*13 and *R*14, but especially by *R*12.
4. No such zone exists in any other part of the crater.
5. From an electrical viewpoint, the deeper lying zone would be considered as a moderate or weak effect.

Chord Lines 1000 Ft. from Crater Center

*Stations C*1 *to C*6.—The curves for these chord lines are similar. From depths of 800 to 1800 ft. the curves are practically identical in form but different somewhat in absolute values. The shallower portions of stations *C*2, *C*3, *C*4 and *C*5 are all similar in form and show a characteristic rise in potential values from the surface to an average depth of 700 ft. *C*1 and *C*6 are very similar and different from the others, in that to a depth of 700 to 800 ft. the absolute values are much higher and the curves show a tendency to hold down, indicating the inclusion of more conductive material near the surface and extending to an average depth of 700 to 800 feet.

Summary.—Each chord section has a corresponding radial section. Comparison of these shows that for the shallow depths a good check obtains between them. *C*1 and *C*6 indicate the shallow conducting zone shown by corresponding radial lines. However, these chord curves do not indicate sufficiently the existence of the deeper zone to be classed as even a weak effect.

Chord Lines, 1500 Ft. from Crater Center

*Stations C*8, *C*12, *C*14.—The curve for station *C*14 is plotted from data of a section taken on the north rim in the location shown, and is regarded

as a characteristic section, in a barren portion of the rim. The range in potential values as shown by the curve is very large. The initial values are low, due to the fact that the talus at this point is not as coarse as elsewhere and has intermixed a large proportion of soil. The maximum value of 400,000 denotes the inclusion of the very resistant fractured rim rocks.

The curves for the other stations are similar to the curve for *C*14, with but two exceptions: (1) they all exhibit much higher values from the surface to an average depth of 500 ft., due to inclusion of much more coarse talus and landslide debris, and (2) curve *C*8 shows an apparent holding down beyond a depth of 1000 ft. This may possibly be due to the effect of the conductive zone shown by the radial lines, but probably is merely apparent and due to the fact that the values in the middle portion of the curve are abnormally high. That the drop is only apparent is also indicated by the fact that this curve, like all the others, tends to approach a constant end value of potential at a depth of 1800 feet.

Summary.—These curves show similar conditions in corresponding points on the north and south rim, and indicate no anomalous conducting zone extending in a direction such as to be shown by these chord lines.

Chord Lines on Rim 2000 Ft. from Crater Center

*Stations C*9 *and C*11.—The curves of these two stations are practically identical (after correcting for contact effects) from a depth of 250 ft. to a depth of 1800 ft., and indicate definitely the same structural conditions at the two corresponding points on the north and south rims.

Summary.—No anomalous conductive zone indicated.

Radial Lines on Rim 2000 Ft. from Crater Center

*Stations R*17, *R*18, *R*19, *R*20.—Comparison of the curves for stations *R*17, *R*19 and *R*20 to that obtained at *R*18 shows that all of the curves are similar. The curves on the south rim show higher initial values due to more highly resistant sandy debris at the surface. From a depth of approximately 350 to 1500 ft., the curves on the south rim appear to hold down somewhat, due to structural effects. However, the general range of values is the same for all. It is significant that all of the curves reach the same end point value of potential, indicating the inclusion of the same material for each.

Summary.—These curves indicate more conductive material at shallow depths on the south rim of the crater. This effect is a combination of (1) more talus here which is more conductive than the rim rocks, (2) greater depth of fill material adjacent to the rim, and (3) structural or conductive effects. The deeper conductive zone shown by the radial curves of the 1000-ft. stations does not show up, the material included beyond 1200 ft. being essentially the same on both sides of the crater, as shown by the

approach of all of the curves to the same end point potential value. The presence of a deeper conductive zone beneath the rim is not indicated definitely, and neither do the curves show the conductive zone indicated by the 1000-ft. radial stations.

Possible Interference from Pipe Lines

Careful study of the locations of the water and oil pipes on the property indicated a slight possibility that the water pipes from the main storage tank on the southern rim of the crater may have influenced the results. Accordingly, these pipes were disconnected from the tank, and also cut in a number of places, and the pipe ends insulated with dry, wood supports. Potential line No. 12 was repeated, with very excellent agreement between values. It was concluded, therefore, that the decided drop obtained on the potential lines in the southwest quadrant of the crater was not affected or caused by artificial conductors.

Electrical Data

Drill-stem Potential Distribution

This work does not represent a complete study, as it was carried on hurriedly during the last day of the survey. The basis for the work was the possibility that the drill casing may give direct connection to the meteoric mass, whereby a study of the current distribution would give data as to the existence and location of the meteoric material.

In the early exploration, the drill hole (marked Churn-drill Hole on Fig. 2) had been sunk to a depth of approximately 1300 ft., where it had encountered a very hard substance. In attempting to drill through this material, the tools had become lodged in the hole, and finally were lost after several prolonged fishing operations. The hole was dynamited in an attempt to free the tools and the casing broken or destroyed at such places. The distribution of current and its effect upon potential distribution could not be predicted. Because of the many unknown factors involved in this work and its generally incomplete status, it does not provide data for definite conclusions.

Conclusions from Electrical Investigations

The electrical studies gave definite, although weak, indications of a better conducting zone in the sector bounded by stations $R11$ to $R14$. In an attempt to verify these effects by supplementary studies, no definite results were obtained. This, however, is not considered as particularly detracting from the radial line results, because the supplementary work was so incomplete and hurried.

Magnetic Investigations

Fig. 5 shows the area covered during the magnetic survey. The work was carried to a distance of 5000 to 6000 ft. from the center of the crater. Sufficient readings were made at greater distances to ascertain the general trend of the regional gradient.

Askania magnetometers of the Schmidt field balance, vertical type, were used during this investigation.[15] Upon completion of the work, a calibration was made of the auxiliary magnets by direct comparison with the standard magnets at the magnetic station of the U. S. Coast and Geodetic Survey station at Tucson, Ariz. This work was done through the courtesy and cooperation of Dr. Albert K. Ludy, in charge of the Tucson station. Scale value was checked by the auxiliary magnets.

Theory

The earth's normal field is uniform over areas of homogeneous structure. If difference exists in the subsurface structure, the normal or uniform field is distorted by an amount which depends, among other things, upon a factor called the magnetic susceptibility of the materials comprising the subsurface, and the relative masses, configuration and arrangement of those various component materials. A study of the earth's magnetic field at the surface, therefore, furnishes a means of predicting the subsurface structure and conditions.[16]

Owing to the relatively simple geology of this area, and the great difference between the susceptibilities of the prevailing sedimentaries and unoxidized meteoric material, a study of the magnetic anomalies should indicate the presence of such meteoric materials, if in sufficient quantity and occurring within a depth of 700 to 800 ft., as indicated in the electrical work.

In the magnetic survey, two factors were closely watched: (1) anomalies that would indicate the presence of meteoric materials and (2) anomalies that would be caused by an igneous intrusion or any change in the basement complex. Data on the second factor were desired in connection with the possibility that a steam explosion caused Meteor Crater. Should such a steam explosion have taken place, its origin or source probably would be indicated by magnetic anomalies associated with deeper structural effects.

External Field of Meteoric Mass

There appeared to be an opinion among those associated with the exploration work at Meteor Crater that the meteoric material, in all

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[15] C. A. Heiland: Theory of Adolf Schmidt's Horizontal Field Balance. Geophysical Prospecting, A. I. M. E. (1929) 261–314.

[16] N. H. Stearn: A Background for the Application of Geomagnetics to Exploration. Geophysical Prospecting, A. I. M. E. (1929) 315–363.

probability, has residual magnetism, and that these particles were heterogeneously arranged at the time of impact. Therefore such heterogeneously arranged particles, possessing residual magnetism, are believed not to have an external field.

Even on the assumption that the particles are magnetized and heterogeneously arranged, it seems that such a mass will cause an anomaly in the normal magnetic field. This is due chiefly to the fact that such permanently magnetized particles are considerably below saturation, and therefore they will have a permeability, for a field as weak as the earth's, of practically the same value as unmagnetized particles. As a result, the mass, even though fragmental, will have a polarity due to induction by the earth's field.

Measurements

Corrections for diurnal variation were made by "checking in" at a base station three or four (usually the latter) times each day. For all the work on the surface, base station No. 00 was used (Figs. 2 and 5). This station was selected because of its general accessibility and apparent freedom from unnatural magnetic effects. During the work in the crater, station $R14$ was used as a base. These two base stations were "tied in" by making readings at each station on four different occasions, and correcting for difference in time and diurnal variation.

Field Procedure

Because of the symmetrical conditions prevailing at the crater, the magnetic measurements were all made along radial lines. The stations within the crater are usually at 400, 600, 800, 1000, 1250 and 1500 ft. from the center, along the radial lines. The stations outside the crater are at 300-ft. intervals. There being a large amount of tramp iron (old cables, casings, pipes, machinery, etc.) in the crater, a number of the stations were omitted or shifted to regions more free from such material. These readings, which were obviously influenced by tramp iron, were deleted from the final data, and not used in drawing the contours.

Masking Effect of the Crater Fill

The magnetic permeability of a material varies with the permeabilities of its constituents and the structure. Pulverizing and shattering of a material causes a decrease in permeability,[17] which is noticeable on all of the magnetic traverses. These show a drop for the readings taken over the debris at the rim. In addition to the drop caused by the debris around the crater, another change is caused by the steep topography.

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[17] L. B. Slichter: Certain Aspects of Magnetic Surveying. Geophysical Prospecting, A. I. M. E. (1929) 246.

Inasmuch as the crater is filled with broken and crushed material, to a depth of 850 ft. in center (Fig. 6), there is an appreciable drop in vertical intensity over the crater itself. This drop is noticeable in the northern part of the crater, while in the southern part there is an increased anomaly. A material of higher permeability is present, therefore, which is responsible for this increase over and above the decrease which normally would be effected by the shattered and fill zone. A comparison of the rate of change for the anomalies on the north and south sides and the decrease in permeability with pulverizing indicates that the maximum anomaly in the south portion of the crater may be 100 gammas, or more, assuming that the magnetic material lies at a depth of 700 ft., as indicated by the electrical work. Due to the unknown masking effect of the crater fill, no calculations are given herein for determining the size of the magnetic material from the observed anomalies.

Regional Gradient

Inspection of the magnetic data discloses an increase in magnetic strength in a general west to east direction. This change is typical of the regional gradient in this area and reflects the regional structural conditions. This change is not associated with any meteoric material which may be present in the crater. In order to show more clearly the anomalies existing in the crater, the regionl gradient is compensated for and anomalies (corrected readings) plotted in Fig. 5 showing change or deviations from the normal regional gradient.

Anomalies in the Crater

The magnetic map shows that the largest magnetic changes are in the southern portion of the crater. Many of the anomalies are of considerable magnitude as compared to the short distances between stations. This, together with the small horizontal extent of the changes, indicates that such changes are due to scattered magnetic fragments fairly close to the surface. The lake beds are approximately 110 to 120 ft. thick (Fig. 6) and the magnetic work indicates that magnetic material is scattered through the debris underlying the lake bed.

In addition to the smaller anomalies mentioned, there is a general high area in the southwestern quadrant of the crater, which is of considerable size in horizontal projection, and is believed to indicate a deeper lying fragmental mass. The magnetic low observed at places along the rim of the crater is caused by the combined effects of topography and the crushed and fill material.

Summary of Magnetic Data

1. The general location of the areas wherein magnetic highs were found agrees with the locations obtained in the electrical work, wherein a highly conductive zone was found.

2. The magnetic high is also in the same general area as the location of the meteoric zone obtained from the geologic data.

3. The magnetic work does not show any changes in the deeper structure or basement complex, such as would be expected if Meteor Crater were the result of a steam explosion.

4. The magnetic effect would be classed as weak.

ACKNOWLEDGMENTS

The writers desire to express their indebtedness to the officials of Meteor Crater, especially to Mr. G. M. Colvocoresses, General Manager, for cooperation and help during these investigations. In addition, particular thanks are due Messrs. Victor F. Hanson of International Geophysics staff and Walter Goeglein, George Harbauer and Frank M. Leonard, Jr., of the staff of Meteor Crater Exploration Co., for their helpful and thorough work during the field investigations.

DISCUSSION

(Donald H. McLaughlin presiding)

H. Lundberg, New York, N. Y.—We have had the opportunity to study the electrical results as presented by Mr. Jakosky, and we have arrived at the conclusion that the procedure adopted for his electrical explorations was not entirely suitable for this investigation.

The survey lines crossed a contact between the solid rock in place and loose fill, and the difference in the shape of the curves I would attribute to the different angle of the slope of the contact. In crossing such a contact, it seems utterly impossible to arrive at any conclusions as to conditions at greater depth; therefore the existence of a meteoric or highly conducting material, according to my opinion, has not been indicated by this electrical survey.

T. Zuschlag, New York, N. Y. (written discussion).—Mr. Jakosky and his coauthors do not explain their five-electrode system and method of surveying in detail. This deficiency of the paper is regrettable, particularly from a theoretical viewpoint, because it makes it difficult to check the geophysical interpretation. Yet no matter how the five-electrode system was used in this investigation, it seems to me that the final conclusions of the paper are not supported by the results of the investigation.

Referring to the radial survey lines, it is clear that in crossing the crater rim in any direction, topographic and lithological disturbances of considerable magnitude are encountered. The authors maintain that these disturbances are "characteristic" for the greater part of the crater rim except for the southwestern part of the crater, where they are modified by the effect of a buried meteoric mass. Considering the magnitude of the change, it is evident that the volume of the meteoric mass, particularly if buried 700 to 800 ft., should be rather large to justify these changes. Assuming that such a mass containing a large percentage of highly conductive iron really existed, it should be expected that this mass would produce considerable disturbances in the earth magnetic field. Yet the accompanying magnetic survey revealed only slight deviations of the vertical component of the earth magnetic field in the suspected part of the crater. This discrepancy between the electric and magnetic indications, therefore, is not in accord with the assumption of a buried magnetic mass, but it would be in accord with the assumption of a slight change in the dip of the buried

part of the crater rim. Assuming that the susceptibility of the crater fill is somewhat higher than that of the Coconino sandstone, a slight increase in dip would produce a slight increase of the magnetic readings within the crater while at the same time it would produce pronounced deviations from the "characteristic" potential drop curve. This statement does not exclude the possibility of the existence of a meteoric mass somewhere within or without the crater, but it definitely implies that such meteoric mass was not located by this particular geophysical survey.

J. J. Jakosky (written discussion).—Mr. Lundberg brings up a point which has had considerable consideration and study on our part, both before the survey was conducted and after the results were obtained and final interpretation attempted.

Considering the relative conductivities of the various materials in the crater area, and other theoretical factors, it is difficult to account for the greater drop and holding down of the curves, unless it be due to a more highly conductive material than that normally found in the area. Mr. Lundberg attributes this drop entirely to the attitude of the bedrock. The attitude or angle of contact of the bedrock and crater fill has an effect, of course, but we believe that the major portion of the drop is caused by the deeper fill material in this portion of the crater. Even allowing for this, however, our interpretation indicated the presence of a still better conducting material.

The magnetic anomalies in the southwest quadrant of the crater also indicate the presence of meteoric material. The crater fill has a masking effect on this anomaly; the magnitude of which we hesitate to predict. From one viewpoint (considering the relatively low susceptibility of the original sandstones and limestones) this masking effect may be unimportant.

We expect subsequent drilling operations to show the presence of fragmental and scattered meteoric material in the southwest quadrant of the crater.

In reply to the discussion by Mr. Zuschlag, we are unable at this time to give the technical details of the five-electrode low-frequency method developed by this company. We are planning, however, on making that method and comparative field data the basis of a technical paper to be published as soon as it is released by our patent attorneys.

Mr. Zuschlag evidently infers that all of the drop is due to the fragmental meteoric material in the southwest quadrant of the crater. This assumption is incorrect, as may be seen in the original paper, or in our reply to Mr. Lundberg's criticism.

Mr. Zuschlag's assumption that the susceptibility of the crater fill is somewhat higher than that of the Coconino sandstone is entirely erroneous. As stated in the paper, the crater fill has a susceptibility less than that of any of the surrounding country rocks. Examination of the meteoric anomalies should show Mr. Zuschlag that these anomalies could not be caused by any "slight change in the dip of the buried part of the crater rim." The anomaly occurs too much in the center of the crater and does not extend to or beyond the rim. Furthermore, it should be noticed that inasmuch as our work indicates that only a small part of the drop in the southwest quadrant of the crater is caused by the fragmental meteoric material, there is no "discrepancy between the electrical and magnetic indications." We believe that the magnetic anomalies are well in accord with the amount of fragmental meteoric material which we believe to exist in the crater.

Mr. Zuschlag's final conclusion, to the effect that "On the other hand, this statement does not exclude the possibility of the existence of a meteoric mass somewhere within or without the crater, yet it definitely implies that such meteoric mass was not located by this particular geophysical survey," removes any force which his criticism may have had, because of the obvious stand which he can take after the work is proved up by drilling operations.

Electrical Exploration Applied to Geological Problems in Civil Engineering

By E. G. Leonardon,* New York, N. Y.

(New York Meeting, February, 1931)

The object of this paper is to describe briefly the practical results obtained in several problems of civil engineering by resistivity measurements of the underground. It is intended for the mining engineer, as well as for the construction engineer; the stratigraphical problems discussed are general and frequently are encountered in mining exploration.

Resistivity measurements have been used in mining and oil exploration for a number of years.[1] Their introduction in the field of civil engineering, on the contrary, is rather recent. It was applied for the first time in the spring of 1928 near Littleton, N. H., in the survey of two dam sites for the New England Power Assn.[2] Since then, the geophysical firm with which the author is connected has had an opportunity to develop this field of activity, and more than 20 investigations at dam and tunnel sites have been carried out in the United States and Canada. A part of this work has been verified by underground or drilling exploration. This paper deals with but a few of the results, which we are permitted to make public now.

The theoretical and technical sides of the processes employed will not be discussed; they have already been described in various technical publications. A few general points will be mentioned, to make clear the practical examples given.

Resistivities of the Rocks

The surveys are based on the measurement of the specific resistance of the rocks. This resistivity is the electrical resistance of a cylinder of

* Schlumberger Electrical Prospecting Methods.

[1] The earliest studies of Prof. C. Schlumberger on the resistivities of the underground were made in 1913. Numerous commercial, stratigraphical surveys, some of them covering large areas, were carried out by the method of "the resistivity map" in 1920 and following years. Some of these studies have been described: La Prospection Électrique par les Procédés Schlumberger, Paris, 1927. E. G. Leonardon and S. F. Kelly: Some Application of the Potential Methods to Structural Studies. Geophysical Prospecting, A. I. M. E. (1929) 180.

[2] Irving B. Crosby, consulting geologist for the New England Power Assn., was, to the best of our knowledge, the first to visualize the interest of electrical exploration in the study of underground conditions at proposed dam sites, and should receive credit for the discovery of this new field of application.

rock, having as height the unit of length and as section the unit of surface. We have adopted the ohm per meter-meter square as a practical unit, because this unit of measurement gives convenient figures, usually varying from one ohm to a few thousand ohms.[3]

The electrical conductivity of the sterile rocks (that is, of all the rocks, with the exception of a few metallic sulfides which possess a metallic conductivity) is purely electrolytic and disappears when they are entirely dry. From this, it results that the resistivity of a rock is inversely proportional to the amount of water it contains, and roughly inversely proportional to the quantity of ionized salts dissolved in this water.

If a rock is very compact it is evident that the amount of moisture it contains is exceedingly small, and that its specific resistance, therefore, is considerable.[4] This is true of rocks like granite, gneiss, marble and, in general, of all metamorphosed or compact rocks, where only a limited number of open spaces or pores exist. On the other hand, rocks with a high water content, like clays, marls, soft limestones, shattered zones and wet faults, show a good electrolytic conductivity. These differences in the conductivities of the rocks are well characterized by some figures.

Clays and unconsolidated clayey formations often possess resistivities as low as 10 or 30 ohms, clayey-calcarous soft terrains show corresponding figures between 20 and 400 ohms; eruptive or metamorphic masses may have resistivities ranging from 200 to 2000 ohms and up, according to their lack of porosity. As to the sands, their resistivities will vary greatly according to their dryness and to the impurities contained therein (clay or organic matter). Pure siliceous sands may have great resistance, because the absorbed water contains a very small percentage of dissolved salts.

These data show the interest of resistivity measurements to civil engineering. They will not only permit a picture of the general underground conditions (thickness of an overburden, study of a contact between two rocks, location of a fault, etc.) but they will even make it possible to predict, with more or less certainty, the mechanical properties of the underlying formations. As pointed out above, there is often a definite relation between the compactness, the mechanical resistance of a rock and its electrical conductivity. It is true that the occurrence of pure sands, which are electrically very resistive and mechanically loose, will complicate the investigations, but even in this case the electrical study may present an interest, since it will permit the differentiation of

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[3] Some authors use the ohm per centimeter-centimeter square, which gives resistivity figures one hundred times as great.

[4] Of course, we assume here that the water of imbibition is ordinary fresh water. The statement is not true if the electrolyte has a high content of dissolved salts. Under such circumstances, even compact rocks may show low resistivities, as in numerous areas where the terrains contain much sodium chloride.

an impervious material like clays, boulder clays, from a pervious one, such as sand.

In determining the resistivities of the rocks, two different procedures can be employed. In the first, an invariable measuring arrangement is utilized. A layer of the soil of uniform thickness is investigated over a given area. This is called a "horizontal exploration." In the second, a series of electrical measurements is carried out at a single station, to determine the electrical properties at various depths. This operation determines the variation of the electrical parameter with reference to the depth of investigation and is called an "electrical vertical drilling." These two processes have different objects and will be discussed successively.[5]

Field Material

Whichever method of exploration is employed, the material and apparatus necessary to carry out the field measurements are exceedingly light, practical, and not at all cumbersome; they comprise a potentiometer, some electrical batteries and a few reels of insulated cable. Each piece can be handled conveniently by a single individual, even under the most trying conditions. This makes it especially valuable when the place of work is difficult of access, as is often the case in a preliminary survey, upon the results of which depends the further development of the project.

Horizontal Exploration

In horizontal exploration, the measuring arrangement (in other words, the length of the line AB into which the current is sent) possesses an invariable dimension. The depth of investigation, therefore, is uniform, and a certain thickness of the soil, always the same, is methodically explored by the electrical measurements. These measurements can be summarized or represented, according to circumstances, in the form of profiles or maps of resistivity. These are comparable to the geological cross-sections and maps prepared by the geologist, except that they are not concerned with purely superficial observations, but with the measurement of a parameter which takes into consideration a certain thickness of the ground. Also, another difference is that the underlying terrains are no longer differentiated by their lithological characters but by their electrical parameters.

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[5] Regarding the technique of the resistivity measurements, their significance and possibilities, as well as their practical application, refer particularly to C. and M. Schlumberger: Depth of Investigation Attainable by Potential Methods of Electrical Exploration and Electrical Studies of the Earth's Crust at Great Depths (this volume, pp. 127 and 134); Memoire sur la méthode de la carte des résistivités et ses applications pratiques. Congrès International des Mines, de la Métallurgie et de la Géologie Appliquée, Liége, June, 1930.

The similarity to the geological map often will be striking in shallow-depth investigation. As a matter of fact, it would be an error to imagine that only the observations at great depths are valuable in the investigation of underground conditions. Often the sound rocks, although practically outcropping, cannot be observed by the geologist because of a thin layer of soil, and a superficial and rapid electrical survey would quickly give evidence of them. The following instance illustrates such a problem at shallow depth.

Exploration at Bridge River Tunnel[6]

In the summer of 1928, the British Columbia Electric Railway Co., Ltd., was driving the Bridge River tunnel at Bridge River station along

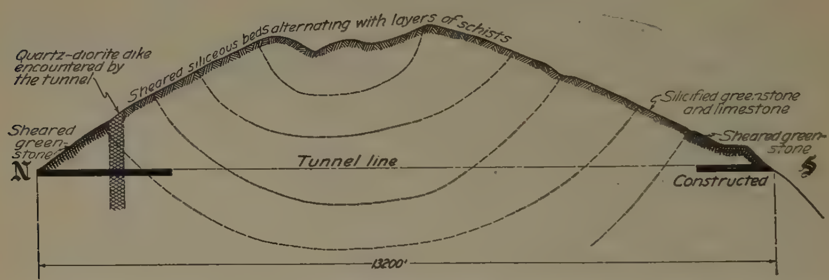

Fig. 1.—Cross-section of rocks across Bridge River tunnel line. (Courtesy of V. Dolmage.)

the line of the Pacific Great Eastern Railway, which runs from Squamish to Lilloet, B. C., for the purpose of bringing water to its proposed new Bridge River power plant. The tunnel is 13,200 ft. long. It had been commenced at both ends, and at that time was approximately in the state shown on Fig. 1.

According to Dr. V. Dolmage, formerly of the Dominion Geological Survey, the underground stratigraphy can be described briefly as follows:[7] At the northern end of the tunnel the rocks dip at an angle of 45° towards the south, while at the southern end the conditions are just the reverse (dip of the rock about 45° towards the north). The same rocks outcrop on either slope of the mountain, but in opposite order, thus showing plainly that they constitute the two limbs of a large syncline.

[6] This section of the paper, for the most part, was contributed by E. E. Carpenter, consulting engineer for The British Columbia Electric Railway Co., Ltd., Vancouver, B. C. The results of this work were briefly summarized by E. E. Carpenter and E. G. Leonardon: Geophysical Study Predicts Rock Conditions at Tunnel Site. *Eng. News-Rec.* (1930) **105**, 364.

[7] Fig. 1, which shows the cross-section of the rocks along the tunnel line, represents Dr. Dolmage's geological interpretation and is reproduced here through his courtesy.

At both ends, the tunneling work was begun in the sheared greenstone. This rock is largely constituted of altered and serpentinized andesite; it contains numerous fissures but does not contain a great amount of water, and is quite firm. No especial difficulty was encountered at the northern end, and the work proceeded satisfactorily. At the southern end, however, a decided change appeared in the constitution of the rock, at a distance of 1020 ft. from the entrance. The rock became much softer and more fractured, and at the same time some small water occurrences were encountered. At 1220 ft. from the entrance, a strong flow of water was discovered, which invaded the tunnel under heavy pressure, crushing the heavy timbering and causing a complete collapse over a length of more than 200 ft. As a result, the end of the clogged

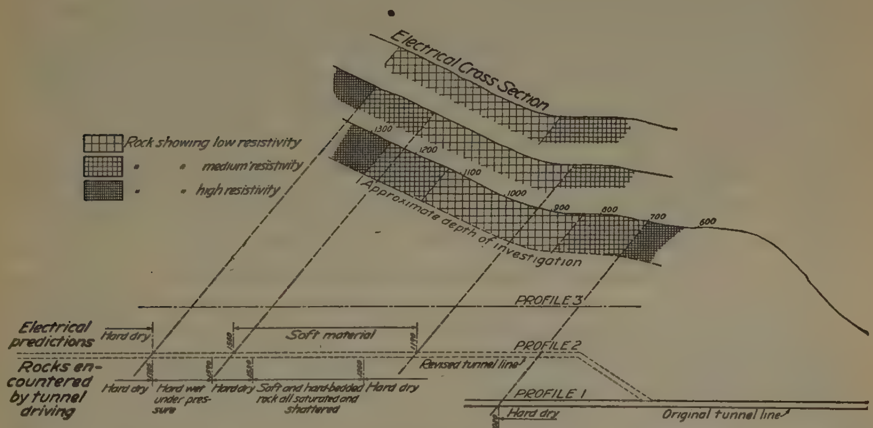

Fig. 2.—Electrical resistivities and record of rocks actually encountered in tunnel driving.

drift had to be abandoned. After some delay, it was decided to resume the tunneling work along a new line, about 100 ft. east of the original tunnel axis (Fig. 2).

At this time the British Columbia Electric Railway Co., Ltd., decided to use electrical prospecting. The purpose was to make a survey of the rock occurrences in the area of the southern end of the tunnel, and to determine beforehand the underground conditions that would be encountered by the new tunnel line. To achieve this, the resistivities of the rocks were studied at the surface, on the slope of the mountain, by means of three resistivity profiles traced along profiles 1, 2 and 3 (Fig. 2). Since the overburden was very thin, the depth of investigation adopted was shallow—about 80 ft. Cross-sections 1, 2 and 3 demonstrate the resistivities obtained. The lighter shade represents the resistivities of the rocks lower than 400 ohms per meter-meter square; the medium one indicates

resistivities between 400 and 700 ohms, and the darker one corresponds to resistivities of 700 ohms and up. With these profiles at hand, and taking into consideration the average dip of the rocks, as observed in the tunnel, it was possible to forecast the conditions that would be encountered along line 2, on which the tunneling work was being resumed. These conditions were presented as follows in the geophysical report submitted to the British Columbia Electric Railway Co., Ltd., in September, 1928:

"For a length of 370 ft. (approximately from point 1190 to point 1560, the drift will cross a bed of particularly soft material. A bed of harder rock will then be encountered from point 1560 to point 1730, but it is not until point 1730, that conditions similar to those encountered before point 1020 can be expected."

Fig. 2 shows the constitution of the rocks that were actually encountered later on by the tunneling work. The discrepancies between the predictions of the electrical survey and the results of the underground exploration are small. Undoubtedly they can be ascribed to the fact that the dip of the rocks is not absolutely constant and the beds are not entirely parallel. The correspondence between the electrical resistivities and the mechanical properties of the rocks is remarkable.

Electrical Vertical Drilling

In "electrical vertical drilling," the technique of the measurements is entirely different. It consists in carrying out at the same station a series of readings with different lengths of lines AB. Thus, the electrical resistivity is studied at different depths along the vertical line passing through the point of station. With this information at hand, the geophysicist will endeavor to predict the underground conditions occurring under the point of station. It must be stressed immediately that the problem thus outlined does not always lend itself to a satisfactory solution. In general, numerous heterogeneities occur at the same time and affect the surface measurements. These heterogeneities are located either near the surface or at depth and are entirely unknown. They may be very irregular and very complicated. Since the geophysicist has no way of differentiating them and studying them separately, a prediction of the underground conditions is often impossible.

Fortunately, there are some simple cases where the electrical vertical drilling can be readily interpreted, among them being one in which there are several homogeneous isotropic, horizontal, parallel layers possessing distinct electrical conductivities. This case can be treated mathematically and is often encountered in geology in its more or less ideal form. In this instance, there are no perturbations in the surface measurements except those caused by the different layers. In applied geology

the solution of this problem is of considerable moment, since it makes possible the underground investigation of horizontal formations.[8]

This electrical study of horizontal layers also finds its application in civil engineering. Very often, for instance, the rocks outcropping at a proposed dam site are made up of unconsolidated formations, upon which it is not safe to erect a dam, and it becomes necessary to ascertain at what depth a solid foundation exists. A similar problem is encountered in tunneling work. If the bedrock does not outcrop all the way along the proposed tunnel line, an exploration is required in order to determine whether there is a safe cover of solid rock above the projected work. If both the surface and the bedrock topography are not exceedingly rough and remain somewhat parallel, the electrical exploration furnishes an expeditious way of computing the thickness of the overburden at numerous places, and of establishing quickly a topographic map of the bedrock. If the overburden is made up of several layers electrically differentiated, often it will even be possible to give an idea of their respective thicknesses.

It is evident that, if the problem is to be solved, the different formations to be distinguished must not possess the same electrical conductivity. It is also necessary that the theoretical conditions of the investigation of homogeneous, horizontal, parallel layers be approximately realized. If the surface and the bedrock topography are decidedly irregular, or to a too great extent not parallel, if the layers are very lenticular, or lack uniform electrical conductivities, etc., the interpretation may become difficult or unreliable. These facts must be borne in mind when discussing the accuracy of the process. It is impossible to give definite figures that will be correct for all practical cases. The accuracy is entirely dependent upon the degree in which the ideal conditions pertaining to the problem of horizontal layers are realized.

In practice, an approximate idea of the correctness of the results is easily obtained after a few depth determinations on the field. The resistivity curves corresponding to the measurements are computed and compared to the theoretical diagrams. From their form, and from the order of magnitude of the discrepancies observed in the two kinds of curves, it becomes possible to see to what extent the problem is amenable to electrical prospecting. The experience proves that the compact rocks,

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[8] For technical information regarding investigation of horizontal layers see:

I. B. Crosby and E. G. Leonardon: Electrical Prospecting Applied to Foundation Problems. Geophysical Prospecting, A. I. M. E. (1929) 199.

J. N. Hummel: Der scheinbare spezifische Wiederstand. *Ztsch. f. Geophysik* **5**, 89; Der spezifische Wiederstand bei vier plan-parallel Schichten. *Idem.*, 228.

S. Stefanesco and C. and M. Schlumberger: Sur la distribution électrique potentielle autour d'une prise de terre ponctuelle dans un terrain homogene à couches horizontales homogénes et isotropes. *Le Journal de Physique et le Radium* (1930) **1**, 132.

which constitute the bedrock, are generally homogeneous over large areas. This can also be said, although to a lesser degree, of the soft formations which constitute the overburden. It is, then, rather unusual that a dam-site problem will not lend itself to the electrical process for solution. In the study of more than 20 dam sites, we have encountered but one case in which the electrical measurements were entirely uninterpretable.

Exploration on the St. Lawrence River, near Morrisburg, Ontario, Canada

In the spring of 1929, the Department of Railways and Canals of Canada was studying the location of a dam on the St. Lawrence River, in the region of Morrisburg, Ontario. The preliminary study of the project was to be accomplished with all possible speed, so as to take full advantage of the summer season. Therefore an electrical survey was quite justified in order to obtain rapidly and broadly an outline of the bedrock conditions and thus to orient the subsequent drilling exploration. A large area at Morrisburg and on Ogden Island (New York State) was systematically covered with electrical measurements and resulted in the establishment of a topographic map[9] of the bedrock surface (Fig. 3).

The geology of the area under consideration has been described.[10] The bedrock in this portion of the St. Lawrence valley is made up largely of nearly flat-lying beds of more or less pure limestones, magnesian limestones, dolomites and shaly limestones, with a small amount of shales. At Morrisburg the bedrock is composed of dolomites and magnesian limestones with a few narrow bands of dark gray shale, and thin films and layers of shale scattered through two of the dolomite beds. The overburden is composed principally of boulder clay. Under such circumstances, it was to be expected that the electrical conductivities of the two formations (bedrock and overburden) would be quite different, the bedrock showing a much higher resistance than the overburden. This is what actually occurred on the field. The resistivity of the overburden was uniform and in the order of 150 to 200 ohms, while the bedrock showed everywhere a resistivity 8 to 10 times as large. Fig. 4 shows the resistivity diagram obtained at station 18. Together with the experimental resistivity curve is shown the theoretical diagram corresponding to ratio of 10 between the resistivity of the bedrock and that of the overburden, and a depth to bedrock of 75 ft. (23 m.).

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[9] Other areas were studied electrically on the Canadian shore at Cardinal, Cornwall and Williamsburg, but much less systematically and less extensively. The results of this scattered work are not discussed here.

[10] J. Keele and L. H. Cole: Report on Structural Materials Along the St. Lawrence River, between Prescott, Ont. and Lachine, Que. Canadian Dept. of Mines (1922).

No less than 101 electrical determinations, which are shown on Fig. 3, were made. All these determinations, as well as the results of some drill holes put down in the neighborhood and in the river, made it possible to

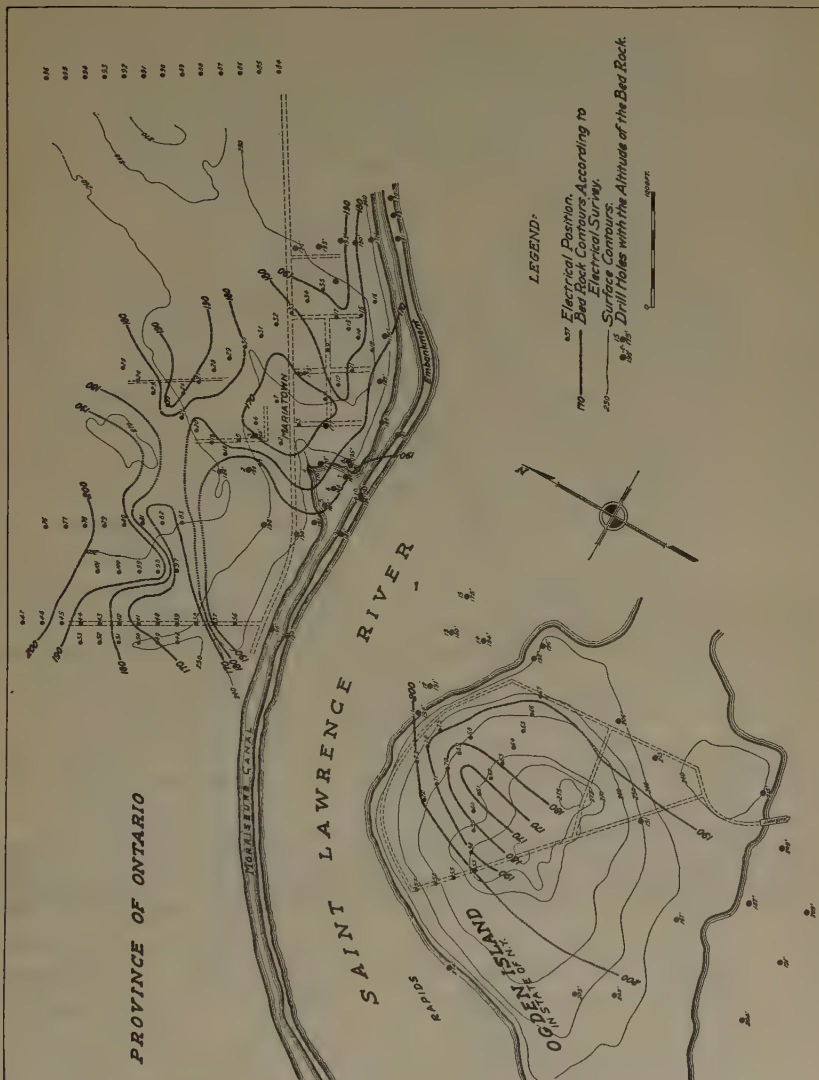

FIG. 3.—TOPOGRAPHIC MAP OF BEDROCK SURFACE AT MORRISBURG AND OGDEN ISLAND.

establish a tentative representation of the bedrock surface by means of contour lines at 10-ft. intervals, covering an area of more than 100 acres. After the completion of the electrical survey, 15 new holes were drilled. They are numbered 1 to 15 on Fig. 3. Some of them are outside of the

area covered by electrical measurements and therefore cannot be utilized to analyze the precision of the results, but drillings 1, 2, 3 and 11 are within the area of the electrical determinations and enabled the preparation of Table 1.

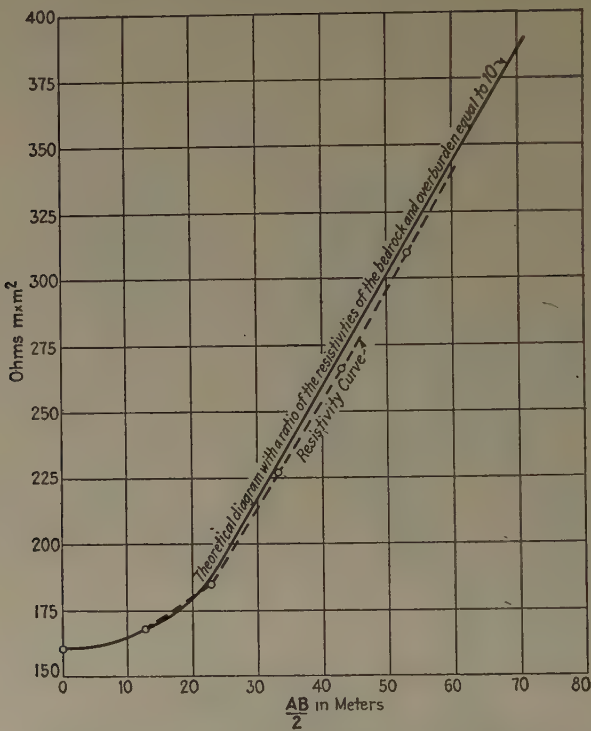

FIG. 4.—RESISTIVITY DIAGRAM AT STATION 18, MORRISBURG SURVEY.

TABLE 1.—*Comparison of Electrical and Drilling Records*

Hole No.	Altitude of Rock Obtained by Drilling, Ft.	Altitude of Rock According to Electrical Contour Lines, Ft.	True Depth to Rock, Ft.	Error of Electrical Prediction, Per Cent
1	186	192	74	−8
2	199	204	54	−9
3	169	170	80	−2.5
11	194	197	44	−6.8

The largest error is 9 per cent. at hole 2 and the average error is 6.6 per cent. This result is remarkable, and even more satisfactory than could be expected, if we consider the fact that the surface of the bedrock is not at all regular. This point is especially demonstrated by the results

of some drill holes. For instance, drill holes 8 and 8^1 are at a distance of only a few feet,[11] yet they give 57 and 43 ft. respectively for the depth to bedrock. The discrepancy between the two figures amounts to 37 per cent. A similar fact occurs at the twin drillings 10 and 10^1, where the depths are 52 and 45 ft., a discrepancy of 15.4 per cent. Under such circumstances, the electrical measurements evidently will furnish an average value of the depth to bedrock in the neighborhood of the station. These data will not necessarily check very closely with the figures obtained by a single drilling, but the general picture of the bedrock obtained by the electrical survey will represent quite truly the underground conditions.

Survey on the Lièvre River, near Masson, Quebec, Canada[12]

During the latter part of 1929, a survey was made for the James Maclaren Co., near Masson (Quebec), on the Lièvre River, a tributary of the Ottawa River. The site of the survey is about two miles from the junction of these two rivers, about 15 miles downstream from Ottawa.

At the time the electrical survey was commenced, the position of the dam site had been chosen (*A*, Fig. 5) and had been studied. It was the intention of the company to direct the waters of the river by means of a tunnel about 5000 ft. long to a power house, which was to be erected at *B*. The purpose of the electrical exploration was to make a quick survey of all the territory in the path of the proposed tunnel line in an endeavor to ascertain whether the tunnel would be safely located in the solid bedrock. Upon the results of the survey would depend the definite project of the dam site at Masson, as well as the final location of the tunnel line.

The area under consideration is underlain by a compact bedrock made up of pre-Cambrian and Paleozoic formations; there are also some igneous intrusions noticeable at various places.[13] The overburden, on the other hand, is composed of Quaternary sediments comprising a series of glacial boulders, sands and clays, overlain by the fresh-water sands, clays and gravels of the Champlain formation. These formations are unconsolidated and quite conductive. This was, then, a favorable problem for electrical exploration.

The work was carried out by means of electrical drillings, regularly disposed about 200 ft. apart. There are 63 stations in all. They are

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[11] Holes 8^1 and 10^1 were drilled before the electrical survey; holes 8 and 10 were made after the survey.

[12] This survey was performed under the supervision and the technical cooperation of Hardy S. Ferguson & Co., consulting engineers, New York City. We wish to acknowledge here the important role which they played in the execution of the work on the field and in the preparation of this part of the paper.

[13] See the geological map of Hull and Labelle countrics, Quebec, Canadian Department of Mines, Geological Survey, 1920.

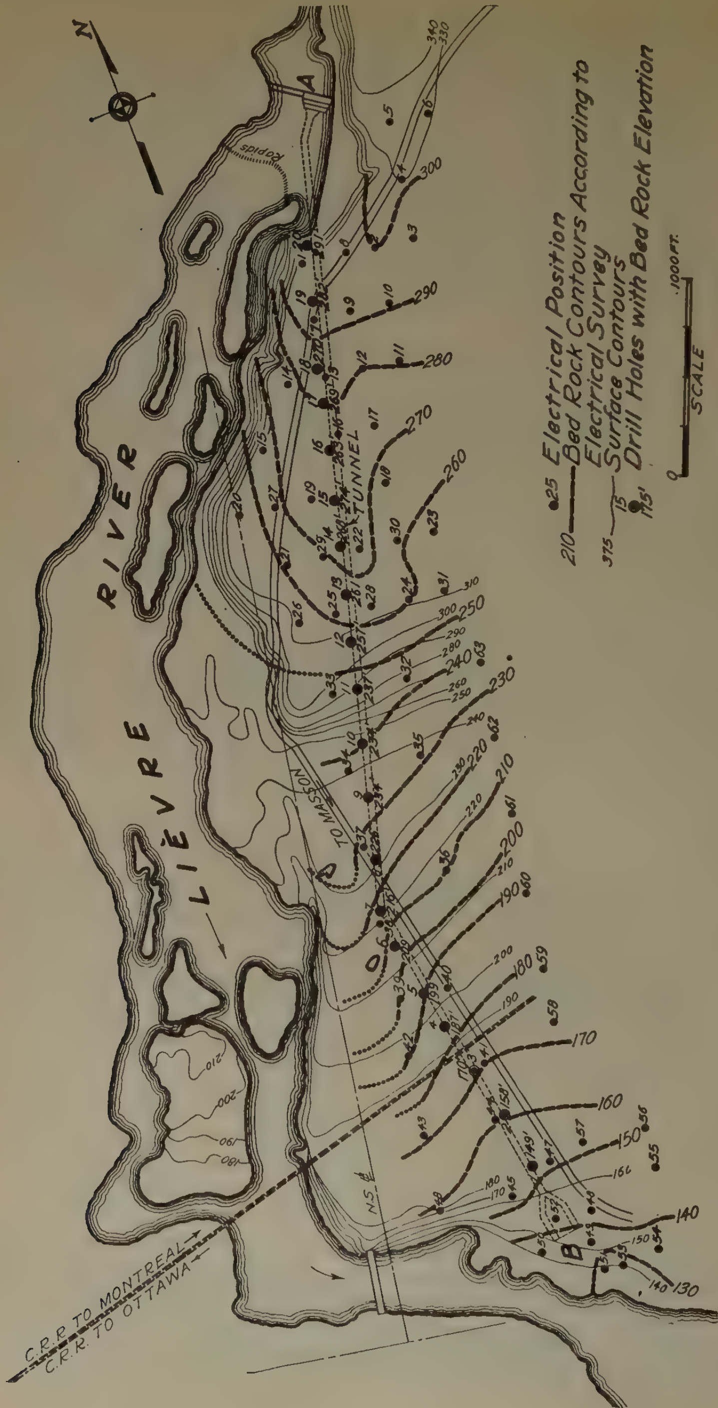

Fig. 5.—Electrical survey on Lièvre River.

shown on Fig. 5 together with the contour lines of the bedrock at 10-ft. intervals, just as these were deduced from the electrical measurements.[14]

As a result of the exploration, the location of the tunnel line was definitely chosen and 20 drill holes were made along the axis of the tunnel, early in the year 1930. The locations of these holes are shown on Fig. 5. The results of the drill exploration are summarized on Fig. 6, on which are also marked the two cross-sections of the bedrock as they can be

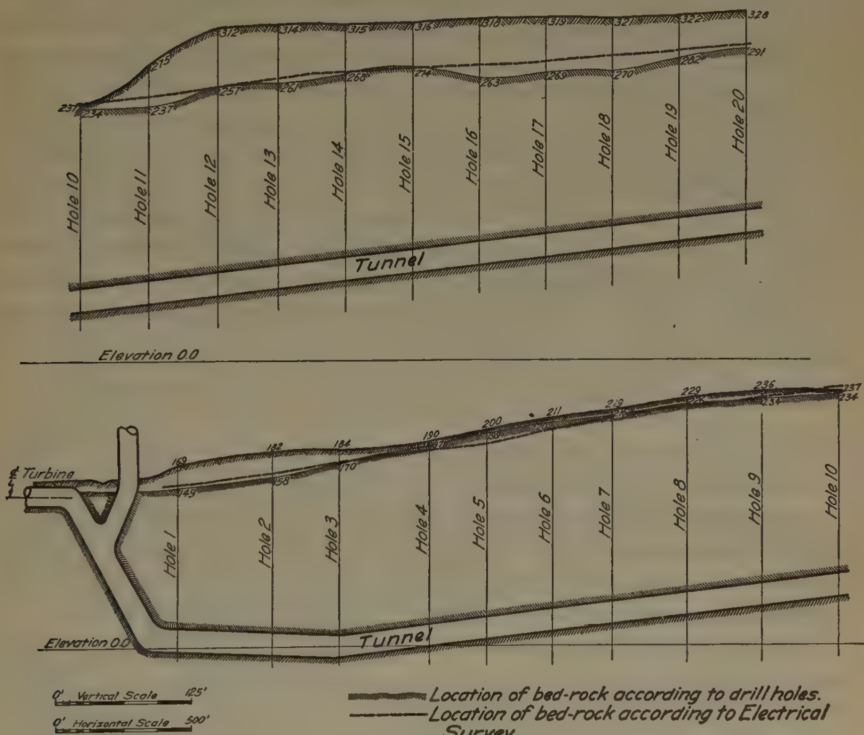

FIG. 6.—SUMMARY OF DRILLING EXPLORATION.

established either from the drill-hole data or from the electrical contour curves. This comparison leads to the following conclusions.

1. The problem was satisfactorily solved, on the whole. The electrical measurements showed that everywhere, in the area surveyed, the overburden was thin and that the tunnel would run through a cover of solid rock all the way along the proposed line.

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[14] Outside of this exploration of the left bank of the river, about one dozen electrical positions were made on the right bank, at the place chosen for the dam site. This small part of the survey is not discussed here because it is independent of the tunnel problem. We have been informed that two drill holes were made on the right bank, which checked very satisfactorily with the electrical determinations.

2. Only some of the electrical positions are located right along the tunnel line, so that the electrical cross-section is not established, most of the time, by means of electrical drillings actually made, but through the aid of the contour curves that were deduced from the net of the electrical measurements. Sometimes the point of intersection of a contour line with the drill-hole line is about 150 ft. or more from the nearest electrical station. There is little doubt that if an electrical determination had been performed at each drill hole, the accuracy of the electrical prediction would have been still more satisfactory. This fact, for instance, is particularly well evidenced at drill hole 10, where the electrical cross-section shows for the bedrock an elevation greater than that of the surface.

3. At drill holes 16, 17, 18 and 19 the results are not accurate. This is explained by the fact that the clays, which constitute the superficial material, in this particular area have irregular resistivities within the first 20 ft. from the surface. These resistivities vary in patches from 30 to 90 ohms, in other words, in a ratio of 1 to 3. This renders the interpretation of the electrical diagrams somewhat uncertain. Inasmuch as the depths to bedrock are shallow, these superficial perturbations strongly affect the electrical measurements.

Conclusions

The practical examples of exploration discussed in this paper show how the resistivity measurements of the underground can efficaciously supplement the geological information in the study of certain engineering projects. According to the problem at hand, the geophysicist has two procedures of exploration at his disposal; namely, the horizontal exploration and the vertical electrical drilling, or vertical exploration.[15]

It must be emphasized here that the studies presented are, strictly speaking, structural studies which are encountered almost in an identical form in mining or oil exploration. It is evident that the mapping of a contact, the tracing of a fault, the following of a series of tilted beds, hidden under an overburden, often will be conveniently solved by a horizontal exploration, and the preparation of a resistivity map. The same process will also enable the location of a buried plug (conductive or resistive) encased in formations of different conductivities, a problem which crops up in the exploration for salt domes. On the other hand, problems like that of outlining buried river channels carrying high values in gold and silver, or locating mineralized pockets near the surface, are not different from the study of the thickness of an overburden at a dam site and will be treated by the same technique.

[15] It goes without saying that these two general resistivity procedures can be supplemented for the investigation of particular points by the various other techniques of the potential and electromagnetic methods.

The advantages of the electrical method are its rapidity and economy, which makes it particularly valuable in preliminary surveys where a general outline of the underground conditions is required immediately. From the results obtained, it is generally possible to orient an underground or drilling exploration with a minimum of cost. It is also well to mention that in numerous cases a mere electrical survey will put in evidence the unfavorable conditions that may exist at a proposed dam site, or tunnel location, etc., thereby saving any further unnecessary expenditure. That the process is rapid and economical is demonstrated by the following figures. The survey at Bridge River required only four working days. The work at Morrisburg and Masson was performed in 30 and 33½ working days respectively. During the course of these last two explorations, 176 depth determinations were made,[16] which cost the companies about $10,000. These data show that an average of 2.8 depth determinations were made by the prospecting party per working day, although one of the surveys was performed during the Canadian bad season, under adverse weather conditions. They further show that the cost of a depth determination averages less than $60, which is a low figure, considering the information furnished. For surveys of very short duration, the cost per vertical drilling is a little higher, although not to any considerable extent.

Another point to be stressed is the fact that the field equipment is very light and can be transported easily. It weighs only a few hundred pounds and the parts are sufficiently small to be conveniently carried about. An observer with the aid of three or four workmen can operate almost anywhere, whatever may be the topographical conditions and the vegetation. This advantage presents a special interest in a preliminary investigation, upon the results of which will depend entirely the further development of the engineering project.

The most complete and satisfactory results will be realized when a close cooperation is maintained between the geologist and the geophysicist. The former with his knowledge of the site to be studied, and of its surface geology, will orient the electrical observer, thus avoiding any unnecessary work in the areas already known. He will also, with his knowledge of the rocks of the region, their lithologic characteristics and mechanical properties, be able to help greatly in the interpretation of the results.

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[16] This figure includes the 12 determinations made at Masson on the right bank of the Lièvre River, which are not discussed in the present paper.

Applying the Megger Ground Tester in Electrical Exploration

By Bela Low* and Sherwin F. Kelly,* New York, N. Y., and William B. Creagmile,† Philadelphia, Pa.

(New York Meeting, February, 1931)

Electrical methods and instruments for geophysical exploration have been almost exclusively applied, during these years of development of the art, by a few companies specializing in this field. The control of patents, the cost of the instruments and the general lack of published information on details of field procedure and office interpretation have given these owners a monopoly in carrying out electrical surveys. The development of the Megger Ground Tester[1] seems likely to change this situation, and to enable mining engineers and geologists, without special training in geophysics, to carry out simple reconnaissance electrical surveys. Nevertheless, the need for such special training and experience in planning surveys and interpreting their results remains, and probably always will remain, indispensable for a successful outcome.

The unique thing about the Megger Ground Tester, which gives rise to this hope for its general use, is its simplicity of operation. The four binding posts of the instrument are connected by insulated wires to four stakes driven into the ground at appropriate points. The crank of the Megger is then turned, and the resistance of the ground between the two inside stakes is read directly, in ohms, by the deflection of a pointer over a scale. The apparatus is shown in Fig. 1.

Fundamentally, the principle of operation is that of determining the resistance of a four-terminal conductor, as applied by Frank Wenner to the measurement of earth resistivity. "A four-terminal conductor is a conductor provided with two terminals to which current leads may be connected and two terminals to which potential leads may be connected. The resistance of such a conductor is the difference in potential between the potential terminals divided by the current entering and leaving

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† Electrical Engineer, James G. Biddle Co.

[1] The word "Megger" is a trade name which, for the past 25 years, has been applied to instruments of a particular type for testing electrical insulation resistance. The Megger Ground Tester is a recent development for testing the resistance to earth of installed electrical ground connections, and also for quickly measuring earth resistivity.

through the current terminals."[2] When the crank of the Megger Ground Tester is turned, it drives a self-contained, direct-current generator. The current thus generated passes first through the current coil, or ammeter element, of an ohmmeter. Thence it goes to a commutator mounted on the same shaft with the generator, where it is changed into alternating current of about 50 cycles per second. Finally, it is led from the current binding posts C_1 and C_2 of the instrument to the two appropriate field stakes. The other two field stakes are connected to the potential binding

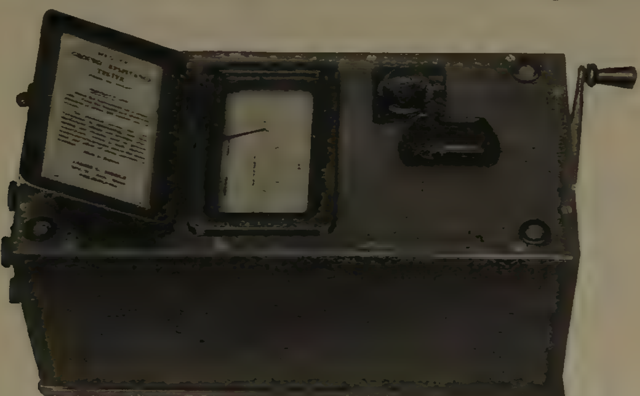

FIG. 1*a*.—MEGGER GROUND TESTER.

FIG. 1*b*.—MEGGER APPARATUS FOR GEOPHYSICAL WORK.

At left, switchboard for facilitating the connecting of the Megger leads to field stakes. Center, the Megger Ground Tester. Right, carrying case, reel of wire and field stakes.

posts P_1 and P_2 of the instrument. The potential drop across these two stakes is measured by leading the current picked up by them through a second commutator, run synchronously with the first, where it is converted back into direct current. It then goes to the potential coil, or voltmeter element, of the ohmmeter. See Fig. 2.

The current coil and the potential coil of the ohmmeter are mounted on a common spindle, so that their torques oppose each other. Thus they automatically perform the division of volts by amperes. According to

[2] F. Wenner: A Method of Measuring Earth Resistivity. U. S. Bur. Stds. *Sci. Paper* 258 (1915).

Ohm's law, resistance equals volts divided by amperes, so the scale of the ohmmeter is calibrated to read directly in ohms.

The arrangment described is doubly advantageous. It retains the sensitiveness and accuracy of the direct-current ohmmeter. By using

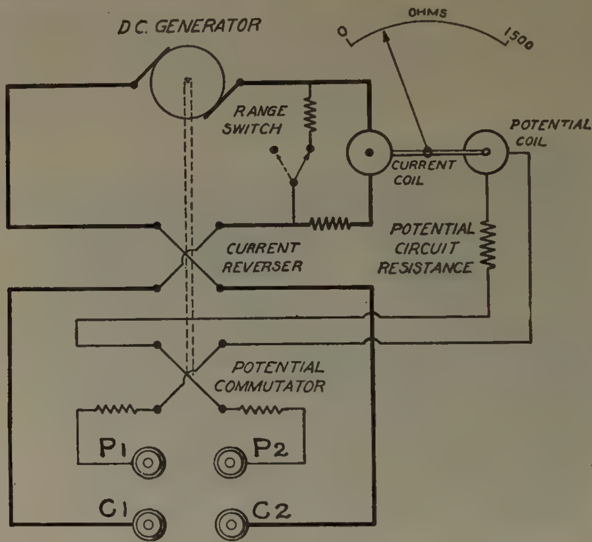

FIG. 2.—INTERNAL CONNECTIONS OF MEGGER GROUND TESTER.

alternating current in the ground circuits, however, the errors that would be introduced in a direct-current circuit by polarization and electrolysis at the ground stakes, or by stray direct currents, are eliminated. Thus the

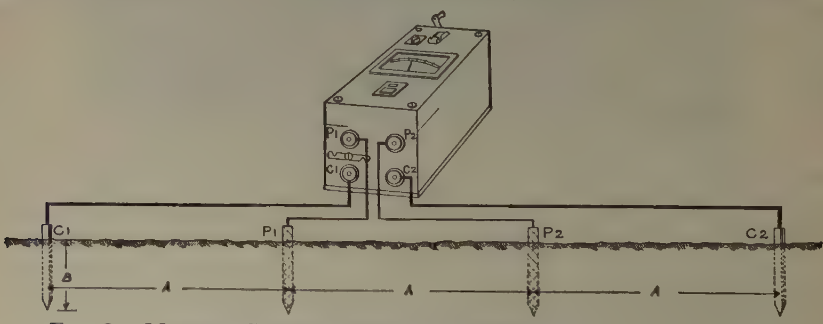

FIG. 3.—MEGGER GROUND TESTER CONNECTIONS TO FIELD STAKES. Note, A must be at least $20 \times B$.

use of nonpolarizing electrodes is avoided, and copper or iron stakes can be employed without having to introduce any manually operated device to compensate for the unwanted currents. Stray alternating currents are not bothersome, as by changing the speed with which the generator-commutator shaft is turned, the Megger Ground Tester circuits may be thrown out of phase with the strays, and so are no longer affected by them.

The four field stakes to which the Megger is connected may be of iron, brass or copper, for instance, about ½-in. dia. and driven into the ground

to a depth of 1 ft. or more. For simplicity and convenience they are best arranged in a straight line, at equal intervals A, Fig. 3. The two end stakes are connected to the current leads from the Megger, and are correspondingly called C_1 and C_2. The two intermediate ones, designated P_1 and P_2, are connected to the similarly named potential leads.

With the apparatus thus disposed, the resistance is obtained of a portion of the earth between the potential stakes. "Roughly speaking, the effect of material at distances equal to the distance between adjacent terminals is found to be so small that the effect of materials beyond this distance is negligible. Thus, the body of earth involved in a single determination has linear dimensions of the same order as the distance between terminals."[3] The depth of measurement consequently is equal to the electrode, or stake separation A, Fig. 3.

The actual resistance read on the instrument will depend on the distance separating the electrodes. Where this distance is not kept constant throughout a given survey, the readings are not comparable. It then becomes necessary to determine the specific resistance, or resistance per unit volume, called the resistivity, of the ground under investigation. This may be expressed as the ohms resistance of a cube measuring 1 meter on an edge, abbreviated ohm-meter. As long as resistivities are expressed in the same units, the figures are evidently comparable. In this paper, because of the small scale of the experimental demonstrations, the unit ohm-inch is adopted for the laboratory experiments. The Megger Ground Tester reading is related to the resistivity by a simple formula:

$$\rho = 2\pi AR$$

Where

ρ = resistivity in ohm-meters,
R = resistance in ohms, as read on the Megger,
A = electrode separation in meters.

If the electrode separation is given in inches, the resistivity will be in ohm-inches. A full development of the derivation of the formula may be found in the article by Wenner already mentioned.

In perfectly homogeneous ground the resistivity, obviously, must be constant, consequently the resistance reading will vary inversely as the electrode separation. If the ground is not homogeneous, this inverse proportion is interfered with. The application of the above formula in such a case gives the average, or apparent, resistivity of the entire volume involved in the measurement. In other words, the presence of a layer or body of better conductivity, between the potential electrodes and within a

[3] W. O. Hotchkiss, W. J. Rooney and James Fisher: Earth-resistivity Measurements in the Lake Superior Copper Country. Geophysical Prospecting, A. I. M. E. (1929).

radius A of the line joining them, will result in a lowered apparent resistivity; conversely, a poor conductor within that volume will cause a rise in the apparent, or average, resistivity. Evidently such material must be of sufficient size to produce an appreciable effect in order to be detected.

By starting with a small interval between stakes, and progressively increasing their separation, successively greater depths of penetration are obtained. Then the electrode separation at which a markedly higher or lower average resistivity is observed gives a rough measure of the depth at which occurs a resistant body, as bedrock under drift, or a conductive one such as a water table or orebody, capable of exerting an important influence on the measurements.

This evidently does not provide a measurement of the true resistivity of the underlying, different layer or body. In order to calculate this value from the observed readings it becomes necessary to utilize certain formulas, or curves, with which the present paper will not deal. The determination of the true resistivities of the formations under observation is often useful, and frequently indispensable for obtaining their depths accurately. Those interested will find complete discussions of this matter in papers by Weaver, Hummel, Ehrenburg and Watson, Lancaster-Jones, and Tagg.[4]

An alternative technique can be used where it is expected that formations of different electrical characteristics may occur within a given depth over a limited horizontal extent. These might be igneous intrusions, salt domes, buried hills and valleys, concealed anticlines or synclines, sinks, and conductive or resistant strata dipping into, or out of, the zone of observation. For such cases the electrode separation can be fixed for the desired depth of investigation, and the given area explored by a systematic grid of observations. A map of resistivities is then drawn up from the results of this survey.[5] This map will show islands of higher or lower

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[4] W. Weaver: Certain Applications of the Surface Potential Methods. Geophysical Prospecting, A. I. M. E. (1929).

J. N. Hummel: A Theoretical Study of Apparent Resistivity in Surface Potential Methods. See page 392.

D. O. Ehrenburg and R. J. Watson: Mathematical Theory of Electrical Flow in Stratified Media. See page 423.

E. Lancaster-Jones: The Earth-resistivity Method of Electrical Prospecting. *Min. Mag.* (1930) **43**, 19.

G. F. Tagg: The Earth Resistivity Method of Geophysical Prospecting. Some Theoretical Considerations. *Min. Mag.* (1930) **43**, 150.

[5] E. G. Leonardon and S. F. Kelly: Some Applications of Potential Methods to Structural Studies. Geophysical Prospecting, A. I. M. E. (1929).

E. G. Leonardon: Electrical Exploration Applied to Geological Problems in Civil Engineering. See page 99.

C. and M. Schlumberger: The Method of the Ground Resistivity Map and Its Practical Applications. Can. Min. & Met. *Bull.* 226 (1931).

G. F. Tagg: Earth Resistivity Surveying. *Eng. & Min. Jnl.* (1931) **131**, 325.

resistivities where formations of poorer or better conductivity than their surroundings are included within the depth of investigation.

Laboratory Experiments

The two field procedures described can be demonstrated by means of a simple laboratory experiment. A glass tank measuring approximately 18 in. long, 7 in. wide and 6 in. deep[6] is filled with water to represent a section of the earth. Contact with the water surface is achieved with four nails attached to the Megger Ground Tester leads. The nails are thrust through holes, in which they fit tightly, bored at intervals of 1 in. in a wooden rod arranged to rest on the ends of the glass tank. A thin sheet of metal serves for the conductive strata, and a piece of glass, bakelite or phenolite for the resistant beds. These sheets fit in the tank, and are ⅛ to ¼ in. thick.

Keeping the electrode separation fixed at, say, 2 in., the conductive stratum may be brought from a depth of 4 in. or more to positions progressively nearer the surface, Megger Ground Tester readings being taken at each step. The same process may be repeated with the resistant sheet. These readings are representative of what would be expected in a gridwork of Megger observations, where a conductive or resistant bed dips up into the zone of investigation.

Table 1 gives figures obtained in two laboratory demonstrations similar to the one just described. The effect produced by the resistant and the conductive sheets on the resistivities is apparent. This effect is

Table 1.—*Figures Obtained in Two Laboratory Demonstrations of Megger Ground Tester*

Electrode Interval A, 2 Inches

Depth of Immersion of Sheet, In.	Experiment 1[a]				Experiment 2[b]			
	Conductive Sheet		Resistant Sheet		Conductive Sheet		Resistant Sheet	
	R, Ohms	ρ, Ohm-in.	R, Ohms	ρ, Ohm-in.	R, Ohms	ρ, Ohm-in.	R, Ohms	ρ, Ohm-in.
4	110	1383	140	1760	125	1572	145	1820
3	105	1320	147	1850	115	1447	165	2075
2	80	1005	168	2112	85	1070	185	2325
1.5	57	717	215	2705	65	817	230	2890
1	26	327	325	4080	25	314	375	4720
0.5	5.5	69	420	5280				

[a] Performed at Harvard University in the course of a lecture on Geophysics as a New Tool for the Geologist, February, 1931.

[b] Performed during the course of a lecture on geophysics at Cornell University, March, 1931.

[6] For demonstration at the February meeting, 1931, of the A. I. M. E. a glass tank was provided through the courtesy of Eimer & Amend, New York City, and a Megger Ground Tester was loaned by James G. Biddle Co. of Philadelphia.

slight at depths greater than the electrode separation, but becomes increasingly pronounced as shallower depths are reached.

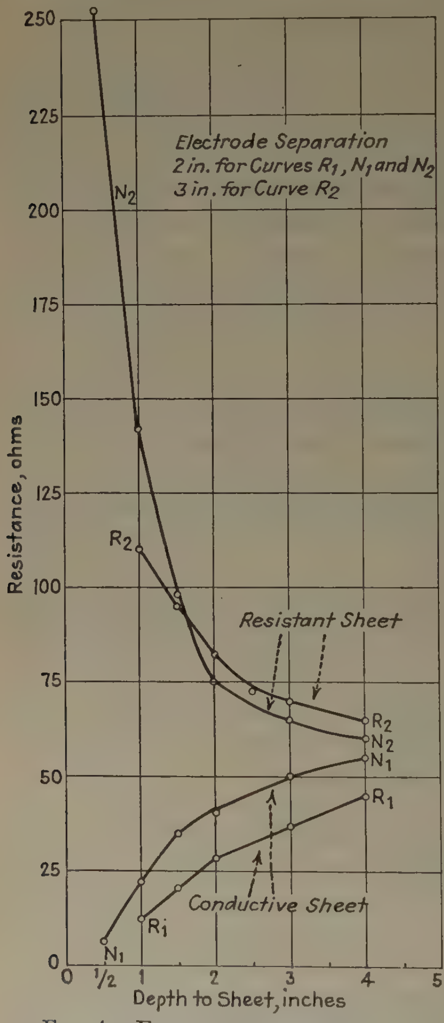

FIG. 4.—EFFECT ON RESISTANCE READINGS OF VARYING DEPTHS OF CONDUCTIVE AND RESISTANT SHEETS.

Since electrode spacings were kept constant, resistances read, instead of resistivities, are given in the ordinate scale.

Curves N_1 and N_2 were obtained during experiments at Northwestern University.

Curves R_1 and R_2 were obtained during experiments at Missouri School of Mines, Rolla, Mo.

Such data as the above may also be expressed in the form of curves, in which resistance, or resistivities, are plotted as ordinates, and the depths to the stratum as abscissas (Fig. 4). This has the advantage of showing more graphically the change brought about in the resistivity when the disturbing factor comes within the depth of investigation. Not infrequently the curves show a break at or near this point, but they may be quite smooth. Both types are shown. Because no break may appear, or if present may not coincide exactly with the depth of transition, it is safer to base the interpretation on the type curves given by Hummel, Lancaster-Jones and Tagg in the works already cited.

To demonstrate the process of determining the depth to a given stratum beneath a certain point, the metal or glass sheet is kept at a depth of 2 or 3 in. The nail separation is then increased from 2 in. to 3 and then 4 in., Megger readings being taken at each stage. The figures duplicate what would happen in a survey designed to determine the depth to a given conductive formation, such as a clay bed, water table or orebody, or to a resistant one, such as bedrock beneath soil or glacial drift, or a limestone or sandstone stratum in shale.

In this case it is necessary to calculate the resistivities in order to get interpretable results, as the electrode separation is not constant. An important fact to be kept in mind is that in a laboratory experiment on such a small scale the resistance of the glass walls and bottom of the

tank interferes with obtaining true values. This may be verified by using nothing but water in the tank and progressively increasing the nail separation as described. Obviously, the water is homogeneous, but in spite of this fact the apparent resistivity rises, owing to the effect mentioned. Tanks of larger dimensions would lessen this error, but are hard to obtain and difficult to handle.

The results of two such experiments are set forth in Table 2. Although the data should properly be drawn up in the form of curves, this has not been done because of the impossibility of obtaining a sufficient number of points when experimenting on such a restricted scale. The

TABLE 2.—*Results of Two Experiments in Determining Depth to Strata*

Electrode Separation			2 In.	3 In.	4 In.
Experiment 3	Conductive sheet at depth of 3 in.	R, ohms	45	20	12
		ρ, ohm-in.	566	375	315
	Resistant sheet at depth of 3 in.	R, ohms	65	60	60
		ρ, ohm-in.	818	1130	1510
	Water	R, ohms	58	48	50
		ρ, ohm-in.	729	905	1258
	Dividing by effect in water only	Ratio of drop, ρ	0.78	0.41	0.25
		Ratio of rise, ρ	1.12	1.25	1.20
Experiment 4	Conductive sheet at depth of 3 in.	R, ohms	40	24	19.2
		ρ, ohm-in.	503	455	482
	Resistant sheet at depth of 2.5 in.	R, ohms	65	72	87
		ρ, ohm-in.	818	1355	2185
	Water	R, ohms	58	62	75
		ρ, ohm-in.	729	1169	1888
	Dividing by effect in water only	Ratio of drop, ρ	0.69	0.39	0.26
		Ratio of rise, ρ	1.12	1.16	1.16

Experiment 3 was performed during a course of lectures on the Application of Geophysics in Geology at Northwestern University, March, 1931.

Experiment 4 was performed during a course of lectures on Geophysics at the Missouri School of Mines, April, 1931.

figures given suffice to show the diminution of resistivity brought about by a conductive stratum when the electrode separation is extended to include it within the depth investigated. Conversely, the increased resistivity brought about under the same circumstances by a resistant formation also is demonstrated.

The effect of the insulating walls of the tank appears in the figures of resistance and resistivity when only the water was measured. For its resistivity to have remained constant at 729 ohm-in., as would normally be the case, it is readily calculated that the resistance should have read 39 ohms at 3 in. separation, and 29 ohms at 4 in. separation. The departure from these figures was less pronounced in experiment 3 than in experiment 4, because the tank used in the former was considerably larger than the one used later. In experiment 4 it was about the size mentioned on page 6.

The figures in the rows headed "Ratio of Drop, ρ" are obtained by dividing the resistivities when using the conductive sheet by those with water only. Those under "Ratio of Rise, ρ" show the resistivities with the resistant sheet divided by the resistivity with water only.

Another phase of the demonstration, concerning which no figures were kept, dealt with the effect of conductive and resistant sheets in a vertical position. From this it readily developed that conductive veins, dikes or vertically inclined strata would have the greatest effect when near and parallel to the line of electrodes. On the other hand, resistant dikes or sharply tilted beds exert the strongest effect when perpendicular to the line of electrodes, lying across it between P_1 and P_2.

The data from these experiments, it should be distinctly understood, do not represent the results of any attempt to carry out a series of scientific investigations. The apparatus used was too crude and too hastily assembled, and the work too sketchily done, to warrant putting any such interpretation upon them. They do show, however, the simplicity with which it is becoming possible to apply the resistivity technique, for these results could be duplicated by anyone with a Megger Ground Tester, a good-sized metal ash tray, a dinner plate and a bathtub of water.

Field Work

In field work, of course, certain precautions must be observed. For example, it may be necessary to measure the stake resistances of P_1 and P_2, if it is expected that they will lie outside a certain range. By stake resistance is meant the resistance the current encounters at the contact of the stake with the earth. This value is easily determined by connecting C_1 and P_1 of the Megger to the stake where the resistance is to be measured. P_2 must be some distance away, say 25 ft. or more, and C_2 as far away again beyond P_2. The reading thus obtained is the stake resistance,

which can then be used in a simple formula to correct the ground-resistance reading. The error introduced by high or low stake resistances is seldom large enough to warrant bothering about, save in exceptional cases.

The question naturally arises, "How can the Megger Ground Tester be applied in civil engineering, mining engineering or geology?" The answer must be somewhat roundabout. First and foremost, the simplicity of the instrument puts it almost in the class with the magnetic dip needle. For many years the dip needle has been used, as a matter of course, to aid in working out geological questions where the formations involved possess markedly different magnetic permeabilities. Similarly, the Megger Ground Tester now gives the engineer and geologist the opportunity to utilize certain electrical methods as everyday tools, in their attacks on earth problems.

Naturally, the actual geophysical technique employed must depend on the type of information desired, and on the geological circumstances of the problem. Sometimes two or more methods must be conjointly applied in order to obtain the desired information. Before deciding to use the Megger Ground Tester, or any other method or combination of methods, the situation must be carefully examined to determine the proper geophysical course of action.[7] If the geological circumstances warrant a belief in the probable efficacy of resistivity (or conductivity) or potential methods, the Megger Ground Tester probably could be applied. Exception must be made for studies to great depths, say about 600 ft. or over, for which the instrument as now designed is not sufficiently powerful.

The conditions that make possible the application of resistivity methods have been detailed in numerous geophysical articles,[8] and need not be repeated here. There is some literature, however, outlining actual field applications of the Megger Ground Tester.

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[7] S. F. Kelly: Geophysics in Exploration; Prospect and Retrospect. *Eng. & Min. Jnl.* (1931) **131**, 11.

[8] E. G. Leonardon and S. F. Kelly: *Op. cit.*

E. G. Leonardon: *Op. cit.*

H. Lundberg and T. Zuschlag: A New Development in Electrical Prospecting. See page 47.

S. F. Kelly: Electrical Methods for Sub-Soil Investigation. *Proc.* Brooklyn Engineers' Club (April, 1930).

I. B. Crosby and S. F. Kelly: Electrical Subsoil Exploration and the Civil Engineer. *Eng. News-Rec.* (1929) **102**.

O. H. Gish and W. J. Rooney: Measurements of Large Masses of Undisturbed Earth. *Terrestrial Magnetism* (1925) **30**.

C. and M. Schlumberger: *Op. cit.*

F. W. Lee, J. W. Joyce and P. Boyer: Some Earth Resistivity Measurements. U. S. Bur. Mines, *Inf. Circ.* 6171 (1929).

H. A. Buehler: Geophysical Prospecting. Biennial Report of State Geologist of Missouri, Appendix III (1931).

The study of subdrift contours is described by Lancaster-Jones,[9] who also goes into theoretical discussions of the technique of interpreting the results. Professor Gilchrist, of Toronto, has used the Megger Ground Tester to study geological structure as well as sulfide orebodies at the Abana mine in Quebec.[10] Eve and Keys discuss the Megger Ground Tester and some of its applications.[11]

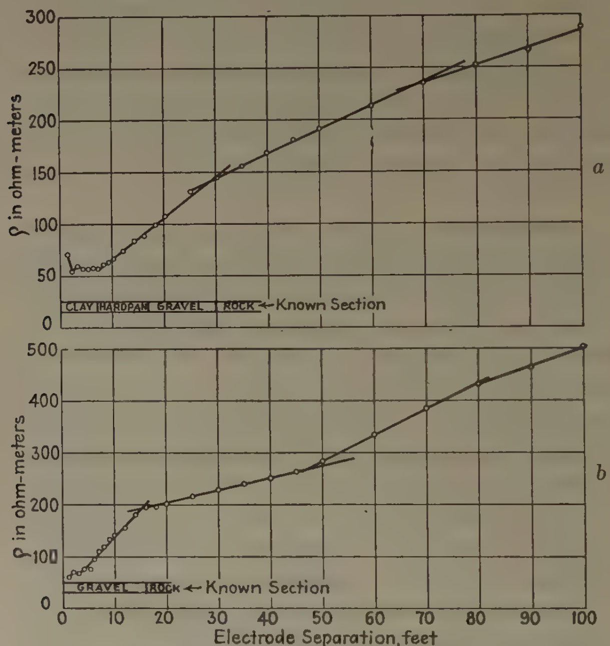

Fig. 5.—Megger observations of depth to bed rock, April 22, 1931.
a. At Gasconade Bridge, Route 63, Missouri.
b. At Roubidoux Bridge, Route 66, Missouri.

The curves drawn up from some measurements made to determine depth of bedrock are given in Fig. 5. The work was done in cooperation with F. C. Farnham of the Missouri Geological Survey, through the courtesy of the director, H. A. Buehler. Two bridge locations were chosen, where the depth of overburden was known from drilling records. The close correspondence of the break in each curve with the depth to bedrock is a striking feature of these graphs. The fact that the observations at each place occupied but a couple of hours makes this an interest-

[9] E. Lancaster-Jones: *Op. cit.*

[10] L. Gilchrist: Measurements of Resistivity by the Central Electrode Method at the Abana Mine, Northwestern Quebec, Canada. A. I. M. E. *Tech. Pub.* 386 (1931).

[11] A. S. Eve and D. A. Keys: Applied Geophysics in the Search for Minerals. Cambridge, 1929. University Press.

ing procedure for engineers concerned with obtaining information regarding subsoil conditions at proposed bridge or dam sites.

CONCLUSION

The study of certain types of geological problems may often be facilitated and speeded, with a saving in expense, by employing the Megger Ground Tester for making reconnaissance, resistivity studies. Problems involving such things as mapping subsurface contours and determining depth of overburden, tracing faults, contacts and intrusions, following key horizons and investigating simpler questions of ore occurrence come to mind as being frequently amenable to treatment by electrical studies, to which the Megger Ground Tester might profitably be applied on many occasions.

DISCUSSION

G. F. TAGG, London, England (written discussion).—I have read this paper with considerable interest, particularly those parts which deal with the interpretation of the results. There are one or two points in connection with this which I should like to mention. In dealing with the effective depth to which the current will penetrate, the authors make the statement: "The presence of a layer or body of better conductivity between the potential electrodes, and within a radius A of the line joining them, will result in a lowered apparent resistivity; conversely, a poor conductor within that volume will cause a rise in the apparent or average resistivity."

These effects are not limited to conducting or resistant bodies at a depth equal to or less than the electrode separation, or to bodies located between the potential electrodes. The effects are present to a greater or lesser extent in any case where such bodies are existing in the earth. The effect, of course, is dependent on the size of the body and its distance from the electrode system relative to the electrode separation. The effect increases gradually for bodies nearer and nearer to the electrode system, reaching a maximum when the electrodes are actually in the body concerned. This is clearly shown for one particular case in the curves given in my paper dealing with theoretical considerations.[12]

The method used to interpret the results obtained is still open to discussion. There is a vast amount of experimental data upholding the theory that if a sudden change occurs in the apparent resistivity curve it indicates the presence of a body having a resistivity different from that of its surroundings, at a depth equal to the electrode separation at which the change occurs. On the other hand, all theoretical and mathematical considerations show that such a sudden change cannot occur. In an experimental survey I carried out to test one thing, no sudden changes in the resistivity were apparent.[13] It thus appears to be safer to base the interpretation on theoretical considerations where this is possible. The method of interpretation based on theory was quite successful in the case quoted above.

Up to the present, however, the theory has been developed for only one or two simple cases of earth structure, and for the more complicated forms of structure it is necessary for the present to be content with some form of empirical method, which has been shown to be successful in many cases.

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[12] *Min. Mag.* (1930) **43,** 150.
[13] *Proc.* Phys. Soc., London (1931) **43,** 305.

W. B. Creagmile (written discussion).—Mr. Tagg begins his criticism by calling attention to a statement on page 117. He explains that the effects described are not limited strictly to materials within the volume mentioned. This is well recognized, but Mr. Tagg seems to have overlooked our quotation from Hotchkiss, Rooney and Fisher, earlier on the same page.

We agree that the method of interpreting results, as referred to on page 118, lines 8 to 12, in the sentence beginning with "Then the electrode separation," has no theoretical or mathematical foundation but has worked out in many cases, and therefore we feel that it must be given some weight.

Naturally, it will be best to work from curves plotted between resistance or resistivity and the electrode spacing, as referred to in the first paragraph of page 120. If a curve shows a break, the spacing of the electrodes at which the break appears may be taken as the depth at which a change in strata occurs. Mr. Tagg seems to have overlooked that part of the paragraph in question, when he says "Not infrequently the curves show a break at or near this point, but they may be quite smooth. Both types are shown. Because no break may appear, or if present may not coincide exactly with the depth of transition, it is safer to base the interpretation on the type curves given by Hummel, Lancaster-Jones and Tagg in the works already cited." Of course, if a sharp break occurs in a curve, the approximate depth at which the change in strata occurs can be read off the curve more easily than any of the theoretical methods of interpretation can be applied.

S. F. Kelly (written discussion).—Mr. Tagg's comments emphasize a point which may not have been sufficiently stressed in our paper, although it was certainly brought out, as Mr. Creagmile has shown. In actual field work, the curves plotted with resistivities as ordinates against electrode separation as abscissas frequently fail to show a break corresponding to the change of formation at a given depth. Even when they *do* show such a break, the electrode separation does not necessarily correspond closely to the depth of transition from one formation to another. Therefore, it should be clearly understood that an attempt to deduce the depth to the lower formation from such a break is admittedly a rough estimate, and not to be applied where accuracy is desired. It will often suffice in reconnaissance work, however. The only way of obtaining a reasonably accurate idea of the thickness of the overlying formation is to utilize some such set of graphs as has been worked out by Mr. Tagg. These should be applied *whether the curves show a break or not.*

Depth of Investigation Attainable by Potential Methods of Electrical Exploration

By Conrad and Marcel Schlumberger,* Paris, France

(New York Meeting, February, 1929)

The object of this paper is to clarify the idea, so important when exploring by potential methods, of the depth of investigation attainable by electrical measurements. After defining, with some precision, the term "depth of investigation" in geophysics, the authors show how it is possible to regulate this depth in potential methods and to obtain an 'electrical drilling" from surface observations. They consider also the difficulties encountered in deep measurements and indicate how it is possible to reach greater depths than those having an industrial interest.

Depth of Investigation

The term, depth of investigation, in its general sense, is easily understood. For instance, a driller who can pull cores from a depth of 2000 m. is capable of working, with accuracy, to that depth. By means of a properly placed drill, he may locate an orebody at a depth of 1999 m., however small this orebody may be.

On the contrary, the term as used in geophysics requires precise elaboration. As a matter of fact, no geophysical method, irrespective of the physical property studied (density, magnetic permeability, conductivity), can discover a very small body at a great depth because, in practice, the physical parameter upon which such a geophysical method is based is never regular enough, throughout the ground, to distinguish the feeble anomaly caused by the small, deep-lying body from the numerous anomalies due to irregularities of the encasing terrains.

Aside from this question of the ratio between the size of the body being hunted for and its depth of burial, a second factor plays its role in the possible depth of investigation; that is the difference, for the parameter under consideration, between the body and its surrounding material. The greater this difference, and the more homogeneous the encasing terrains, the greater is the possible depth of investigation. This being generally established for all geophysical methods, it is possible to distinguish two main classes of these methods, from this same point of view

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of depth of investigation, namely, (1) the class which, such as the gravimetric and magnetometric, utilizes natural fields of force not generated by the operation of the method itself, and (2) the class which studies the actions of artificially provoked perturbations (electrical methods utilizing an outside source of current; seismic method).

The systems that study natural fields have, aside from the general limitations indicated above, theoretically no limit to their depth of investigation. Thus, if we consider two differentiated horizontal formations, A and B, cut by a fault F (Fig. 1), it is sufficient for the throw of the fault to increase proportionally with the depth to permit studying the step B by these methods, no matter what its distance from the surface.

At the same time, this theoretical advantage is more than counterbalanced by the inconvenience of being unable to control the depth of investigation. In fact, attention should be directed to the point that one of the greatest difficulties in the geological interpretation of measurements of this type comes from the impossibility of distinguishing deep-lying from superficial activity. Also, it should be noted that, for the same reason, no work can be undertaken in horizontal terrains by these methods (for instance, determining the depth of a horizon marker).

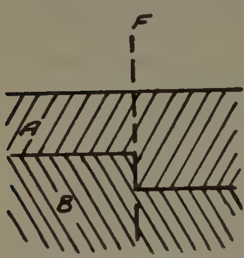

Fig. 1.

The methods which themselves create the disturbance they study, on the contrary, are faced at once by this question of the depth of investigation, since, on the one hand, they have at their disposal a technique for controlling such depth, and, on the other hand, there are practical limits to this depth because of the augmentation of the power required. From these two points of view, then, let us examine the possible depth of investigation with respect to the potential methods.

Theoretical Depth of Investigation by the Potential Method

Suppose that we have to deal with an ensemble of horizontal geological formations. This is the simplest case, and the one for which the expression "depth of investigation" has the clearest significance.

The technique utilized in such a case is not that, often described, of the deformation of the equipotential curves, but the one of absolute resistivity measurements. This method was described in the authors' French patent of Sept. 15, 1925, which was the outcome of a long period of experimentation. O. H. Gish and W. J. Rooney, without knowledge of the authors' previous work, also conceived the idea of using[1] this tech-

[1] See *Terrestrial Magnetism and Atmospheric Electricity* (Dec., 1925). Although his is not very important, these dates settle a small point of priority.

nique. In view of the fact, however, that these methods have been described but little, it is worth while to recall the principles:

1. If alternating or direct current of intensity, i, is passed through homogeneous soil between two ground electrodes A and B, connected to a generator G (Fig. 2), and the resulting difference of potential is measured between two searching electrodes M and N, the resistivity of the earth is given by the formula:

$$\frac{1}{\rho} = \frac{i}{2\pi\Delta V}\left(\frac{1}{AM} - \frac{1}{AN} - \frac{1}{BM} + \frac{1}{BN}\right)$$

In this formula, AM designates the distance between the points A and M; BN that between points B and N, etc. For practical reasons $AMNB$ are usually placed on the same straight line.

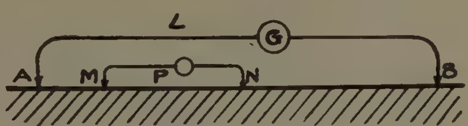

FIG. 2.

2. If the terrain assumed to be homogeneous at depth is covered by a superficial layer, itself homogeneous but of a different nature, this formula remains valid for the superficial material when the measuring arrangement $AMNB$ is small in comparison with the thickness of the overburden; for the reason that the great majority of the current paths going from A to B in the ground remain in the upper, homogeneous terrain and are not influenced by the material below. The situation is as though the lower terrain did not exist, and the resistivity of the overburden is obtained.

If, on the other hand, the measuring arrangement $AMNB$ is long in comparison with the depth of overburden, the section available for current flow in the superficial cover becomes negligible beside that offered by the subjacent formation. The upper layer no longer exerts an appreciable effect on the total current flow or on the distribution of potentials. Measurements therefore give the resistivity of the deep-lying terrain.

If, now, a diagram be made of the variations in resistivities observed as the length of the measuring arrangement is increased, a curve is obtained which has, as its origin, the resistivity of the superficial terrain and as its asymptote the resistivity of the subjacent formation (Fig. 3). The thinner the overburden, the more rapidly does the curve approach this asymptote. This curve may be determined when the resistivity of the surface material, that of the subjacent material, and the thickness of the overburden are known. Inversely, given a curve of this type,

it is possible to obtain therefrom the thickness of the overburden, its resistivity and the resistivity of the underlying formation.

By the use of this diagram, greater definition can be given to the idea of "depth of investigation." For instance, take the case of a lower formation five times more conductive than the overburden (curve given in Fig. 3). From this it will be seen that if the resistivity curve has been traced with electrode separations increasing from zero to a length

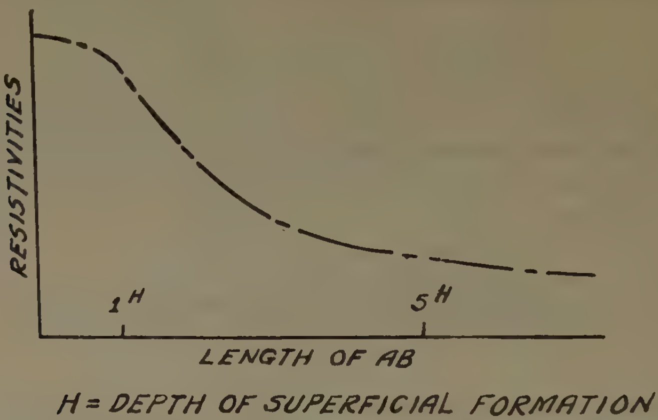

FIG. 3.—VARIATIONS IN OBSERVED RESISTIVITIES WITH LENGTHENING *AB* LINE.

of *AB* double the depth to the lower formation, there will be observed a resistivity which is regular, but which begins to decrease at the last measuring points.

The anomaly due to the deeper material begins to make itself felt at this length of measuring line. In the case considered we can say that the limiting depth of exploration is equal to one-half of *AB*. The drop of the curve, however, is still too slight to be practically interpreted with security, so it needs to be continued. The drop in resistivities becomes clear when *AB* reaches a length equal to four times the thickness of the overburden. Therefore, we may admit that one-fourth the length of line *AB* gives a fair idea, in this case, of the practical depth of investigation.

An optical comparison will illustrate the point. The measuring arrangement can be compared to a telescope, the enlargement or diminution of the measuring lines being compared to the process of focusing the lens. We begin to perceive the electrical anomaly (in this case the plane of separation of the two formations) when the measuring line is as long as the overburden is thick. This boundary is clearly evident when the length of *AB* is doubled. The nature of the lower formation (with respect to its resistivity) is clearly seen, without further interference by the overburden, when the measuring arrangement is still further lengthened.

Therefore, we can regulate at will the depth of investigation, which is proportional to the length of measuring arrangement employed.

3. In the case of a more complicated arrangement of horizontal strata, these ideas are still valid. The diagrams become more complicated but the thicknesses and characteristic resistivities of the beds can be determined by comparing the diagram obtained in the field with the ones calculated, or worked out experimentally in the laboratory. As far as the depth of investigation is concerned, it is usually necessary to push it further than would be strictly necessary in the simpler cases, and it may be useful to utilize lines AB as much as eight times as long as the depth to be studied in detail.

These studies at the surface finally result in giving a scale diagram of the real resistivities of the formations in the geological column. It is an "electrical drilling" comparable to a core drill, but one in which the traversed formations are classified according to their resistivities instead of according to the geological characteristics of the rocks.[2] The remarks of a general nature, made at the beginning of this paper, should be recalled here; namely, the better the terrains are differentiated vertically and the more regular they are horizontally, the better will be the electrical drilling. Although the latter does not make possible the detection of small details, as can be done with a core drill, it does, however, provide a perspective of the formations which avoids the error, for example, of mistaking a small sand lens for a continuous horizon.

4. The above explanation assumes geological ensembles of indefinite horizontal extent. The same reasons for the diminution of action at a distance which control the depth of investigation in a vertical direction, also intervene to limit the distance of investigation in a horizontal direction. Practically, it suffices that a geological ensemble shall have a length at least double the longest AB line used, and a width equal to that AB, to be considered of indefinite extent.

The study of the question of the depth of investigation in the case of more complex geological conditions would lead us too far; it would result, moreover, in the same conclusions—namely, that the depth of investigation can be simultaneously regulated vertically and horizontally by the employment of larger or shorter measuring arrangements. In view of the fact that these can be moved about over the surface of the ground, it is theoretically possible to study, from the surface, the real resistivities of all the points in the volume of the earth under investigation.

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[2] For practical illustrations of these electrical drillings, see E. G. Leonardon and S. F. Kelly: Some Applications of Potential Methods to Structural Studies; and I. B. Crosby and E. G. Leonardon: Electrical Prospecting Applied to Foundation Problems. Geophysical Prospecting, A. I. M. E. (1929).

Practical Limits to Depth of Investigation

Theoretically, therefore, there is no limit to the depth to which studies may be pushed, since, by sufficiently enlarging the measuring arrangement, as great a depth as desired may be studied.

Practically, aside from the question of laying longer and longer lines and reeling them up again, it is evident that the technical difficulties in the way of correctly measuring the drops of potential increase with the length of the lines. These difficulties will be examined, and it will be shown that the electric current necessary for making correct measurements remains fairly weak, taking certain precautions, even for very great depths of investigation.

Skin Effect

When a noncontinuous current transverses the ground it tends, through the ohmic effect, to distribute itself among the conductive paths according to the laws governing direct current, but the variations of the current in one path induce an electromotive force in the neighboring paths, so that the current thus induced tends to oppose variations in the magnetic flux.

From this it results that the variations of the current in one path are hindered by induction from the neighboring paths; consequently the electromotive force resulting from the inductive effect of the surrounding current paths, upon any one of them, opposes current variations in the latter.

This influence is less marked in the paths near the surface, since they have neighbors on one side only. The flow of variable current is hindered more within the ground than at the surface; therefore it tends to localize itself at the surface to a greater extent than would a direct current. This effect becomes more marked as the electromotive forces induced by the intensity variations become larger compared to the ohmic drops of potential.

Alternating Current

It is possible to calculate exactly the skin effect in the case of flat, homogeneous soil of resistivity, r, when transversed by sinusoidal current of frequency, $\frac{1}{T}$, sent betweeen points A and B an infinite distance apart (with respect to the resistivity and frequency under consideration). Under such conditions, the law for the middle part of the electrical field is the following: The maximum density of the current Do decreases from the surface in an exponential ratio and is equal to $\frac{Do}{2.7}$ at a depth a (called depth of penetration) and to $\frac{Do}{2.7^2}$ at double that depth. The

total intensity remaining sinusoidal lags one-eighth of a phase behind the surface current (which is in phase with the electro-motive force) and is the same as though the current were uniformly distributed, with the same density as at the surface, throughout a thickness of terrain equal to $\frac{a}{\sqrt{2}}$.

The depth of penetration, already defined, is given by the formula:

$$a = 500\sqrt{\rho T}$$

in which a is given in meters, ρ in ohms per meter cubed, and T in seconds.

In practice, the electrical current is sent into the soil between two points which are not at infinite distance; the penetration is then greater than that given by the above formula. Nevertheless, the skin effect hinders the use of high frequencies, and even of audiofrequencies in every problem requiring a great depth of investigation or concerned with very conductive ground.

Induction Phenomena

On these phenomena of skin effect are superimposed, in the case of a noncontinuous current in the ground, the phenomena of induction between the current circuit, formed of the insulated line AB and its ground return, and the searching circuit formed by the line MN and its supposed return circuit in the ground. These phenomena are practically impossible to compensate because the circuits contain a portion, the ground return, whose form, as we have seen, is continually variable.

For these two reasons, it is necessary, in order to obtain readings at great depths, to use direct current and to measure the drops of potential only after this current has become stable, or else to use alternating currents of very low frequency. By taking account of the preceding observations, in setting up the measuring apparatus, it is possible to make measurements to great depths, utilizing currents of the order of only one ampere.

As a matter of fact, in 1928, the authors carried out methodical observations with lines AB increasing to lengths of 125 miles, and thus achieved depths of investigation of several tens of miles. These measurements, involving blocks of ground containing more than 100,000 cubic miles, are evidently delicate, but the fact that they are actually possible and yield information about the constitution of the deep parts of the earth's crust, shows that the depths of investigation attainable by electrical prospecting greatly surpass mining requirements.

Electrical Studies of the Earth's Crust at Great Depths*

By Conrad and Marcel Schlumberger,† Paris, France

(New York Meeting, February, 1930)

In order to explore electrically a terrain composed of a succession of horizontal beds, a current of known intensity i is caused to flow between two grounds A and B, and the resultant drop of potential ΔV, caused by the resistance of the earth, is measured between two given points in the soil, M and N. In practice it is convenient to choose the two points M and N so that they lie on the line AB and their midpoint coincides with the middle O, of that line, although this is not imperative.

When the lengths L and l of the two lines AB and MN are varied, equations are obtained which may be solved for the resistivity ρ of the subsoil at various depths. Recall briefly that the formula

$$\rho = \frac{\pi}{4} \times \frac{\Delta V}{i} \times \frac{L^2 - l^2}{l}$$

gives the average resistivity, in the horizontal direction, of a layer of earth which extends to a depth[1] about equal to $\frac{L}{4}$. This elementary concept is adequate for numerous practical applications. Nevertheless, it must be said that, in many cases, a rigorous mathematical analysis may be made, provided it is always a question of homogeneous, superposed horizontal beds. The authors expect to publish, later on, appropriate methods of calculation for some specific cases.

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* Some technical details regarding the depth to which electrical studies may be carried by potential measurements were discussed in a previous paper before the American Institute of Mining and Metallurgical Engineers in February, 1929 (C. and M. Schlumberger: Depth of Investigation Attainable by Potential Methods of Electrical Exploration), consequently in the present communication they are merely summarized.

† Société de Prospection Electrique, Procédés Schlumberger.

[1] In this connection, the following observation is of interest. If the current is caused to flow not between A and B but between M and N, the drop of potential between A and B is exactly the same as is observed when the normal disposition is used and the drop measured between M and N. In other words, there is complete interchangeability between the two lines AB and MN, either one being available for passing the current into the ground and the other for the measurement of the potential drop. This mathematical theorem is always applicable, no matter what heterogeneities are found in the ground.

If the formations conserve, over a sufficient area, the character of regular, horizontal strata, there is no particular difficulty in operating with AB lines whose length L may reach 5 km. This corresponds, as explained above, to a depth of investigation of over 1000 m. This depth is sufficient for most of the problems encountered in practice.

In the spring of 1928 the authors undertook, in France, to push their studies to much greater depths. The purpose of this work was not to solve any particular problems of practical mining interest, but to make a scientific study of a considerable thickness of the earth's crust. They employed a series of lines increasing in length from 2 km. to 200 km. As mentioned, this means that the maximum depth of investigation was of the order of 50 km. This paper describes the difficulties encountered in this investigation, the means of surmounting them and the results obtained.

Difficulties Encountered

The use of very long lines requires access to existing circuits that are strung on poles, well insulated from the ground. For this purpose, the authors had recourse to the lines of the Administration des Postes et Telegraphes, which kindly allowed the use of its circuits. The telephone and telegraph lines utilized were not, of course, arranged in straight lines, and therefore did not permit the application of the simple scheme described above. This is of no real importance, however, since the calculations are not greatly complicated thereby.

The geological constitution of the terrain of experimentation is of much greater import. In order to pursue a rigorous reasoning and carry out exact calculations, it is imperative that the entire region under exploration be constituted of beds practically homogeneous and horizontal. With an AB line of 200 km., the ground involved is approximately a rectangle 250 km. long and 100 km. wide. It is evidently difficult to find, in France at least, a region of these dimensions where the formations present themselves as horizontal beds practically regular at the surface, to say nothing of the geological constitution of the deeper material, which is generally quite unknown.

Being faced with the impossibility of finding a really satisfactory area for exploration, the authors were guided by the following considerations. If there is a great difference between the resistivities of the deep-lying rocks and of the superficial ones, due to high pressure and temperature, and if in particular, the deep-lying, semifluid layers are very conductive in comparison with the surficial ones, the geological homogeneity of the latter is no longer of prime importance. Under these circumstances, the electrical differentiation of the crustal layers follows especially the isothermal and isobaric planes, which are approximately horizontal. It is advantageous to operate on great masses of

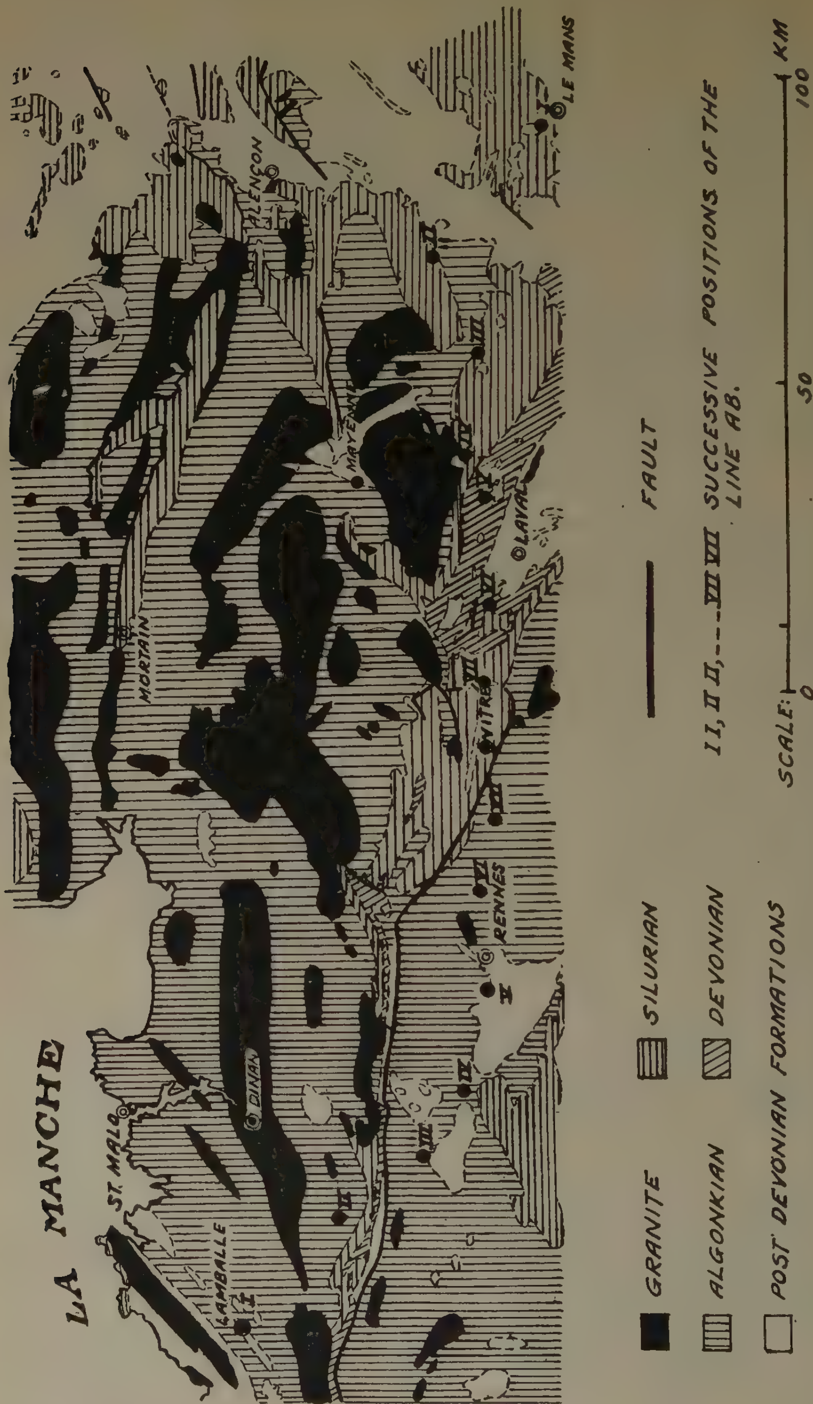

FIG. 1.—NEIGHBORHOOD OF VITRÉ, WHERE DEEP DETERMINATIONS OF RESISTIVITIES WERE MADE.

crystalline rocks (particularly granitic) or at least on very ancient formations, because probably there are fewer varying horizons between the surface and depth that are capable of introducing perturbations. It is certainly necessary to avoid the great basins of recent sedimentation, such as the Paris Basin, which form superficial plates, 1 to 2 km. thick, of very conductive beds quite different from the deep rocks of the lithosphere.

These various points, as well as the locations of the available telephone and telegraph lines, led the authors to choose Brittany as the region of experimentation. The center of the measuring arrangements was placed at Vitré (Ille et Vilaine) about halfway between Laval and Rennes. The lines *AB* paralleled the railroad from Laval to Rennes, extending beyond Rennes to the west and Laval to the east (Fig. 1).

The terrain consists of a great Silurian syncline with an approximately east-west axis and comprising Devonian and Carboniferous strata, bordered on the north and south by pre-Cambrian metamorphics and by granite. The beds of this fold are steeply tilted, which is very far from the ideal case of horizontal strata discussed above, and which is the only one lending itself to exact calculations.

The technique adopted must take into account the following three facts:

1. Practically, it is almost indispensable to limit the intensity of the current to about 1 or 2 amp. These figures could not be increased tenfold except through a rather important expenditure of energy, a high voltage and the generation of prohibitive perturbations on the neighboring telegraph circuits.

2. The skin effect of the current flowing in the ground becomes relatively important for the large dimensions of the lines used. At the moment of closing the circuit *AB* the current flows especially at the surface of the ground, because of mutual induction between the current paths, and it is only after a certain lapse of time that equilibrium is reached and the permanent régime of current flow is established. This interval, which is very brief with the usual short lines employed in ordinary electrical exploration, amounted to several seconds in the experiments with lines stretching 200 km. and more in length. This delay would have been even longer if the exploration had been in conductive soil instead of in the resistant rocks that were utilized.

In practice, the skin effect makes itself evident in the following manner. The drop of potential ΔV, between M and N has a certain first value ΔV_0 *immediately after closing the circuit* (concentration of the current at the ground's surface); it then diminishes progressively and in a short time reaches a limiting value of ΔV_1, which is that of the régime of equilibrium. Incidentally, it must be noted that to the skin effect (mutual reaction of the current paths in the earth) there is added the

capacity effect of the ground. Here it is a question of volume capacity derived from the small condensers made up of fragments of dielectric (grains of quartz, etc.) immersed in an electrolytic conductor (water absorbed by the rocks). These condensers store up a certain amount of energy at the instant of closing the circuit, which they *slowly* give back after the current ceases to flow. Naturally, the differences of potential registered between M and N take into account this whole series of complicated phenomena.

3. The points M and N, between which the drop of potential is measured, are necessarily distant from each other (10 to 20 km. in this case). As a result, the action of the telluric currents on such a long base assumes a considerable role and produces a much higher ΔV than the one to be measured. As these telluric currents are unstable, we find ourselves faced with the delicate problem of measuring a small variation, ΔV, superposed on a stronger electrical field which is, itself, irregular.

Principles of Experimental Technique Adopted

The method of overcoming these difficulties is described below.

In the first place, the importance of the skin effect led the authors to operate with direct current and to measure the potential only after the establishment of equilibrium. As a source of energy a battery of dry cells was utilized, which was capable of giving a maximum electromotive force of 200 volts and of furnishing up to 2 amp. without excessive polarization. The current was periodically reversed after a time T, which varied from 0.1 to 5 seconds.

The potentials were measured with a potentiometer provided wi h a very sensitive galvanometer. This potentiometer was reversed by the same commutator that served for the current, and with the same period, T. The potentiometer was not put in continuous action, however. It was connected in circuit only with a lapse of time t, after establishing the current flow, and was disconnected just before opening the circuit. By this means it was possible to avoid the direct inductive effect of the AB circuit on the line MN, an effect which is practically instantaneous. By varying the delay t, between commencing the current flow and connecting the potentiometer in the circuit, it is possible to study the law governing the establishment of potentials at the surface. Attainment of the permanent régime is retarded by the skin effect, as already explained. In practice, t varied from 0 to 4 seconds.

In order to eliminate the action of telluric currents a galvanometer with high damping effect and a long period of oscillation was used in the potentiometer. The readings were taken only after several minutes—that is, after numerous inversions of the battery current—so as to eliminate, by means of an average extending over a long period of time, the action of irregular telluric currents. It is well known, in fact, that a

rapid inversion ($\frac{1}{20}$ sec., for example) usually permits an easy elimination of earth currents. In the present case, the skin effect made it necessary to take a period of inversion 100 times longer and the elimination therefore required the recording of an average concerned with a considerable duration of time. This was realized by the employment of a slow galvanometer.

Naturally, the earth currents were studied beforehand and compensated, as completely as possible, by an auxiliary potentiometric arrangement.

Results Obtained

The results obtained were of two kinds:

1. As far as the skin effect was concerned, when closing the circuit AB on a constant voltage, the difference of potential ΔV, between the

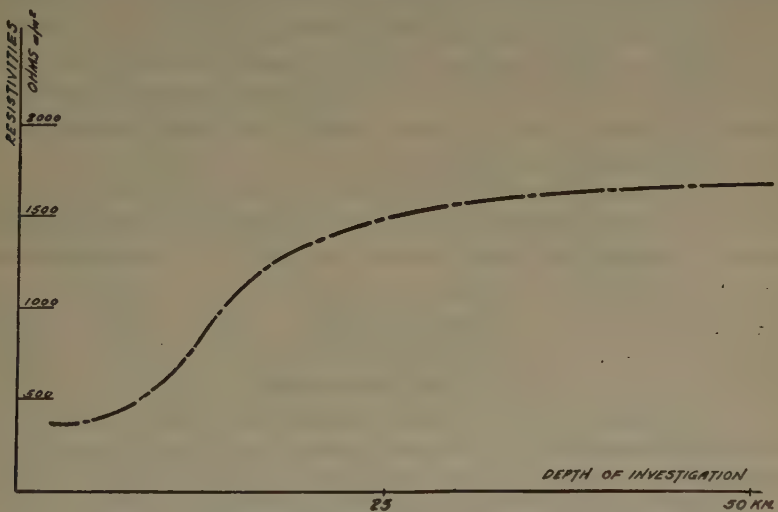

Fig. 2.—Electrical resistivities obtained at Vitré.

two points M and N, assumed an initial value ΔV_0, and then decreased according to a roughly exponential law until it attained a stable value ΔV_1. The time constant (duration of time for ΔV to fall to about $\frac{2}{3}$, or more exactly to $1 - \frac{1}{e}$ of its initial value) of this exponential law takes the form $\frac{K. AB^2}{\rho}$, K being a constant coefficient, AB the length of line and ρ the average resistivity of the soil.

In the authors' experiments this time constant has attained a value of about 2 sec. For K an approximate value of 2.5×10^{-7} can be assumed, AB being expressed in meters and ρ in ohms per meter-meter square. This formula makes it evident that these investigations cannot be pushed to great depths on conductive ground (resistivity less than 10 ohms, for example) because the time constant would reach a prohibitive value.

2. From the point of view of the resistivity of the ground, we found that the superficial formations (500 m. thick) had a resistivity of the order of 400 ohms per meter-meter square at the Vitré station. This value increased progressively as the depth of investigation was augmented and reached a maximum of 1800 ohms for depths in the neighborhood of 20 to 30 km. and more (Fig. 2).

The authors had expected to find a lowering of resistivity at very great depths, due to the rise in temperature of the rocks, but this was not the case. Perhaps it will be necessary to reach much greater depths in order to discover such a diminution.

Résumé

The authors' experiments at Vitré in 1928 should be considered as only a first attempt to measure, by surface potentiometric observations, the resistivities of deep-lying formations of the earth's crust. Although the figures obtained give only an order of magnitude, they demonstrate the possibility of solving the problem. It must be noted that a small expenditure of energy, not over a few hundred watts, suffices to electrify a volume of ground containing over one million cubic kilometers so as to produce a measurable drop of potential.

The authors are convinced that these studies should be resumed and enlarged, since they are one of the few possible ways of investigating the deep regions of the earth's crust.

DISCUSSION

(*Allen H. Rogers presiding*)

A. S. Eve, Montreal, Que.—There is one suggestion I have to make—that we should adopt a common method of expressing resistivity. Today we have had three different samples. I expressed mine as ohm-centimeter, which I claim to be correct. Schlumberger writes ohm-meter-meter. Sundberg uses ohm-centimeter cube, which I believe to be wrong. We may as well settle the point and get it right.

The resistance is equal to the resistivity times the length divided by the area; therefore the resistivity is equal to the resistance multiplied by the area and divided by the length. Now this is not centimeter squared or cubed. So resistivity is ohm-centimeter, or ohm-meter if you like, but not ohm-meter square, or ohm-centimeter cube.

S. F. Kelly, New York, N. Y.—Dr. Eve misunderstood the expression in Professor Schlumberger's paper. It is ohms per meter per meter square.

A. S. Eve.—The paper does not say that.

S. F. Kelly.—Because it is ohms meter, then a line, meter square. That is the correct mathematical expression.

A. S. Eve.—Why put in the two meters?

S. F. Kelly.—Because the resistivity is concerned with the volume; it corresponds to the resistance of a cube of ground, which is one meter on a side.

A. S. Eve.—If you double the linear dimensions you have to multiply by two merely. That is why we do not want to put in the cube at all.

A Uniform Expression for Resistivity*

By Sherwin F. Kelly,† New York, N. Y.

The need for geophysicists to adopt a uniform mode of expressing the electrical resistivity of geological formations has been stressed by Dr. A. S. Eve.[1] The present paper is to emphasize the point he raised, to bring the weight of disinterested authority to bear on the matter, and to urge the adoption henceforth, save in exceptional cases, of a standard expression for resistivity; all in the hope that the needed reform will thus actually be brought about.

The general definition of the resistivity of a material is the resistance in ohms between two opposite faces of a unit cube of that material. No account need be taken here of the cases in which the unit volume is not a cube, and the units of length and cross-section are different, encountered principally when dealing with metallic conductors such as wires. The idea of the resistance of a unit volume of earth or rock has found expression in geophysical literature in a number of forms and with a variety of units. Fortunately, the metric system has been adhered to in most cases, so ordinarily the meter or the centimeter, usually the latter, has been the measure chosen. The resistivity is then the resistance of a cube of the given rock, or soil, one centimeter on an edge. This is far from always being the same thing, however, as the resistance of a cubic centimeter. A long thin wire could contain a cubic centimeter and have a high resistance, whereas the same volume formed into a flat plate would have an entirely different, and quite low, resistance between faces. Consequently, it is not proper to express resistivity in ohms per cubic centimeter, inch, etc., as has been done occasionally by some writers.

In an effort to emphasize the idea of cubic volume, some geophysicists, the writer among them, have used the phrase, ohms per centimeter cubed. This is a graphic way of picturing the measure, but is not in favor among highly qualified authorities. In this connection, an appeal was made to the U. S. Bureau of Standards for advice, and the Director, George K. Burgess, kindly replied in detail as to the practice adopted by his Bureau: " . . . such expressions as 'ohms per centimeter cubed' or any similar expressions are objectionable, because they certainly carry the implication that the resistivity is found by dividing ohms resistance by the volume in cubic centimeters." Actually, of course, resistivity is found

* Scheduled for presentation at the New York Meeting, February, 1932.

† Low and Kelly, Consulting Mining Engineers, Geologists and Geophysicists.

[1] See page 140.

by multiplying the ohms resistance by the cross-section, and dividing by the length, of the conductor under consideration. This follows naturally from the law that the resistance of a substance is proportional to its length, and inversely proportional to its cross-sectional area, which may be written

$$R = \frac{L}{A} \times \text{Constant.}$$

The constant is the resistivity of the material, R is the resistance in ohms, L the length, and A the cross-section. Therefore,

$$\text{Resistivity, or } \rho = \frac{A}{L} R.$$

To bring out this relationship, some authors have used the rather cumbersome phrases: ohms per meter per meter square; ohms per meter meter square; ohms per centimeter per square centimeter; ohms/cm./cm.^{2} This last expression provides the clue to a desirable simplification. Writing it out thus,

$$\frac{\text{ohms}}{\text{cm./cm.}^2}, \text{ and simplifying, ohms} \times \frac{\text{cm.}^2}{\text{cm.}},$$

the final expression becomes, ohms × centimeters. Resistivities, therefore, can be expressed as ohm-centimeters, ohm-meters, etc.

This is the form favored by Dr. Eve, and adopted by the U. S. Bureau of Standards. The Director of the Bureau, in the letter referred to, says, "for uses such as soil measurements the 'ohm-centimeter' is very convenient, since values expressed in it will usually range from a few hundred to a few thousand units. We have therefore regularly used this unit in our recent work on soil resistivity. It would, of course, be entirely logical if one wished to do so to use a similar unit based on the resistance of a cubic meter rather than a cubic centimeter of material."

For geophysical work, ohm-meter would be preferable. Although the values in ohm-centimeters are not large when working on soil measurements, they become unwieldy when the electrical characteristics of rocks are being studied. Sandstones, for example, may show resistivities around 3000 to 40,000 ohm-centimeters when moist, and up to a million, and more, when dry. Some shales have values of nearly 15,000 ohm-centimeters, conglomerates of about 130,000, and lavas in the millions of ohm-centimeters. Glacial drift, even, will vary from about 500 ohm-centimeters to nearly 400,000 ohm-centimeters. To convert these to ohm-meters, it is necessary only to divide by 100, the number of centimeters in a meter, as can readily be seen by examining the formula given above.

The figures thus obtained are smaller, and therefore more readily handled in writing, speaking and calculating. For example, in ohm-

meters, the resistivities of the sandstones mentioned above will be from 30 to 400 for wet samples, and up to 10,000 for dry ones. Conglomerates mentioned would measure 1300, and the drift would vary from 5 to 4000 ohm-meters. The shales would have resistivities of 150 ohm-meters, instead of the more cumbersome figure of 15,000 ohm-centimeters. This linear relation between resistivities expressed in different units, by which the transformation from one to the other is accomplished by a simple division or multiplication, is brought out by the form of expression recommended.

It may be objected that the substitution of the ohm-meter for the ohm-centimeter will give cumbersomely small decimals when dealing with the resistivities of ore minerals that are metallic conductors. In connection with fairly pure specimens, this would be true, but it should be recalled that the comparison of the resistivities of such minerals serves no useful purpose in the present state of the development of applied geophysics. The degree of concentration of the sulfides in the rock affects the resistivities so profoundly that thus far no hope is extended of the possibility of distinguishing the ore minerals on the basis of their relative conductivities. The comparison of resistivities finds its great field of application in the differentiation of rock and soil formations, and for this purpose the ohm-meter provides the most easily handled unit.

A further advantage follows from the use of the meter as the chosen unit. The yard is very nearly equal in length to the meter, so one unaccustomed to the metric system can easily visualize ohm-meters as the ohms resistance of a cube one yard on an edge, as a rough equivalent. The frequent use of the cubic yard in rock and soil excavation makes this a more graphic and familiar equivalent than could be obtained for any other unit of the metric system. Ohm-inches, ohm-feet, and even ohm-centimeters, are less desirable units that can be reserved for special cases, such as small demonstrations in the laboratory and on the lecture table.

Therefore, because of its simplicity, the linear relation it emphasizes, and the easily handled figures that result from its use, the recommendation is urgently made that henceforth geophysicists adopt the expression and unit "ohm-meters" when speaking of electrical resistivities.

Mapping Oil Structures by the Sundberg Method

By Theodor Zuschlag,* New York, N. Y.

(New York Meeting, February, 1930)

Electrical prospecting is the art of exploring the structure of the subsoil in regard to conductivity variations and interpreting the results of such exploration as to their geological meaning.

Electrical prospecting has been applied during the last few years with outstanding success, not only to prospecting[1] for ore but also to structural studies.[2] Both applications have proved helpful to the geologist, and the latter application especially promises to become of still greater importance in the future.

The mapping of structures is the most important function of the oil geologist. Oil is lighter than water, therefore in water-bearing beds it will migrate upward until the migration is stopped by a bed impermeable to the oil. Geologic structures such as domes, anticlines, faults and terraces, offer good prospects in the search for oil accumulations and can be traced easily by electrical prospecting, provided they are composed of a series of sedimentary layers of which at least one possesses fair electrical conductivity and follows the general trend of the structure.

Conditions for Successful Mapping

The first condition, fair electrical conductivity, is met in almost any oil-bearing series, because generally the electrical resistance of a sedimentary bed is identical with the resistance of the more or less saline waters, filling partly or entirely the pores of the rocks.[3] Briefly, the electrical resistivity of a bed can be expressed for all practical purposes by the product $R \times P$, where R is the specific resistance of the waters in the rock pores and P a factor depending upon the percentage of water by volume in the rock. The specific resistance of any solution can be computed from the chemical analysis, being primarily a function of the

* Swedish American Prospecting Corporation.

[1] H. Lundberg: Recent Results in Electrical Prospecting. Geophysical Prospecting, A. I. M. E. (1929).

[2] E. G. Leonardon and S. F. Kelly: Some Applications of Potential Methods to Structural Studies. Geophysical Prospecting, A. I. M. E. (1929).

[3] K. Sundberg: Prospecting by the Swedish Geo-Electrical Methods. *Bull.* Inst. of Min. and Met. (1929).

K. Sundberg and H. Hedstrom: Communication sur les Recherches Electriques de Minérai et d'Huiles: II Congrès Internationale de Forage, Paris, September, 1929.

chlorine content of the waters. The percentage of water by volume differs for different types of rocks and must be determined for individual conditions. For geoelectrical purposes, all underground waters can be classified as surface waters, ground waters and subsurface waters. The specific resistance of these waters varies tremendously, on account of their difference in chemical composition. Very often subsurface waters possess less resistance than surface waters, while many surface waters are concentrated brines of extremely low resistance values, which sometimes are even lower than those for metallic ore deposits.

The second condition for a successful application of electrical prospecting to subsurface contour mapping—namely, that the layers must follow the general trend of the structure—is generally found true of beds at the greater depths below the surface. Below the ground-water level, waters of certain concentration seem always to follow certain beds, the configuration of which represents an accurate picture of the geological structure.[4] On the other hand, layers at the very surface or at shallow depth cannot be relied upon for this purpose, because the disposition of these beds might have no correspondence with the actual structure; therefore structural investigations by electrical methods should always be extended to the deepest conducting layers that can possibly and economically be reached.

Potential Electrical Methods

The known electrical methods for mapping structures can be divided into potential and electromagnetic methods. Potential methods are based upon the potential drop between two exciter electrodes caused by subsurface conductivity variations.[5] The exciter electrodes are connected to a source of either alternating or direct current and the potential drop is determined by means of one or more search electrodes, grounded in certain relation to the exciter electrodes and connected to a compensating or indicating arrangement. Observations are taken for various spacing of the exciter electrodes, the assumption being that, with increased spacing, an increased effect of the deeper layers upon the variations in potential drop takes place. The theory of these variations is based upon the electrostatic image theory of Maxwell, which can be developed, theoretically at least, to determine the variations due to

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[4] A. C. Veatch: Water Conditions in Northern Louisiana, Southern Arkansas and Adjacent Regions. Louisiana Geological Survey (1905).

H. E. Minor: Chemical Relations of Salt Dome Waters. Amer. Assn. of Petr. Geol. (1926).

[5] W. O. Hotchkiss, W. S. Rooney and J. Fisher: Earth-resistivity Measurements in the Lake Superior Copper Country. Geophysical Prospecting, A. I. M. E. (1929.)

F. W. Lee: Measuring the Variations of Ground Resistivity with the Megger. U. S. Bur. Mines *Tech. Paper* 440 (1928).

any number of parallel conducting sheets of different resistivity or to determine from the variations of a series of observations of systematic potential drop the location and resistivity of parallel horizontal conducting beds.[6] Practically, however, these methods and their variations are handicapped by the fact that, in many cases, the results represent the geological conditions at shallow depth only. The potential distribution is influenced relatively less by large changes of resistivity than by small changes of resistivity. Accordingly, almost every slight change in resistivity of the sedimentary beds will affect the potential distribution at the surface, thereby obscuring not only important changes in greater depth, but also making it difficult to obtain an accurate interpretation on account of the great number of minor reactions. Furthermore, the values of the potential readings measured are very much dependent on local conductivity variations, such as are encountered in staking the exciter electrodes or taking the readings of potential drop. Such local variations can easily become the cause of incorrect interpretation in regard to the deeper layers. These difficulties are increased if electrostatic potential theories are applied to readings obtained by alternating current exploration—for instance, by means of the ground megger—and these difficulties cannot be eliminated by the use of alternating current of low frequency.

In spite of these limitations, the potential methods have proved of great value for the solution of many structural prospecting problems, as, for instance, in foundation problems[7] and in determining the contact between different formations. The Schlumberger resistivity method has also to be credited with the discovery of salt domes in Rumania and Alsace.[8]

Electromagnetic Method

The first successful electromagnetic method for structural studies was developed by Karl Sundberg about five years ago. This process consists in setting up a primary alternating magnetic field and observing at the surface the resultant field components or field vectors caused by the reaction of conducting sheets of different resistivity upon the primary field. The reaction is due to the fact that any alternating magnetic field induces secondary alternating currents in any conducting medium within its range, and that these secondary currents set up secondary alternating magnetic fields which in turn influence the exciting field.

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[6] W. Weaver: Certain Applications of the Surface Potential Method. Geophysical Prospecting, A. I. M. E. (1929).

Hummel: *Ztsch. f. Geophysik* (1929) **Heft** 3 to 6.

[7] I. B. Crosby and E. G. Leonardon: Electrical Prospecting Applied to Foundation Problems. Geophysical Prospecting, A. I. M. E. (1929).

[8] G. Carrette and S. F. Kelly: Discovery of Salt Domes in Alsace by Electrical Exploration. Geophysical Prospecting, A. I. M. E. (1929).

The primary field may be produced by an electric current flowing through an insulated copper cable connected to a source of alternating current and laid out in a large rectangle; for instance, 6000 by 2500 ft. The resultant horizontal and vertical field vectors are measured along transverse lines at several distances from the cable, the transverses crossing the cable at regular intervals. The measuring apparatus consists of a search coil of several hundred turns of copper wire connected to a compensating arrangement by means of which the electromotive force induced in the horizontally or vertically placed search coil is measured in complex values. The arrangement is calibrated in microgauss per ampere primary current (one microgauss = one millionth part of a gauss) and the real and imaginary components of the resultant field strength are measured. The designations "real" and "imaginary" indicate that the corresponding components of the field vectors are either in phase or 90° out of phase compared with the alternations of the primary current.

The theory of the vector changes due to conductivity variations of parallel horizontal beds can be based upon the computation of current *line* images, just as the potential drop theory is based upon the computation of electrode *point* images. The fact that the vector changes are based upon the computation of image lines rather than upon that of image points results in the elimination of the disturbing effects of minor local conductivity variations, because their total influence upon the image lines and, in turn, upon the resultant field vector is negligible. Furthermore, the reactions due to the conducting beds are approximately proportionate to the specific conductivity of these beds. The electromagnetic method, therefore, does not reflect small conductivity changes in the same degree as the potential method, but, on the other hand, gives strong reactions from pronounced conductivity differences, such as are encountered between surface water, ground water and subsurface water levels in general. Accordingly, we have to consider in electromagnetic structure the prospecting of only a relatively small number of effective conducting beds which, by a standard nomenclature adopted in electrical prospecting, have been classified as surface conductors, intermediate conductors and subsurface conductors, the last ones being, of course, the most important for geological purposes.

The problem to compute the influence of these conductors upon the resultant magnetic field at the surface has been solved completely and the theory and elaborate mathematical formulas, with the help of which these computations are carried out, have been checked by numerous laboratory experiments and extended field tests. The final solution of the problem characterizes the different conducting layers in regard to their depth t below the surface and in regard to their so-called induction factor p. The value of the induction factor is dependent on the frequency of the alternating current and the effective thickness and specific resist-

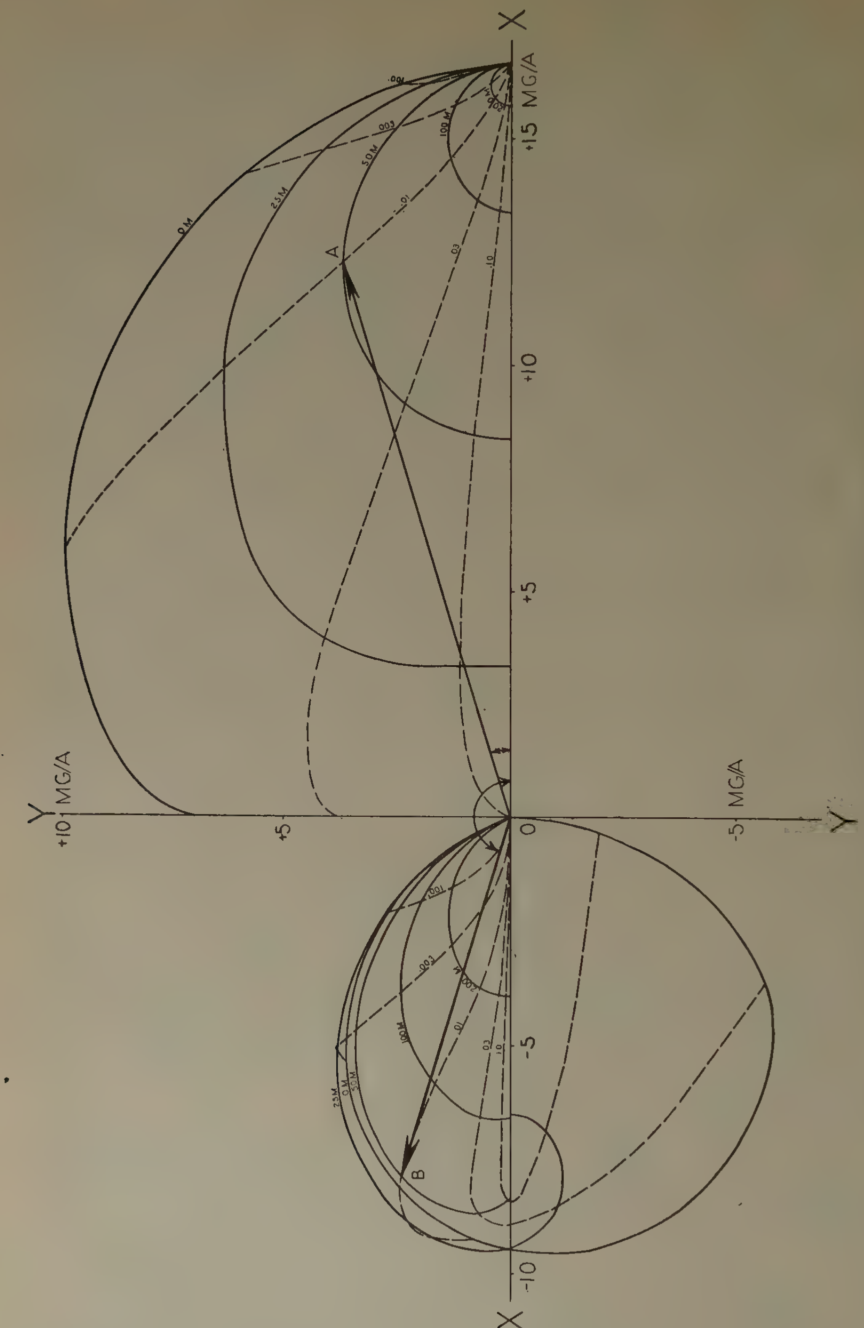


FIG. 1.—VECTOR DIAGRAM OF ELECTROMAGNETIC SURVEY.

ance of the different layers. In other words, the induction factor is the individual mark of the different electrical layers and can be varied at will by changing the frequency.

Assuming a single horizontal conducting sheet below a long straight current-carrying cable stretched parallel to the conducting sheet, all possible vectors of the resultant horizontal as well as vertical field components at a certain distance from the cable can be represented by so-called vector diagrams. Such a diagram is illustrated schematically in Fig. 1. The abscissa axis $X'OX$, called the real component axis, contains the parts of the field vectors in phase with or opposite in phase to the primary current. The ordinate axis $Y'OY$, called the imaginary component axis, contains the parts of the field vectors which are either 90° leading or 90° lagging in regard to the phase of the primary current. The axes are divided into positive and negative microgauss. The solid-line curves represent lines of constant depth and the broken-line curves represent lines of constant induction factor. The right side of the diagram contains the vectors of the vertical field component, while the left side contains those of the horizontal field component. The side of the vertical vectors is larger than the side of the horizontal vectors, due to the fact that, in this case, the vertical component represents the geometrical sum of the primary and the secondary fields, while the horizontal component is due to the secondary fields only. The diagram is valid for fields of all frequencies without radiation component. Diagrams have been computed for all standard distances.

The practical meaning of the diagram is the following: Assuming a single conducting sheet of a certain induction factor p at a certain depth t below the cable, then the resultant vectors of the vertical and horizontal field components are represented by the lines OA and OB, which connect the origin of the diagram with the intersection points A and B for the assumed p and t values. The length of the lines is called the amplitude of the vectors and the angle they form with the real component axis their phase or phase displacement. The amplitudes as well as the phase displacements AOX and BOX are simple geometrical functions of the coordinates of the two intersection points. The coordinates, in turn, are identical with the complex readings of the compensating arrangement. Therefore, if only one conductor is present, it is evidently possible to determine the depth as well as the induction factor with the help of the correct diagram and of only one observation of either the horizontal or the vertical field vector. Generally, however, we have to deal with a series of several conducting layers below one another. In this case, the resultant readings have no direct relations to the vector diagrams. The problem is then solved by a mathematical analysis of a series of readings for different components or different frequencies. Another complication is introduced by the fact that normally the readings

are not taken in the plane of the loop but at different elevations above or below the cable and must be corrected to a standard elevation.

It may happen sometimes that the specific resistance of the surface or intermediate conductors is extremely low or, in other words, that the shielding effect of the upper beds prohibits an effective reaction of the

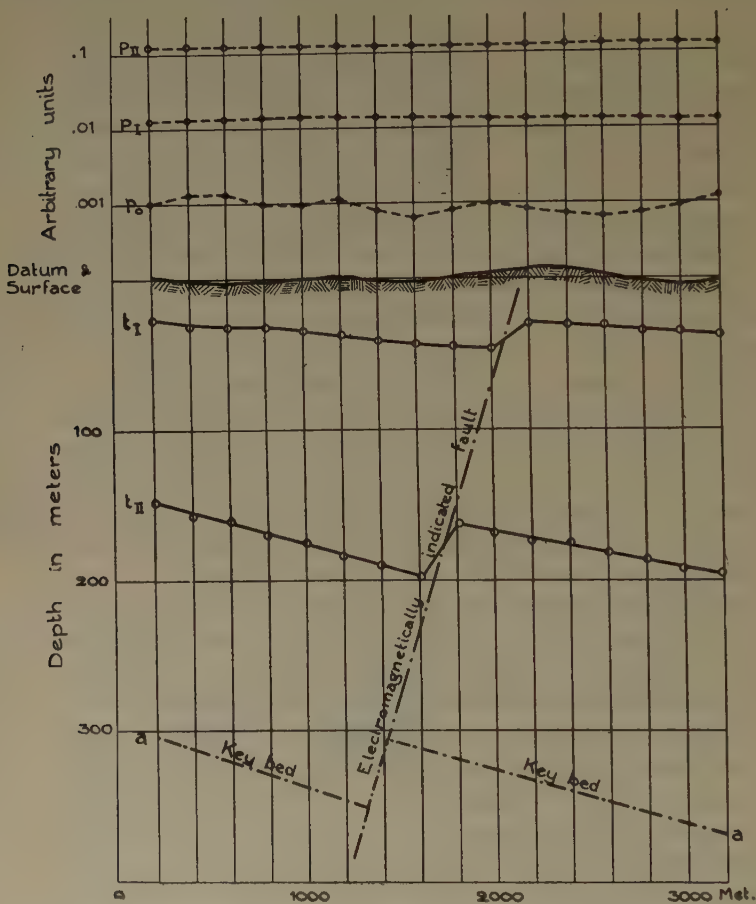


FIG. 2.—ELECTROMAGNETIC INDICATIONS ACROSS A FAULT.

lower conductors upon the resultant field at the surface. This condition can be remedied easily by a lowering of the frequency of the primary current, because this operation automatically lowers the induction factor and thereby the shielding effect of the upper layers. In this way, it is nearly always possible to penetrate to greater depths, even if the surface or intermediate conductor possesses low specific resistivity. The most suitable frequencies for electromagnetic structure investigations lie between 500 and 100 cycles. Higher frequencies are seriously handi-

capped by the correspondingly high shielding effect of the upper beds, while lower frequencies require more complicated measuring arrangements. The depth that can be reached depends on the individual geological conditions and the degree of sensitivity and accuracy to which the

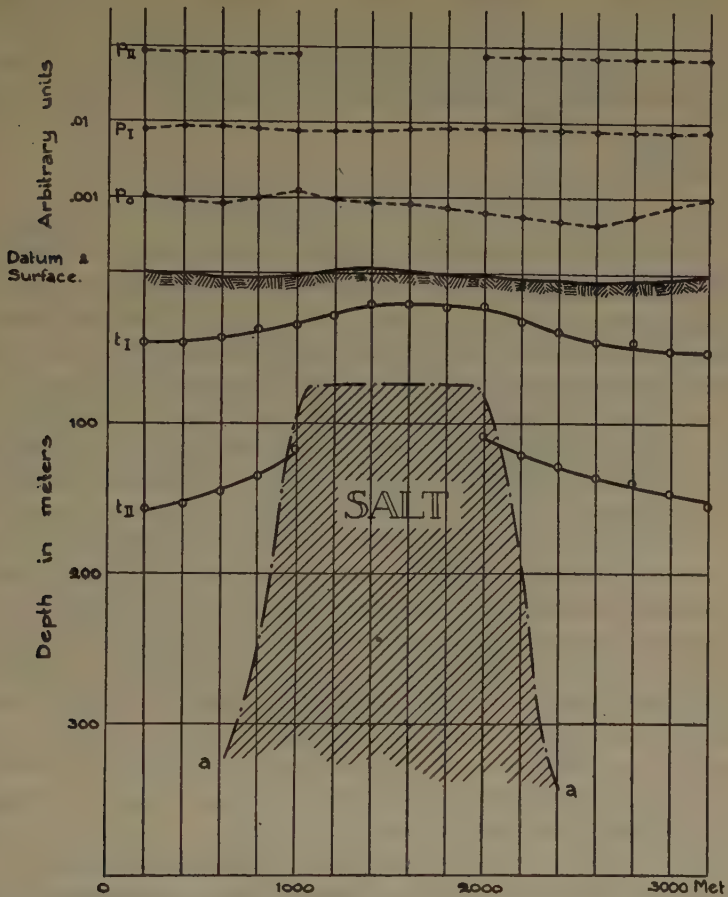


FIG. 3.—ELECTROMAGNETIC INDICATION ACROSS A SALT-DOME STRUCTURE.

compensating arrangement has been adjusted. Up to the present time, actual readings have been obtained from a depth of 1500 ft.

Before considering examples from the field, two typical electrical indications across certain basic geological structures are given in Figs. 2 and 3. The following abbreviations are used: $p_0 - p_0$ induction factor of surface conductor, $t_I - t_I$ and $p_I - p_I$ depth and induction factor of intermediate conductor, $t_{II} - t_{II}$ and $p_{II} - p_{II}$ depth and induction factor of subsurface conductor. Fig. 2 represents the indications across a fault. The fault is reflected electrically as a jump in

the depth lines of the two conductors, the induction factors remaining at constant value.

Fig. 3 represents the results of an electromagnetic investigation across a salt-dome structure. The intermediate as well as the subsurface conductor is pushed upward by the dome. The subsurface conductor disappears within the salt. In many cases it is preferable to present the final result in the form of a contour map of the subsurface conductor, the maps being easily constructed from the individual electromagnetic cross-sections.

Results of Some Actual Surveys

It is well known that during the last few years an astonishing number of salt domes have been discovered in the Gulf Coast region by geophysical, chiefly seismic, methods. The task of location has been surprisingly simple but, when discovered, the obtaining of information as to details, dimensions, etc., has not been so easy. In some cases, it has been obtained successfully by torsion balance surveys and the dome is sometimes detailed by seismic methods also. In a good many cases, both these methods, and particularly the torsion balance, fail to give the detail desirable to avoid the necessity of drilling a number of wells for the purpose of locating the flanks of the dome where oil will accumulate. The map of the Moore's field dome (Fig. 4) shows how electromagnetic prospecting gives these details.

A notable feature is the location of the fault *C-D*. The ability of the electromagnetic method to locate the faults, which are believed to be an almost invariable feature of salt-dome structure, is an important advantage over other geophysical methods.

Fig. 5 is a map showing the electrical indication of the Bruner fault. Cross-sections I to V, Fig. 6, are constructed to compare the electrical results with well data later obtained.

Here we have to deal with a general electrical structure survey. The oil occurs at depths of more than 2500 ft. below the surface. The oil, as well as the salt waters immediately associated with this oil deposit, lies far too deep to have any noticeable effect on the electromagnetic field created on the surface. The electromagnetic field that is measured is, however, largely influenced by electrically conducting clay beds that dip at a rate of about 4° toward the southeast and lie between thick sand beds.

Since there is no unconformity between the Austin chalk (in which part of the oil is accumulated) and the Wilcox, which forms the surface in this area, any sudden change in the position of the shallow clay beds must correspond, under normal conditions, to some sudden change in the deeper strata.

As shown in the five cross-sections, the electrical picture, which corresponds to a good-conducting clay bed found at a depth of about 500 ft., indicates over miles a slight dip toward the southeast. Suddenly

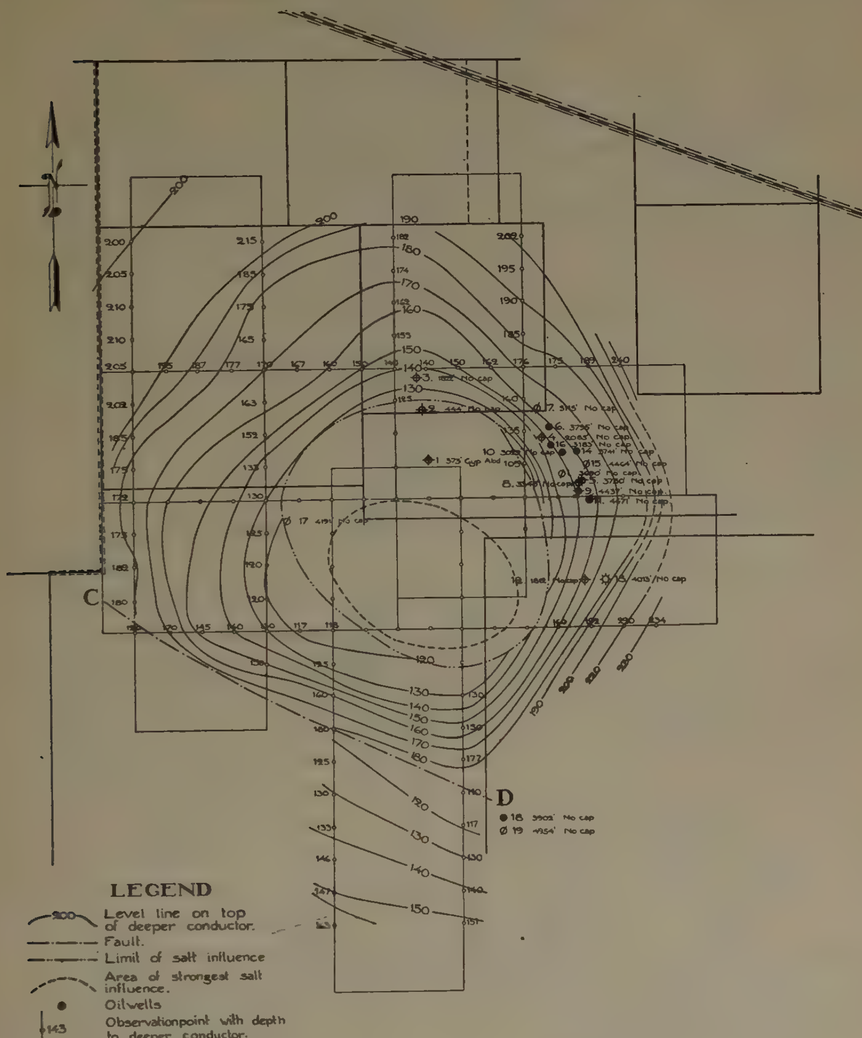


FIG. 4.—GEOLECTRICAL INVESTIGATION AT MOORE'S FIELD, TEXAS.

the electrical picture of the clay bed is interrupted and reappears a few hundred yards farther southeast on a considerably higher plane. These displacements in the electrical picture correspond with faults dipping 20° to 35° and extending to the Austin chalk and deeper strata.

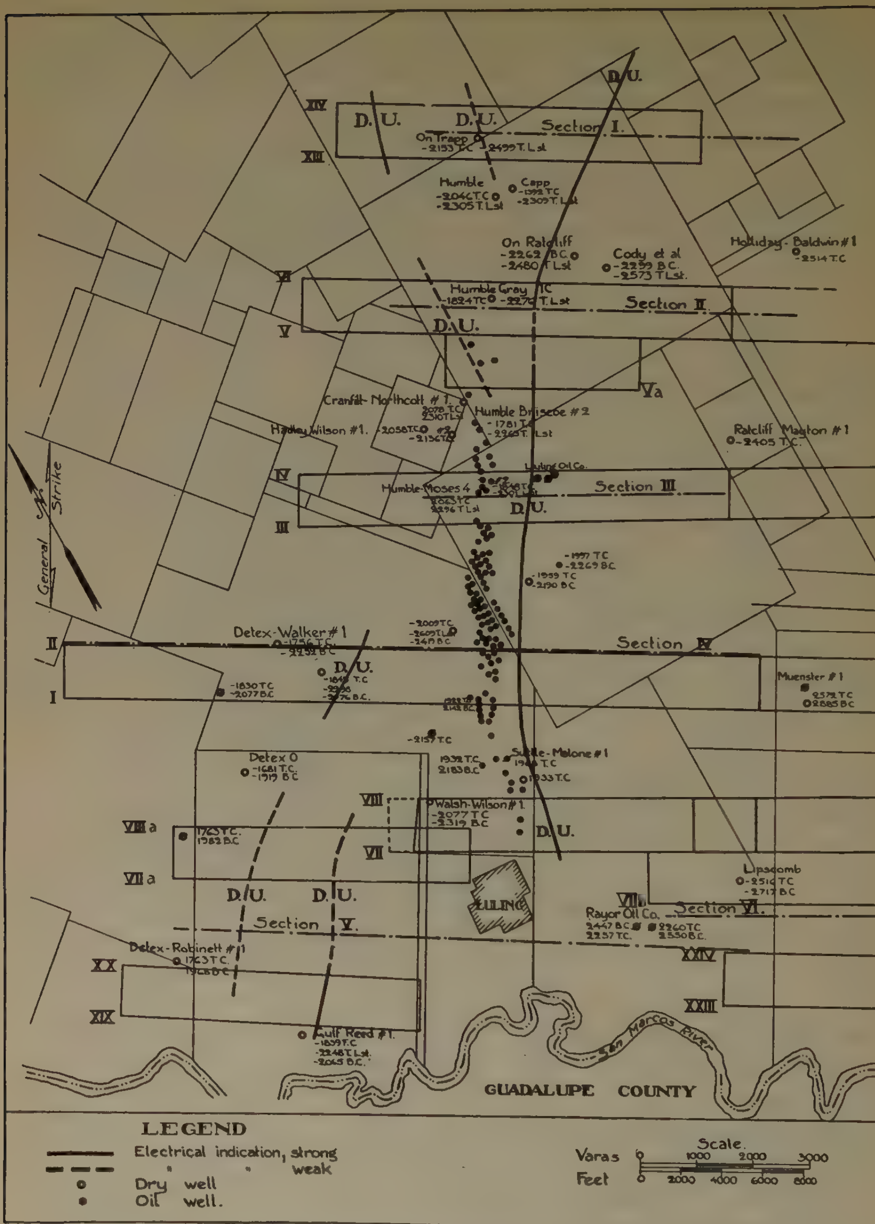


FIG. 5.—GEOLECTRICAL SURVEY ALONG BRUNER FAULT, TEXAS.

The distance between the electrical marker and the top of the Austin chalk, as determined by the drill holes is, on all five cross-sections, accurately 2000 ft. on both sides of the fault.

This fault was mapped electrically in October, 1928. When the electrical survey started, only one well, Joe Bruner Lutex No. 1, was producing from the Austin chalk. The fault was mapped in about one month and subsequent drilling proved that it is parallel with the area now producing, as shown on the map.

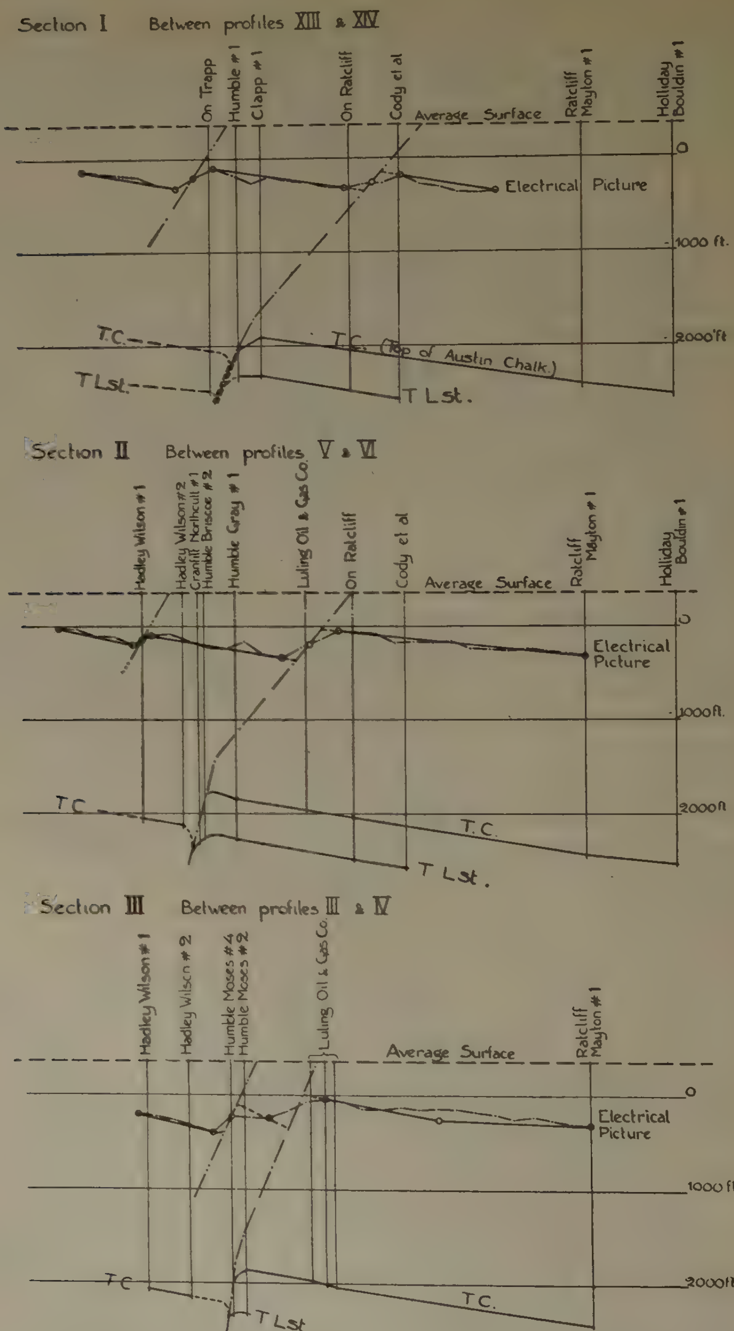
Conclusion

The Sundberg method has resulted, in a large number of instances, in determination of details of structure to a large degree of accuracy, probably greater than that so far attained by any of the other geophysical methods. In any structure the location of the first test is extremely important; indeed, in many cases, an error in its location amounting to only a few hundred feet results in failure. Knowledge of a structure in close detail is therefore of advantage and the electromagnetic method can be used to give as close detail as may be desired. The cost, of course, depends on the degree of detail demanded. The work is usually carried out by running profiles along which observations are made at 200-meter intervals. Where a rough topography or other difficulties do not prevent, one troup can run about 30 miles of profile per month, making observations at this interval and, if the profiles are spaced one mile apart, the cost works out to about 35 cents per acre. Recent improvement in field technique has bettered this rate materially, with the probability of doubling the speed. The depth of measurement attained so far is about 1500 ft., which means that elevations of an electrically conductive bed, an "electrical marker" as it may be called, which exists within that depth from the surface may be determined and used for delineation of the structure. Our work indicates that without exception usable markers occur within that depth.

Summary

The basic principle of electromagnetic prospecting in its application to geologic structure is the measurement of the electromagnetic field resulting from currents set up in strata which, due to their content of saline solutions, are of high electrical conductivity. This permits the determination of the depth of these strata and consequently their elevation can be determined at a sufficient number of points to give their dip and strike and thus determine the form of geologic structure present.

Following a brief non-technical discussion of the principles involved, diagrams of the electrical indications resulting from typical geological structures are given, followed by maps showing the actual results of electrical surveys over a salt dome and a faulted area, respectively. These results are then compared with the evidence derived from drilling and the electrical prospecting results are shown to be consonant with the geology.


Section I Between profiles XIII & XIV
On Trapp
Humble #1
Clapp #1
On Ratcliff
Cody et al
Average Surface
Ratcliff Mayton #1
Holliday Bouldin #1
0
Electrical Picture
1000 ft.
2000 ft
T.C.
T.C. (Top of Austin Chalk.)
T Lst.
T Lst.
Section II Between profiles V & VI
Hadley Wilson #1
Hadley Wilson #2
Cranfill Northcutt #1
Humble Briscoe #2
Humble Gray #1
Luling Oil & Gas Co.
On Ratcliff
Cody et al
Average Surface
Ratcliff Mayton #1
Holliday Bouldin #1
0
Electrical Picture
1000 ft.
2000 ft
T.C
T.C.
T Lst.
Section III Between profiles III & IV
Hadley Wilson #1
Hadley Wilson #2
Humble Moses #4
Humble Moses #2
Luling Oil & Gas Co.
Average Surface
Ratcliff Mayton #1
0
Electrical Picture
1000 ft
2000 ft
T.C.
T.C.
T Lst.

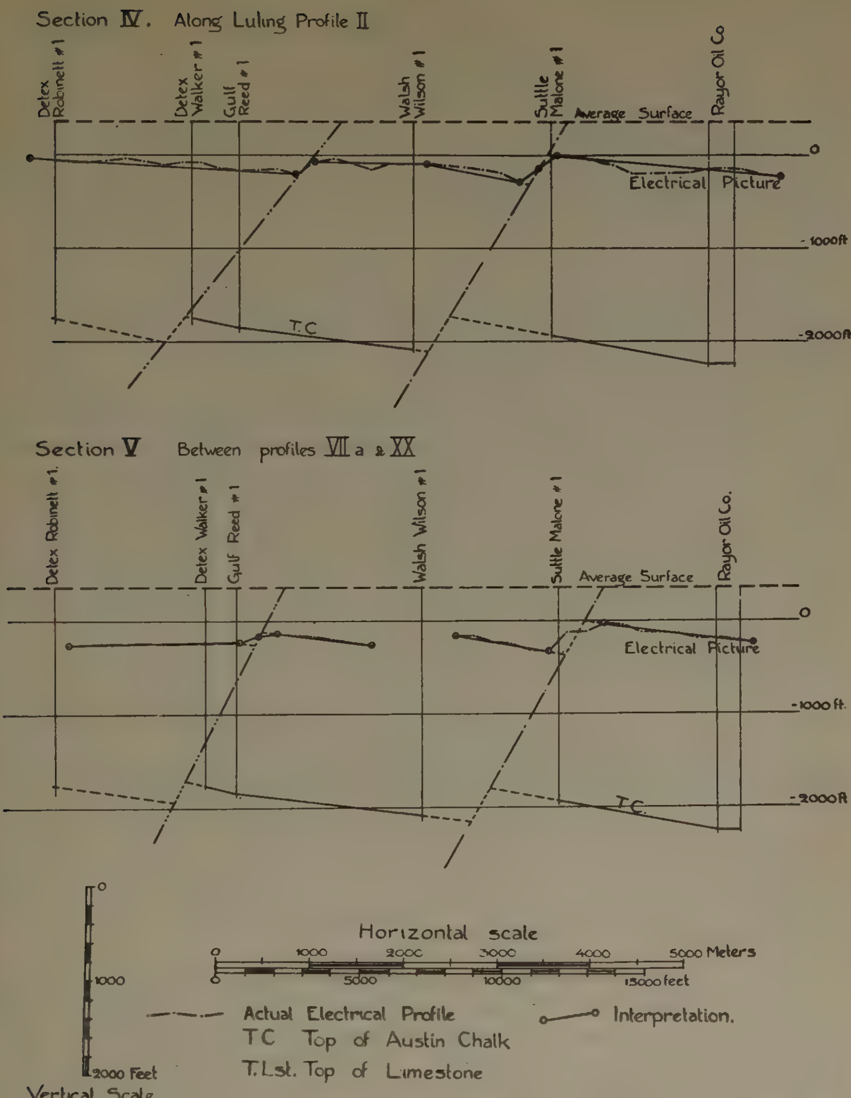


FIG. 6.—SECTIONS OF GEOLECTRICAL SURVEY ALONG BRUNER FAULT.

DISCUSSION

(Allen H. Rogers presiding)

D. Hager, Wichita, Kans. (written discussion).—Mr. Zuschlag's paper presents a method new to most American petroleum engineers and geologists, but a method, judged from results, that will prove of increasing importance in defining and in discovering structural features favorable for oil accumulations. The cross-sections and surface map outlining the Bruner fault near Luling, Caldwell County, Texas, are very interesting and indicate a highly developed technique. The accuracy in delineating such a fault and its changes in direction presents a method far superior to any geophysical or geological methods formerly used, especially in that immediate district where surface geology is at best very sketchy. The ability to define the change in direction of the fault line, thus giving closure, is important.

The interesting application of the Sundberg method in mapping Moore's salt dome, and defining the faulted condition, indicates a valuable method for defining saline domes, and, if such results can be obtained consistently, will supplant torsion balance methods in exploring for domes, and be used as an auxiliary to seismograph methods, especially in defining domes where seismographic methods give a "kick."

The Sundberg method seems to be an important auxiliary to geological work in giving detail in areas where surface geology shows some structural indications, but its usefulness should be more important when extended to areas covered by heavy sands such as the belt of Carrizo beds in southeast Texas, and in areas where no surface outcrops are found or where correlations are difficult.

Illinois, Michigan, western Kansas, western Oklahoma, western Texas, eastern and southern Texas, eastern New Mexico, Arkansas, Louisiana, Mississippi and Alabama all possess areas covered with gravel or late deposits which hide the earlier beds. The areas should furnish a wide field for the Sundberg method. However, in some of those states, notably western Kansas, western Texas and eastern New Mexico, salt beds cover the areas, and in some instances occur at depths of 500 to 1000 ft. and carry saline waters in places. Would the Sundberg method be applicable there, especially when the salt-water horizons vary in position to a marked degree? Also in those areas, the folding is far gentler than in east and south Texas, and low dip folds of Mid-Continent type predominate. An error of 50 ft. would be large enough to vitiate the value of such work. Such is not the case in eastern and southern Texas.

If a technique could be developed to cope with the local variations in saline basins, the Sundberg method would supplant the more expensive core-drilling methods used in parts of Kansas and Oklahoma; and would also largely supplant the other geophysical methods used on saline basins. Until further studies are made or published, the application of the Sundberg method would seem to be limited to areas where there are thicknesses up to 1500 ft. of sedimentary beds without salt measures at shallow depths. Before, however, accepting the Sundberg method in toto, any tests drilled on new structures defined by this method should be completed. Enough has been accomplished to indicate that electromagnetic methods will come into much wider use, and may largely supplant most of the geophysical methods now used, excepting, perhaps, seismographic methods in the Gulf Coast area.

Mr. Zuschlag has presented a clear and interesting exposition of a highly technical subject.

H. Hedstrom, New York, N. Y. (written discussion).—Mr. Hager, in his discussion of the applicability of the Sundberg method in areas where saline beds cover the surface, has hit upon a difficulty that in several cases was found very real during

field practice of the method. In the areas investigated in Texas, there exist in some places at or near the surface clay beds with exceedingly high electrical conductivity, which is due to the high saline character of the contained moisture. At an early stage of development of the Sundberg method, these surface conductors were found to possess a high screening effect, preventing the electrical investigation from penetrating to a depth such that the electrical key horizons could be expected to be conformable with the geological structure. This difficulty was overcome by the use of low frequency, which causes a deeper penetration by the electromagnetic field and in present-day field practice such layers of good conductivity at shallow depths, therefore, offer no serious obstruction.

The cases referred to by Mr. Hager, where salt water horizons are present which vary in position to a marked degree, would seem to make the application of the Sundberg method impractical, since the basic principle for this method is the conformability between the electrical key horizons and the geological key beds. If, therefore, the salt water horizons are the only electrical markers and do not conform with the geological structure, naturally the result of the electrical survey is erroneous. It must be remembered, however, that generally the electrical key beds dealt with by this electromagnetic method will consist of a series of fine-grained sedimentary beds of good conductivity of more or less thickness. Such beds, in all likelihood, will be of great regularity over large areas, and the occurrence of an occasional local thin conducting sheet, such, for instance, as a small lens of salt water, while possibly causing a local irregularity in the measurements, would have no appreciable effect on the general result of the electromagnetic mapping of the structure.

In regard to the accuracy of the method, which is referred to by Mr. Hager, it can be stated that the limit of error of the depth determination from one observation will be within 5 per cent of the average value obtained from a large number of observations. By taking several observations instead of one, naturally the accuracy is increased, and the same effect is obtained in working over structures of low dips by locating the observation points along the profiles closer together than would be necessary for a reconnaissance survey. In this way structure dips as low as 30 ft. to the mile have been determined by the electromagnetic method of Sundberg.

Absorption of Electromagnetic Induction and Radiation by Rocks*

By A. S. Eve,† Montreal, Que.

(New York Meeting, February, 1930)

This paper gives a brief summary of theory on radiation waves and describes experiments by the United States Bureau of Mines and by the Canadian Geological Survey at the Mammoth Cave, Kentucky. Dr. D. A. Keys, Dr. F. W. Lee and the writer did the experimental work, and Dr. L. V. King some of the theoretical work.

In prospecting for orebodies underground it is a usual practice to employ a loop of wire carrying an alternating current that induces an electromotive force in good conductors beneath the earth. The conductors are usually of such low resistance and impedance that currents of considerable magnitude may result in these orebodies or conductors, and these currents in turn induce an electromotive force in a detecting coil placed near the surface of the ground.

By using amplifiers it is possible so to enhance the effect of this electromotive force in the detecting coil that the currents in the underground conductors are detected in spite of the powerful effects coming directly to the coil from the primary loop.

In such cases distance or depth has primary importance, for the effects have to pass down to the orebody and the secondary effects induced therein have again to pass back through the ground to the observer's detection coil on the surface.

However, there is another factor to consider—absorption of the electromagnetic disturbances by the intervening soils and rocks. This absorption will vary with their composition.

Thus the main factors that determine the possibility of detecting ore underground are:

1. Its depth beneath the surface,
2. The magnitude of the conducting orebody,
3. The excellence of the conductivity of the orebody as compared with that of the surrounding strata,
4. The frequency of the alternating current,

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* Published by permission of the Director, U. S. Bureau of Mines and of the Director of the Geological Survey of Canada.

† McGill University.

5. The loss of effect by absorption, due to the properties of the strata that overlie the orebody and extend to the earth's surface. This absorption depends upon

a. The frequency of the alternating current, f
b. The conductivity of the strata, σ
c. Their permeability, μ
d. Their dielectric constant, K.

Radiation Waves

When the loop and coil are a few hundred feet or less apart it is usual to speak of induction between loop and coil. When however the sending station is at a great distance, such as a few miles or more, from the receiving apparatus, it is customary to use the terms "radiation," "wireless," or "radio." In all cases, however, the transmission through space is in accordance with Maxwell's equations for an electromagnetic field and is expressible in terms of electric and magnetic vectors; so that the distinction between induction and radiation is one of convenience rather than fact.

In the usual Heaviside notation we have

$$\text{curl } E = -\frac{1}{c}\dot{B}; \quad \text{curl } H = \frac{1}{c}(\dot{D} + 4\pi I)$$
$$\text{div } D = 4\pi\rho; \text{ div } B = 0$$

while $D = \epsilon E$, $B = \mu H$, $I = \sigma E$. Hence

$$\text{curl } E = -\frac{\mu}{c}\dot{H}$$
$$\text{curl } H = \frac{1}{c}(\epsilon\dot{E} + 4\pi\sigma E)$$
$$\text{div } E = \frac{4\pi\rho}{\epsilon}$$
$$\text{div } H = 0$$

The solution for a plane wave gives for the electric vector

$$E_x = Fe^{\frac{-2\pi\nu\kappa z}{c}} \cos\left(2\pi\nu t - \frac{nz}{c} + \gamma\right) \qquad [1]$$

with a similar expression but a different γ for the magnetic intensity H_y along the y axis. The advance of the plane wave is here supposed to be along the z axis. In the equation (1) κ is defined by

$$\kappa^2 = \frac{\mu}{2}\left(\sqrt{\epsilon^2 + \frac{4\sigma^2}{\nu^2}} - \epsilon\right)$$

and we are wholly concerned with the attenuation factor

$$e^{-\frac{2\pi\nu\kappa z}{c}}$$

where ν is the frequency, e the Napierian logarithmic base, c the velocity of light. The other symbols have been defined.

The permeability of the earth μ is very nearly equal to 1, and if we take the value of the dielectric constant to be about 10, we have

$$\epsilon = \frac{10}{4\pi} = 0.80$$

The resistivity may be taken as about 10^5 ohm-cm. or 10^{14} e.m.u., so that we should expect to put $\sigma = 10^{-14}$, but *we have to use electrostatic units* and write $\sigma = \dfrac{9 \times 10^{20}}{10^{14}} = 9 \times 10^6$, and this is the chief pitfall in the calculation.

If the wave length $\lambda = 30$ m., we have a frequency ν of 10^7 cycles per second and so

$$\kappa^2 = \frac{1}{2}\left[\sqrt{0.64 + \frac{4 \times 81 \times 10^{12}}{10^{14}}} - 0.80\right]$$

whence the attenuation factor becomes

$$e^{-\frac{2\pi \times 10^7 \times 0.765 \times 10^4}{3 \times 10^{10}}} \quad \text{or} \quad \frac{1}{10^7}$$

and this is the reduction of amplitude after penetration through 100 m., when the wave length is 30 m.; the amplitude becomes virtually nothing, as our experiments in Mount Royal tunnel[1] proved. Similar calculations for each frequency have been made, and the results are listed in Table 1 for $z = 100$ m. $= 10^4$ cm., and with permeability 1, dielectric constant 10, resistivity $= 10^5$ ohm-centimeter. It is seen at once from the column on the right that short waves should not theoretically penetrate 100 m. (328 ft.) of rock, that audiofrequency waves should suffer but 1 per cent. reduction through the same thickness, and that 10-kc. radio waves should have fourfold better transmission than 100-kc. waves.

TABLE 1.—*Penetration for Different Frequencies*

λ, Meters	Frequency ν	Kilocycles	κ	Attenuation per 100 M.	Percentage Transmitted
30	10^7	10,000	0.765	$1/10^7$	almost 0
300	10^6	1,000	2.93	1/500	0.2
3,000	10^5	100	9.50	1/7.4	13.5
20,000	15,000	15	24.5	1/2.15	47
30,000	10,000	10	30.0	1/1.87	53
600,000	500	0.5	134.5	1/1.01	99

[1] *Proc.* Inst. Radio Engrs. (1929) **17**, No. 11.

An alternative treatment may be briefly considered. When radio waves are moving horizontally over the sea with the plane of the wave vertical there is little penetration—only a few feet, in fact—into the conducting sea water, and the electric vector is vertical. Over dry land the case is different, for the base of the wave drags behind. There is a horizontal as well as a vertical vector, and as these are not in phase with each other, there is elliptical polarization, with the shape of the ellipse different in the air and in the ground. A full discussion was originated by Zenneck.[2]

The particular point we need to consider is the depth to which radio waves, say from a broadcasting station a few hundred miles away, will penetrate into the earth. The value of a quantity d is determined from a relation

$$d = A \sin \psi$$

where

$$A = \frac{\sqrt{2\pi f}}{3 \times 10^{10}} \frac{\sqrt{s^2 + 4\pi^2 f^2 k^2}}{\sqrt[4]{s^2 + 4\pi^2 f^2 (k + k')^2}}$$

and

$$\psi = \varphi_1 - \frac{\varphi_2}{2} - \frac{\varphi}{2}$$

where

$$\varphi = 90°, \tan \varphi_1 = \frac{2\pi f k'}{s}, \tan \varphi_2 = \frac{2\pi f(k + k')}{s}$$

Here f is the frequency, s is the conductivity of the rock in electrostatic units,

k is (the dielectric constant of air) / 4π

and

k' is (the dielectric constant of rock) / 4π

Having found d we state that $\frac{1}{d}$ is the depth at which the effect is reduced to $\frac{1}{e}$ times the surface value, where e is the Napierian base and $\frac{1}{e}$ has the numerical value 0.367, so that the waves have then fallen to about 37 per cent. of their surface value. All this is rather complicated, therefore the calculations were made and then the *thickness of rock required to reduce the surface effect to half value* was determined.

For example, suppose that the dielectric constant is 8 and the resistivity 10^5 ohm-centimeter. Then for 1000 kilocycles we see that at a depth of 13 m. the effect is reduced to ½ value, and therefore at 26 m. to ¼ value, at 39 m. to ⅛, at 52 m. to 1/16, at 104 m. to 1/256, and so on.

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[2] A. P. M. Fleming: Principles of Electric Wave Telegraphy and Telephony, 2d ed., 623, 730–744. New York, 1919. Longmans, Green & Co.

The depth attained is very sensitive to frequency, and also is dependent on the dielectric constant and conductivity. In Table 2 and Fig. 1 the results of these calculations are stated and shown.

Z_1 is the depth to half value in meters for vertical plane waves traveling horizontally, when the dielectric constant is 8 and the resistivity 10^5 ohm-cm.; Z_2 is the corresponding result for dielectric constant 4 and resistivity 10^6 ohm-cm. Again, V is the depth to half value for

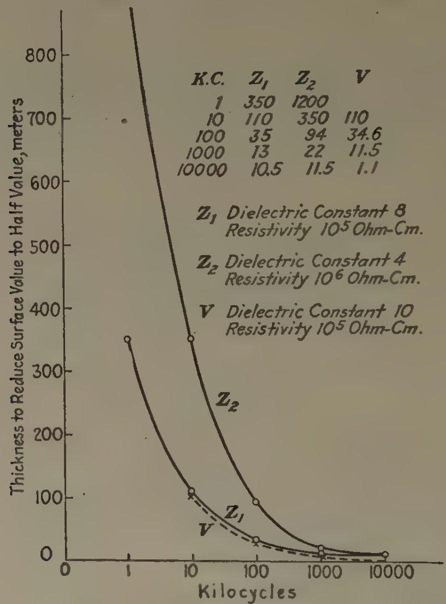

FIG. 1.—PENETRATION OF RADIO WAVES IN EARTH, SHOWING THICKNESS IN METERS TO HALF VALUE (Z_1, Z_2 FOR HORIZONTAL, V FOR VERTICAL PLANE WAVES).

horizontal plane waves plunging vertically into the earth, when the dielectric constant is 10 and the resistivity 10^5 ohm-cm. The low value in this case for 10^7 cycles per second is remarkable.

TABLE 2.—*Depth of Penetration of Radio Waves*

Frequency	1,000	10,000	100,000	1,000,000	10,000,000
Kilocycles	1	10	100	1,000	10,000
Wave length, meters	300,000	30,000	3,000	300	30
Z_1, meters	350	110	35	13	10.5
Z_2, meters	1,200	350	94	22	11.5
V, depth to ½ value		110	34.6	11.5	1.1

Z_1 is the depth in meters at which the surface effect is reduced to *half value* when the dielectric constant is 8 and the resistivity of the rock is 100,000 ohm-cm., and Z_2 is a similar depth when the dielectric constant

is 4 and the resistivity is 1,000,000 ohm-cm. Here Z_1 and Z_2 are for vertical plane waves traveling horizontally; but V is for a horizontal plane wave traveling vertically when the dielectric constant is 8 and the resistivity 100,000 ohm-centimeter.

INDUCTION

Case 1.—To calculate the mutual inductance between a small vertical loop and a small vertical coil beneath it, separated by a slightly conducting medium such as 100 m. of limestone and sandstone.

Solution by Dr. L. V. King

Let a current I flow in the primary loop and an induced current I' in the secondary coil (impedance Z) at a distance r from the loop. Let M_0 be the mutual inductance between loop and coil in air, and M be the mutual inductance between them in the medium (rock), where $M_0 = \frac{nn'AA'}{r^3}$, n, n' are the number of turns, and A, A' are the areas.

Then

$$I' = \frac{MIp}{Z} \text{ where } p \text{ is } 2\pi \text{ times the frequency,}$$

and

$$Z = \sqrt{R^2 + L^2p^2}$$

This result holds only when the plane of the loop passes through the plane of the coil, and if the conducting medium is of infinite extent.

When a separating plane exists between two media, one of finite and the other of infinite resistance, the problem becomes very complex. The above, however, gives a fair approximation if the resistivity of the rock is not too large.

The effect of the medium is to introduce an attenuating factor multiplying M_0 so that $M = M_0 F(r)$ where $F(r)$ is a function of the distance r between the loop and coil.

This function is not a simple exponential function but is given by

$$F(r) = \frac{e^{-\frac{r}{c\sqrt{2}}}}{(\cos\alpha - \sin\alpha)},$$

where c is the attenuation constant of the medium given by

$$c = \frac{\sqrt{(\rho \text{ in ohm-cm.} \times 10^9)}}{\sqrt{8\pi^2 f}}$$

If

$$\rho = 10^5, f = 10^4, \text{ then } c = 110 \text{ m., and with}$$
$$\rho = 10^5, f = 10^5, \qquad c = 35 \text{ m.;}$$

while the auxiliary angle α is defined by

$$\tan \alpha = \frac{\dfrac{r}{c\sqrt{2}}}{1 + \dfrac{r}{c\sqrt{2}}}$$

Using this result of Dr. King's, $F(r)$ may be calculated as follows:

ρ, ohm-cm.	f, kilocycles	c, meters	$F(r)$
10^5	10	110	0.98
10^5	100	35	0.49

and this means in plain language that only 2 per cent. of the 10-kc. effect would be absorbed, but that 51 per cent., or about one-half the 100-kc. effect, would be absorbed in passing through about 100 m. (328 ft.) of rock of resistivity 10^5 ohm-centimeter.

Case 2.—A horizontal loop vertically above a horizontal coil in an infinite slightly conducting medium.

This closely approximates to a loop on the ground with alternating current in it, while reception is made with a horizontal coil straight beneath the loop in a cave or mine.

If I is the current in the loop, the induced current in the parallel coil straight beneath it is given by

$$I' = \frac{MIp}{\sqrt{R^2 + L^2p^2}}$$

where p is 2π times the frequency, M is the coefficient of mutual inductance, and L, R are the self-inductance and resistance of the receiving coil.

In these calculations the self-inductance of the coil has been ignored; therefore in future measurements the different turns of the receiving coil should be loosely coupled, or the coil tuned with capacitance.

The value of M has been calculated as

$$M = \frac{2AA'}{z^3}(1 + x)(1 + x^2)^{1/2}e^{-x},$$

where $x = \dfrac{z}{c\sqrt{2}}$ and z is the depth of the coil beneath the loop, while c is again given by

$$c = \sqrt{\frac{\text{resistivity in e.m.u.}}{8\pi^2 \text{ frequency}}}$$

As to A, A', they are again the areas of loop and coil, in this case, however, already multiplied by the number of turns.

As c is an important constant in all these problems its values have been calculated and tabulated for two different resistivities.

$$\rho = 10^5 \text{ ohm-cm.} = 10^{14} \text{ e.m.u.}$$

and

$$\rho = 10^6 \text{ ohm-cm.} = 10^{15} \text{ e.m.u.}$$

These are stated in Table 3.

TABLE 3

Frequency........	10^3	10^4									10^5		10^6	10^7
Kilocycles........	1	10	20	30	40	50	60	70	80	90	100	110	1000	10,000
$c(\rho = 10^5)$, meters	354	113	80	65	56	50	47	42.5	40	37	355	34	11.3	3.55
$c(\rho = 10^6)$, meters	1127	354	253	206	177	158	149	135	127	117	113	108	35.5	11.3

The values given for c are in meters; the lowest row gives these values when the resistivity is 10^6 ohm-cm., and the row above gives the values of c when $\rho = 10^5$ ohm-centimeter.

It is interesting to note that the function $(1 + x)(1 + x^2)^{\frac{1}{2}}e^{-x}$ has a maximum when $x = 1$, its values being 1 when $x = 0$, and 1.04 when $x = 1$. This slight rise in its graph occurs when the depth is $c\sqrt{2}$. In the case when $c = 71$ m., the depth z would be 100 m., so that, when $e = 10^5$ ohm-cm., the best frequency for penetration through rock would be about 25 kilocycles.

The theories here given for radiation and induction form a guide with a view to future experiment and research.

EXPERIMENTS IN MAMMOTH CAVE, KENTUCKY

These experiments were made in June, 1929, under the joint auspices of the U. S. Bureau of Mines and the Geological Survey of Canada. A preliminary announcement of the work has appeared in *Nature* (Aug. 3, 1929) and in the *Proceedings* of the Institute of Radio Engineers (Nov., 1929). A full report will be issued by the Geological Survey of Canada and by the U. S. Bureau of Mines. It is sufficient, therefore, to give only a brief note of the work here.

The Mammoth Cave was selected by Dr. F. W. Lee on account of the entire absence of metallic conductors. The entrances were far removed from the stations selected for research. The exits were water-sealed by the Echo River. The overburden consisted of horizontal layers of sandstone and limestones about 300 ft. thick. The resistances of these materials were determined in place by the one-electrode probe method. The limestone had a resistivity of about 100,000 ohm-cm. and the sandstone of about 200,000 ohm-centimeter.

In River Hall, 300 ft. beneath the surface, we received Morse signals from about six long-wave stations bearing N. 65 E. Using a long horizontal antenna (300 ft.) we also received audible broadcasting of speeches and music from a loud speaker. The sending stations were Cincinnati (429 m., 700 kc.) 200 miles away N. 36° E.; Louisville (366 m., 820 kc.) 90 miles away S. 10° W.; and Nashville (461 m., 650 kc.) 100 miles away N. 36° E. These signals unquestionably came through the rocks.

Horizontal and vertical loops were placed on the surface of the ground and were stimulated with alternating current of frequencies varying by steps of 10 from 20 to 110 kc. The received voltage on the coils immediately beneath in the cave 300 ft. below was measured in microvolts, and the superiority of the lower frequency, and of the longer wave length was confirmed.

Finally audiofrequency (500 cycles) alternating current (2.3 amp.) was passed round a 10-turn loop (100 ft. dia.) of insulated wire resting on the earth's surface.

With a receiving coil and head-phones, without amplification, it was possible, 300 ft. underground, to find the point under the center of the loop, thus verifying our survey of the cave. On traversing the cave with the coil we were able to detect the effects of the loop 800 ft. away horizontally, so that the electromagnetic effects were passing through 900 ft. of rock.

These tests show, then, that Morse signals could be sent readily enough to large areas of a mine if those underground were supplied with such simple apparatus as a receiving coil and head-phones only. Of course familiarity with the Morse code would be necessary.

It is highly desirable that this work should be continued and that accurate measurements should be made of the strength of the carrier waves of various frequencies underground, with a well arranged program of signals from reliable sources. Our work last year, satisfactory as it was, might fairly be deemed of a pioneer character.

In conclusion we now have ready means of finding the resistivity of rocks in place, but for the equally important question of dielectric constant we have to rely on Ambronn's tables, obtained from rocks in Europe and copied from book to book. These values naturally diverge widely, depending on the water content. It is desirable that a quick method of measuring the dielectric constant should be evolved, so that t may be used for specimens freshly taken from the ground.

Practical Geomagnetic Exploration with the Hotchkiss Superdip*

By Noel H. Stearn,† St. Louis, Mo.

(New York Meeting, February, 1931)

To the successful functioning of the geomagnetic method of exploration in engineering and geological practice there are two prime prerequisites: the *measurability* and the *interpretability* of significant magnetic anomalies. A magnetic anomaly may be defined as a *permanent* distortion of the earth's magnetic field, in order to distinguish it from the distortions, either cyclic or irregular, which vary with time. So far as known, time fluctations have no directly assignable geologic significance in terms of rock formations, while anomalies may be considered to be directly caused by the difference in the magnetic properties within and between the formations of the substances that make up the earth's outer shell. As is effectively pointed out by L. B. Slichter,[1] the effects which these differences produce on the magnetic field may be extremely minute or extremely great, depending upon the magnetic properties of the formations. Thus the actual anomalies may range from the infinitesimal to a magnitude considerably greater than that of the earth's field.

The geomagnetic method of exploration has shared in the deluge of experimental effort which has lately swept geophysical prospecting to its present important place in engineering and geologic practice. But to date more of this effort has been expended upon the measurability of anomalies than upon their interpretability. The tendency has been to strive for the development of instruments and especially field techniques capable of measuring minute anomalies with a precision far beyond that for which there can be any hope of adequate interpretation other than as interesting phenomena having numerous possible causes. Such efforts, of course, are highly commendable in the field of so-called pure science, which includes the accumulation of undigested detailed observations and admits of interpretation by multiple hypotheses, but in engineering practice the factor of economy must enter. Those instruments and techniques are most desirable which can most rapidly measure the interpretable range of anomalies with the precision necessary

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* Published by permission of W. C. McBride, Inc.

† Consulting Geologist.

[1] L. B. Slichter: Certain Aspects of Magnetic Surveying. Geophysical Prospecting, A. I. M. E. (1929) 238.

for reliability of results. In exploration problems the goal is not the compilation of magnetic data, but the reconstruction of a geological picture.

The range of magnitude of anomalies now measurable extends from a fraction of 1 gamma[2] to over 100,000 gammas. The greater the magnitude of the anomaly, the less frequently it is likely to be encountered in nature, and generally speaking, the more easily it can be correctly interpreted in terms of geology. On the other hand, anomalies with a magnitude of only a few gammas are innumerable in nature and except in rare cases may have a confusing multitude of possible geological explanations. This does not mean that any anomaly can be automatically ruled out as valueless on the basis of its magnitude, but it does mean that the chances of usefulness of anomalies of the lowest order of measurable magnitude are in general negligible. Consequently, instruments which specialize in such refined measurements are the least applicable to practical geomagnetic exploration in regions where extrapolation beyond known geological factors is necessary.

Thus is the attention of the economic geologist and mining engineer focused on anomalies of the higher order of magnitude, measurable at the very least in tens of gammas. Instruments that can measure anomalies of this order of magnitude with speed and reliability are at a premium.

The ordinary dip needle is by far the most rapid of magnetic instruments in general use. It can be read by an expert operator in less than 15 seconds. But its sphere of usefulness is limited by its lack of sensitivity. The average response of 14 ordinary dip needles in good working order was found to be 1° change in instrument reading to about 343 gammas change in magnetic intensity. Obviously the instrument is applicable only to the higher range of usable magnetic anomalies.

Several fieldworthy instruments have been designed which are capable of measuring the range of useful anomalies whose magnitude is below that measurable by the ordinary dip needle. The Hotchkiss Superdip is perhaps unique among these in its economy of operation brought about by low first cost and high speed of manipulation; in combining the dual function of dipping circle and magnetometer in a single instrument; and in the equal facility with which it can be made to measure anomalies throughout the whole range of magnitude from those measurable in tens of gammas to those measurable in thousands of gammas. It is not recommended for use in connection with anomalies smaller than 10 gammas.

A description of the salient features of this instrument, together with examples of its application to various problems in the field of exploration, has been requested for the benefit of mining engineers and economic geologists, who now find themselves under the necessity of categorizing the multiplicity of geophysical methods and instruments available.

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[2] 1 gamma equals 0.00001 gauss; c. g. s. unit of magnetic force.

History

The Hotchkiss Superdip magnetometer has been evolved as a result of an attempt to carry over into the field of magnetic exploration in the range of anomalies of a low order of magnitude the outstanding advantages of speed and simplicity which are inherent in the ordinary dip needle. W. O. Hotchkiss, now president of the Michigan College of Mining and Technology, is responsible for the original conception. He hit upon the principle by which an instrument of the general type of the ordinary dip needle can be made to attain theoretically infinite sensitivity. The mathematical basis of this principle was confirmed by Gordon S. Fulcher, and the first model was built by J. P. Foerst, technician for the Department of Physics at the University of Wisconsin. After the first model had proved the workability of the principle, Mr. Hotchkiss was forced by his duties as State Geologist of Wisconsin to abandon the development of the instrument. In the fall of 1925, H. R. Aldrich, Assistant State Geologist of Wisconsin, took up the development; he directed the attention of the Gulf Oil Corpn. to the instrument. After some desultory experimental work by that company, the writer, a former assistant of Hotchkiss and Aldrich, introduced the instrument to W. C. McBride, Inc. of St. Louis, Mo., who proceeded to organize the efforts of those responsible for the development of the instrument, and to initiate a scientific program of research into the applicability of the instrument to geologic problems. The mechanical work was done by J. P. Foerst, the laboratory work by H. R. Aldrich, and the field work and general supervision of development by the writer. The developmental period covered approximately three years, during which time the instrument was tested in connection with a varied array of geological problems throughout large parts of Texas, Oklahoma, Arkansas, Kansas, and in certain regions of Mexico, New Mexico, Arizona, Missouri, Tennessee, Illinois, Wisconsin, California and Minnesota. More than 40 changes from the original model were effected during the progress of this research. The resulting sensitive, swift and fieldworthy instrument has been patented by W. C. McBride, Inc., and is being manufactured for that organization by the Eugene Dietzgen Co. of Chicago, Ill., and J. P. Foerst of Madison, Wisconsin.

Working Principle

The working principle of the instrument involves balancing the force of gravity against magnetic force. A small bar magnet free to move in space invariably aligns itself with the earth's magnetic field, or becomes parallel to the "lines of force" which are conceived as defining the earth's magnetic field. The magnet can therefore show changes in the direction of this field without being able to show changes in intensity.

However, if it is forced into a position out of alignment with the earth's field, a certain magnetic force, varying in magnitude with the strength of the magnet and the intensity of the field, will immediately be exerted to bring it back into alignment. If the magnet is balanced in a position out of alignment with the earth's field at one place, and then moved to another place where the intensity of the earth's field is different, the balance no longer exists and the magnet changes its position until the balancing force is again exactly compensated by the force of the magnetic field. Such a change in the position of the magnet, therefore, indicates a commensurate change in magnetic intensity.

This principle is fundamental to many different magnetic instruments.[3] The ordinary dip needle, for instance, consists merely of a bar magnet on fixed bearings so arranged that it can move only in a single plane, and supplied with an adjustable counterweight mounted on the magnet, serving to balance it at an angle to the inclination of the earth's magnetic field when the instrument is properly oriented in the plane of the magnetic meridian. The counterweight introduces the force of gravity, which is assumed to be relatively constant from place to place; but its effect on the balanced magnet varies because it is applied as a turning moment opposed to that of the magnetic field.

Diagram *a* of Fig. 1 illustrates this point. Let M be considered to be the magnetic moment which is the product of the strength I of the earth's field (with an inclination of Θ degrees) and the magnetic moment of the magnet, which in turn is the product of the pole strength P of the magnet and the effective length a of its axis. Thus $M = IPa$; and it is clearly at its maximum when the axis of the magnet is at right angles to the inclination of the earth's field. But the turning moment of gravity G required to hold the magnet in that position is the product of the weight of the counterweight W and its effective distance d from the fulcrum. Thus $G = Wd$; and this value is not at its maximum unless the magnet is horizontal. In any position of rest $M = G$ or $IPa = Wd$. An increase in the intensity of the earth's magnetic field causes the magnet to move toward the horizontal position, with the result that two changes take place. The factor a is decreased, thus decreasing the value of M, and the the factor d is increased, thus increasing the value of G. This change in proportions of M and G acting against each other materially impairs the sensitivity of the instrument, especially with increasing deflections.

If in the same balanced system the counterweight is so mounted that G and M are simultaneously at their maximum, the effective lever arms d and a will experience the same proportionate change with any movement

[3] For example, see C. A. Heiland: Construction, Theory and Application of Magnetic Field Balances. *Bull.* Amer. Assn. Petr. Geol. (1926) **10**, 1189; N. H. Stearn: Dip Needle as a Geological Instrument. Geophysical Prospecting, A. I. M. E. (1929) 349.

of the magnet. Such an arrangement can be effected by making d horizontal when a is normal to the earth's magnetic field. Diagram b in Fig. 1 illustrates this arrangement. $M = G$ or $IPa = Wd$ as before. But when the magnet turns through any angle x the effective lever

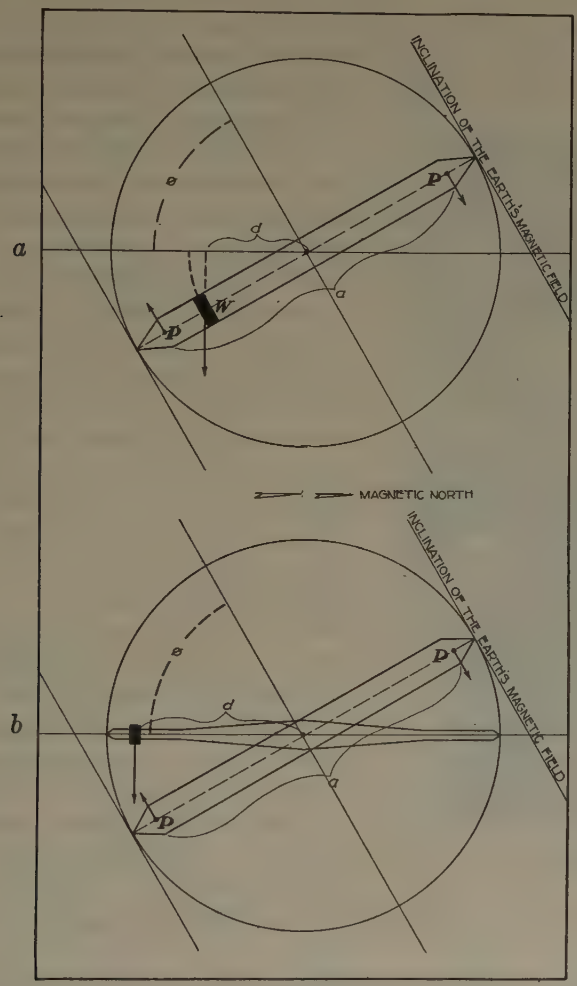

FIG. 1.—DIAGRAM OF WORKING PRINCIPLES OF (a) ORDINARY DIP NEEDLE AND (b) HOTCHKISS SUPERDIP.

arms are reduced to $a \cos x$ and $d \cos x$. The turning moments are $IPa \cos x$ and $Wd \cos x$, and both sides of the equation are diminished by the value of a common factor, $\cos x$. The magnet is thus in unstable equilibrium and any change in M, either increase or decrease, produces a 90° change in its position of rest. Obviously any degree of practical sensitivity, from that of the ordinary dip needle to that of theoretically infinite

sensitivity limited only by friction, can be obtained by varying the angle between d and a. This is the principle of the Hotchkiss Superdip.

Construction

In working out the mechanical embodiment of this principle every effort has been focused on maintaining the utmost simplicity compatible with quick and reliable results in its application to exploration problems. Fig. 2 shows the instrument mounted on its specially designed leveling head, and demonstrates the complete visibility and the ready accessibility of all essential working parts.

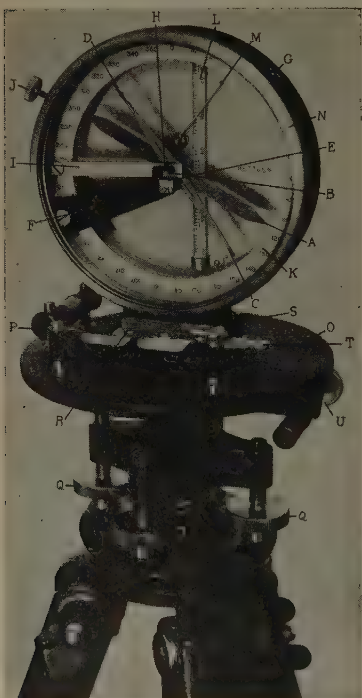

Fig. 2.—Construction and mounting of Hotchkiss Superdip.

The bar magnet A is made of a special tungsten-cobalt steel which possesses the combined characteristics of maximum magnetic susceptibility and retentivity. The magnet is swung at its center of gravity on a pivot B in such a way that it is free to move in a vertical plane. To the same pivot is attached a counterarm C at its center of gravity, in such a way that it can be adjusted to any angular relationship to the axis of the magnet in the vertical plane. The angle between the counterarm and the magnet is measured by a specially designed protractor. The magnet and the counterarm are affixed to the pivot at their centers of gravity in order that the whole assembly may be in perfect mechanical balance. Then to one end of the counterarm is added the counterweight D, which can be adjusted by varying its effective lever arm.

This swinging assembly, consisting of magnet, counterarm, counterweight and pivot, is mounted in the instrument by resting the pivot ends on two parallel and horizontal agate edges E supported by arms F, affixed to the case G. The swinging assembly is lifted from the agate edges, centered and clamped securely against two centering pivot guides H by a release device composed of two lever arms I operated by a thumb screw J. Variations in the position of the magnet are read from the circular scale K graduated into degrees of arc, which is so placed in the case that the magnet swings in its plane. A thermometer L is installed

in the case to register temperature changes for the necessary temperature corrections.

To secure greater speed the instrument is read on the swing from the zero position. To place the magnet at the zero position a mechanical finger *M* is fitted into the back of the case.

The cylindrical brass or aluminum case *G*, which contains all these parts, is provided with a glass face *N*, which can easily be unscrewed so that the interior of the instrument, while almost dustproof when it is closed, is always visible and quickly accessible.

The instrument is mounted on an extension tripod with a special leveling head, consisting of a circular plate *O* provided with two level bubbles *P* affixed at right angles to each other, which can be leveled by means of four screws *Q*. Four pegs *R* on the plate fit into four sockets in the base *S* of the instrument. The instrument is held firmly in place by a clamp *T*. The level bubbles are so oriented that the axis of one is parallel with and that of the other normal to the plane of the instrument when it is in position.

A compass for determining the plane of the magnetic meridian, in which the instrument must be oriented, is fitted to the same leveling head, and a micrometer screw *U*, mounted beneath the plate, serves to facilitate this adjustment.

Table 1 gives an idea of the bulk and weight of the apparatus.

TABLE 1.—*Specifications*

	Weight, Pounds	Dimensions, Inches
Superdip (brass)	2⅞	6 × 5¼ × 1½
Superdip (aluminum)	1½	6 × 5¼ × 1½
Leather case	¾	6¼ × 6¼ × 2½
Compass (brass)	1	4 × 2⅜ × 1
Compass (aluminum)	½	4 × 2⅜ × 1
Leather case	¼	4 × 3⅞ × 1¾
Leveling head	5	5¾ × 5¾ × 5
Tripod	10	4½ × 4½ × 36
Tripod (extended)		4½ × 4½ × 60

BEHAVIOR

The relative sensitivity of the instrument is determined by the angle between the axis of the counterarm and that of the magnet. The limiting angle is established by the angle of inclination of the earth's field. This can be read directly from the instrument, which is essentially a *dipping circle* until the counterweight is applied. Suppose the inclination Θ of the earth's field to be 60°, as shown in Fig. 3. The limiting angle *A* between the counterarm and the magnet would then be 30° for the maximum sensitivity of the instrument, for in this position when the magnet is at right angles to the earth's field the counterarm is horizontal. Thus

the angle of maximum sensitivity, A, is the complement of the angle of inclination (Θ). Because maximum sensitivity is seldom desirable for field work, the angle which is actually set off, T, is usually less than the limiting angle by a determinable amount. This amount is the angle sigma or the sensitivity angle ($\Sigma = 90° - \Theta - T$) which is adjustable to any desired degree of sensitivity.

The procedure for setting the instrument in the northern hemisphere for field use at any specific sensitivity is as follows:

1. Set up the tripod and mount the leveling head. Level the plate and mount the compass so that the north-south axis is parallel to the long level bubble and the short level bubble is south. Release the com-

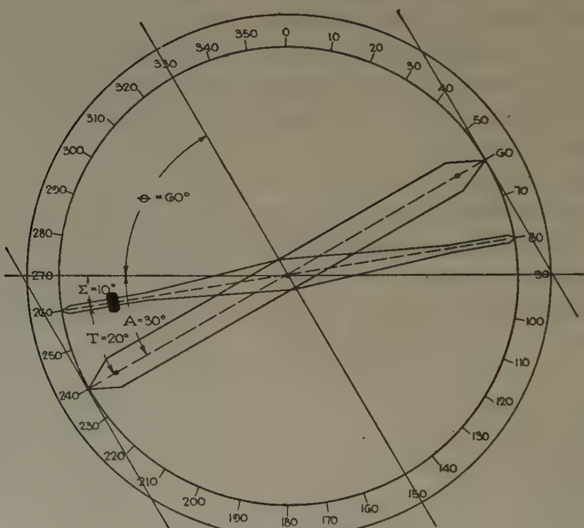

Fig. 3.—Setting of Hotchkiss Superdip.

pass needle and turn the plate until the north end of the needle is at 0° on the compass. Lock the plate. This establishes the magnetic meridian and properly orients the leveling head with respect to it.

2. Remove the compass and mount the Superdip. Take off its glass face, press down on the release arms and remove the swinging assembly from the agate edges. Unscrew the counterweight and set the counter-arm parallel to the magnet. Press down on the release arms and replace the swinging assembly. Replace the glass face. Turn the release thumb screw, lowering the swinging assembly to the agate edges. The instrument now functions as a dipping circle because the magnet will align itself with the lines of force of the earth's magnetic field. Read on the circular scale the position of the north end of the magnet. To get the angle of magnetic inclination Θ subtract 90° from this reading.

3. Decide on the sensitivity required (*i. e.*, the sigma value) and compute the necessary angle T between the magnet and the counterarm from the expression $\Sigma = 90° - \Theta - T$ or $T = 90 - \Theta - \Sigma$. Remove the swinging assembly from the instrument. Place it on the protractor (to be found in the carrying case of the instrument) in such a way that the axis of the magnet coincides with the zero line of the protractor and the south pole of the magnet is at the graduated end. Move the counterarm to the determined angle.

4. Apply the counterweight to the threaded counterarm and adjust it until the position of rest of the swinging assembly is approximately

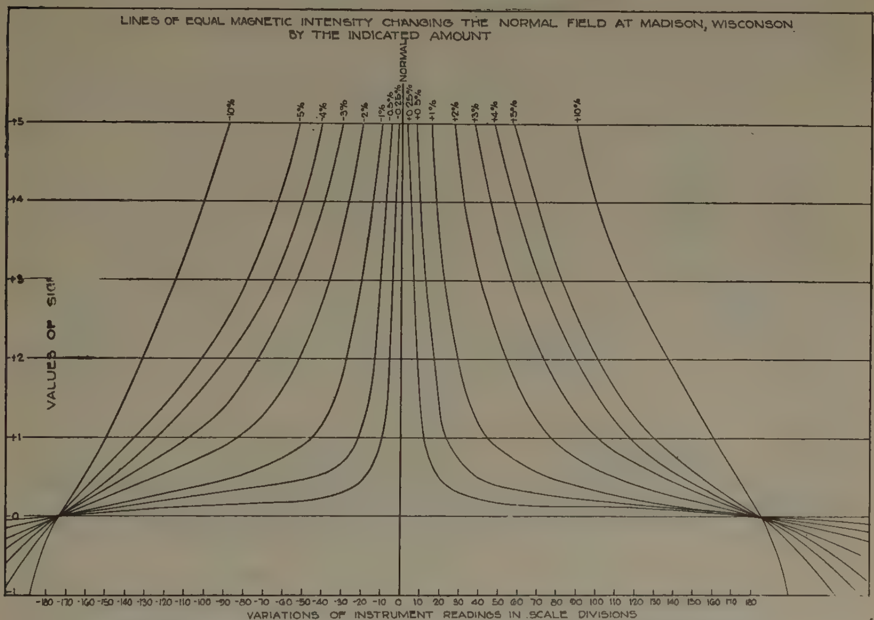

Fig. 4.—Behavior of Hotchkiss Superdip.

at right angles to the inclination of the earth's magnetic field, or until the magnet swings from the zero position to approximately twice the angle of inclination.

This completes the adjustment of the instrument for field use, a process requiring from 10 to 20 min. under normal conditions.

In order to observe the complete behavior of the instrument with various sensitivity settings under conditions simulating those to be found in actual field practice, special testing apparatus was built, the essential features of which are two sets of Helmholtz coils so oriented that their axes both lie in the plane of the magnetic meridian and bisect each other at right angles. One axis is vertical while the other is horizontal. Through each set of coils a separate current can be made to

flow in either direction, so that the horizontal and vertical components of the earth's magnetic field may be altered at will, either simultaneously or separately, either by augmentation or diminution. This simple apparatus, suggested by L. Y. Faust and built by M. D. Harbaugh, serves to establish a complete range of variations in the intensity and inclination of the earth's magnetic field. In this apparatus the behavior of each instrument is carefully tested.

Fig. 4 shows a typical graph of the instrument behavior throughout its principal working range of sensitivity. The ordinate shows values of sigma from −1 to 5. (A negative value of sigma is defined as that

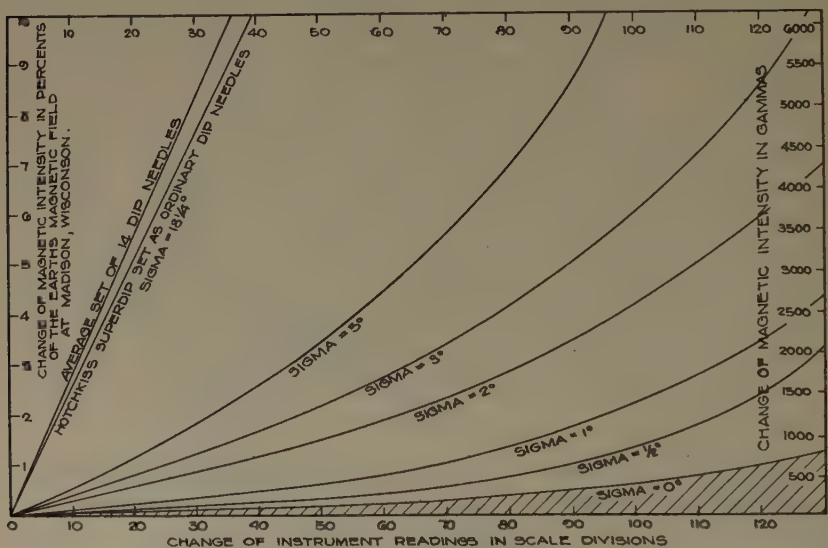

FIG. 5.—SENSITIVITY OF HOTCHKISS SUPERDIP.

obtained when the T angle or the angle between magnet and counterarm exceeds the angle of maximum sensitivity A.) The abscissa shows variations in the instrument readings resulting from the augmentation and diminution of the normal field in which the instrument was balanced. The curves are lines of equal intensity variation from the normal earth's magnetic field at Madison, Wis., a field with a total intensity of approximately 61,278 gammas in 1927. The percentages of the intensity variation are indicated for each curve.

Fig. 5 shows a graph of the practical working sensitivity of a typical instrument together with the average sensitivity of 14 ordinary dip needles. It is interesting to note that when the Hotchkiss Superdip is set as an ordinary dip needle (with the counterweight on the axis of the magnet) it still shows a slightly superior sensitivity, due, no doubt, to the difference in the construction of the two instruments. Another

interesting feature in this graph is the curve marked Sigma = 0°. Theoretically this should coincide with the zero abscissa. The amount of departure therefrom indicates the amount of the departure from infinite sensitivity introduced by the mechanical features of the instrument including friction. This curve shows that the nearest approach to the theoretically infinite sensitivity inherent in the principle of the instrument is the sensitivity shown by a change in the instrument reading of 1 scale division for a 3-gamma change in magnetic intensity.

The sensitivity lines for the various values of sigma are curved rather than straight, because the values are really in terms of a function of an angle. For practical purposes this fact has proved advantageous in permitting the automatic damping of the effects of extreme anomalies so that they can be measured with no diminution of speed. Although the sensitivity lines are curved, it can readily be seen that for the ordinary working range of the instrument the difference between the arc of the curve and its chord is negligible. Thus for ordinary anomalies a simple linear coefficient of sensitivity suffices (*e. g.*, 12½ gammas per scale division for a sigma of 1°). And with the value of sigma known, the gamma value of any large anomaly can readily be obtained from the graph, which includes extreme conditions in showing an instrument variation of 130 scale divisions and an intensity variation of over 6000 gammas. Still more extreme values can be obtained by extrapolation.

Field Procedure

Because a geomagnetic survey is merely a supplement to a geological survey, the field procedure must necessarily depend upon the geological problem to be solved, and upon individual ingenuity. A certain general mode of approach, however, is applicable to all problems. The scale and character of the geological problem determines the desirable distance between magnetic observations, or the station interval. With this interval a traverse is made across the area to be surveyed, with the instrument functioning as a dipping circle. From the results of this traverse can be obtained an idea of the order of magnitude of anomalies to be encountered, from which the desired sensitivity setting can be estimated.

The setting of the instrument is then made at some point near the center of the area to be surveyed or in some spot believed to be magnetically normal for the area. The instrument reading at the setting station establishes the datum plane to which the whole survey is referred. Although this reading is arbitrarily established in the process of setting the instrument, it can be referred to the nearest government station if an absolute value is desired, just as a relative topographic survey can be tied in to a government bench mark. From the setting station the sur-

vey is carried out over the area, with readings taken at space intervals required by the survey. The actual reading involves the following procedure:

1. Set up the tripod and level the mounting plate.
2. Mount the compass, orient the plate in its proper relation to the earth's magnetic meridian and clamp it.
3. Remove the compass and mount the Superdip.
4. Move the north pole of the magnet to the zero position on the circular scale.
5. Carefully lower the swinging assembly to the agate edges and note the end of the swing, taking the reading from the position of the north pole of the magnet relative to the circular scale at the end of the swing.
6. Clamp the swinging assembly and record the reading, temperature, time and location of the station.

(This procedure can be accomplished in slightly less than two minutes by an expert operator.)

PROJECT Reconnaissance Blank Co. Kansas
INST. #21 SETTING $\theta = 69\frac{1}{2}°$ $T = 19\frac{1}{2}°$ $\Sigma = 1°$
T.C. .5 BASE 75° F OPERATOR J.D. DATE 4/1/30

STA.	LOCATION	TIME	TEMP.	READING	COR.	COR. R D'G	NOTES
	S.E. 21	7^{35}	72	130	-1.5 T	128.5	Cloudy
1B	T.14 S.R.19 W.				-2.5 D	126	Base value of
"					0 L	126	1A = 126
1A							
2B	S.E. 16	8^{32}	74	138	-.5 T	137.5	
					-1.7 D	135.8	
					-.9 L	134.9	
3B	S.E. 9	9^{20}	75	142	0 T	142	
					-1.1 D	140.9	
					-1.9 L	139	
4B	S.E. 4	10^{15}	76	136	+.5 T	136.5	
					-.4 D	136.1	
					-3 L	133.1	
5B	N.E. 4	11^{04}	76	124	+.5 T	124.5	
					+.3 D	124.8	
					-4 L	120.8	
6B	S.E. 21	12^{00}	77	124	+1 T	125	
"					+1 D	126	
1A					0 L	126	
	S.E. 28	1^{30}	78	122	+1.5 T	123.5	
7B					-.1 D	123.4	
					+.9 L	124.3	
8B	S.E. 33	2^{10}	78	121.5	+1.5 T	123	
					-.6 D	122.9	
					+1.8 L	124.7	
9B	S.E. 34	3^{05}	78	121.5	+1.5 T	123	
					-1.3 D	121.7	
					+1.5 L	123.2	
10B	S.E. 21	4^{00}	77	127	+1 T	128	
"					-2 D	126	
1A					0 L	126	
1	2	3	4	5	6	7	8

FIG. 6.—CORRECTED FIELD NOTES.

Each morning the instrument is set up at the control station and allowed to become acclimated (care being taken to keep sunlight from shining directly into the face of the instrument). The control station is revisited during the day at time intervals whose length depends upon the degree of precision required in the results of the survey. During the progress of the survey the control station may be shifted for convenience to any station for which a value has been established relative to the original setting station.

The locations of the stations are plotted on a base map and the instrument readings recorded for each station. Various graphic presentations of the magnetic results (or magnographs) can be made, including contouring with lines of equal magnetic anomaly (see Fig. 13), contouring with lines of equal magnetic intensity, or plotting the results as a series of profiles. For greater facility in visualizing the magnetic anomalies the isometric system has been used for at least eight years in connection with

both profile (see Fig. 14) and contour (see Fig. 16) magnographs. The isometric profile scheme was used by Aldrich in 1922, in order to prevent interference of extreme variations when plotted to a scale large enough to show clearly the minor variations.

Fig. 6 shows a typical page of data recorded on a special form designed to offer a concise and complete system of notation. The field observations are recorded in the first five columns to the left. In many instances an adequate geological interpretation can be made from the readings just as they come from the field, after they have been properly plotted; but to secure more precise results certain corrections must be applied to the readings.

Corrections

To secure results of maximum precision six different corrections may be applied to the readings. These have differing degrees of importance, and the importance of each one should be carefully evaluated in the light of its possible influence on the interpretability of the results. In some cases all corrections are unnecessary.

The six possible corrections are for: (1) temperature, (2) diurnal variation, (3) day-to-day variation, (4) latitude, (5) longitude, (6) correction to base. The processes of making these corrections have been so arranged that three steps serve to make them all. These three groups of corrections, in the order of their importance, are classed as (1) temperature, (2) daily variation, (3) latitude (Fig. 6, column 6).

Temperature Correction

The temperature coefficients for each instrument for the various values of sigma are determined in the laboratory. These can be readily checked in the field by observing the variations in the instrument readings which occur concurrently with temperature changes. This checking can be done every morning while the instrument is becoming acclimated The laboratory data are furnished in the form of a curve (Fig. 7*A*) with sigma values as the abscissa and temperature coefficients in scale divisions per degree as the ordinate. The implication is that for each sigma value the temperature coefficient is a straight-line function. This is not precisely the case because the correction actually varies slightly with different ranges of temperature. However, over the temperature range normally encountered in field practice this variation is so slight as to be considered negligible, and so each temperature coefficient as shown on this curve is really the chord of the arc of variation in temperature correction over the normal working range of temperature changes.

The procedure for making temperature corrections is as follows: First determine the temperature coefficient for the sigma value at which the instrument is set. This can be secured either from the curve supplied

with each instrument or from actual field tests. Next determine the base temperature to which all readings are to be reduced, usually the approximate median temperature for the survey. Then for each reading multiply the figure which represents the difference between the base temperature and the reading temperature by the determined coefficient and apply the result to the reading. A rise in temperature produces a decrease in the instrument readings; therefore corrections for readings taken at temperatures below the base temperature should be subtracted and those for readings above it should be added. For example, suppose

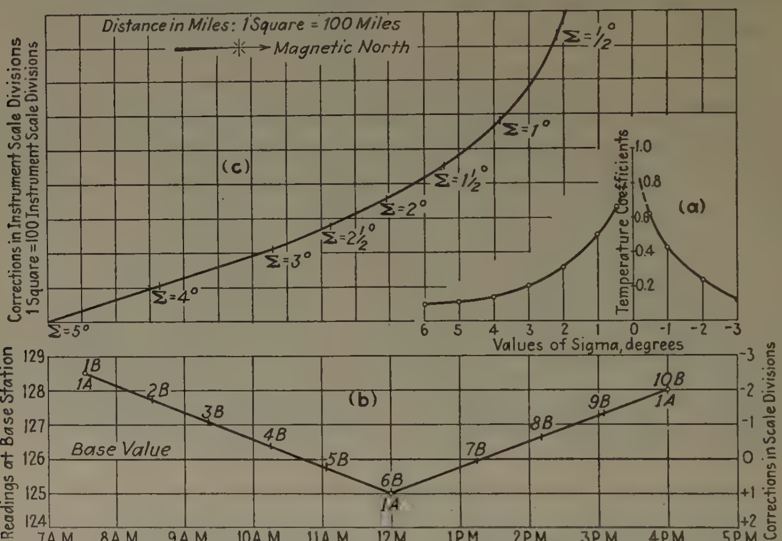

FIG. 7.—TYPICAL CORRECTION CURVES FOR HOTCHKISS SUPERDIP.
a. Temperature correction coefficients for various sigma values.
b. Daily variation curve.
c. Latitude correction curve.

the coefficient to be 0.5 scale divisions per degree change in temperature, and the base to be 75°. If the reading temperature is 72° the correction would be $72 - 75 = -3 \times 0.5 = -1.5$ scale divisions. If the reading temperature be 78° the correction would be $78 - 75 = +3 \times 0.5 = +1.5$ scale divisions. The correction for each reading is recorded on the line marked T in the notes (Fig. 6, column 6).

On a special slide rule, designed by the writer in 1927 to facilitate the process of making temperature corrections, the correction can be read directly from a finder which has been set at the reading temperature.

Daily Variation Correction

A cyclic variation in the earth's magnetic field occurs daily. Its intensity differs. If it approaches the order of magnitude of the mag-

netic anomalies it should be corrected. For this reason the time of each reading is recorded in the field (Fig. 6, column 3).

The following procedure serves to correct not only daily variation, but also day-to-day variation, any mechanical variation in the instrument and correction to base. First the values for the control station, which has been revisited at intervals during the day, are corrected for temperature. These corrected values are plotted on cross-section paper with time as the abscissa and instrument readings as the ordinate. Fig. 7*b* shows a diagrammatic daily variation correction curve. The base value for the control station was established at the setting of the instrument. The plotted points show departures from that value with time. It is assumed that these departures are gradual, so that the curve drawn through the plotted points represents the correction curve. Obviously, the more frequently the control station is revisited during the day, the more points of control there will be for the curve. From this curve the correction for each station can be read by noting the difference between the curve and the base line value at the time of the reading. The correction for each station is recorded on the line marked D in the notes (Fig. 6, column 6), and is applied to the value which has previously been corrected for temperature.

Latitude Correction

If it were easy to visualize anomalies referred to an inclined plane or a curved surface, this correction would be entirely unnecessary. Its function is merely to reduce results to a horizontal datum plane and it is useful only for surveys which embrace many square miles. For this instrument the latitude variation is in the form of a curve, due to the fact that the angle of inclination of the earth's magnetic field varies with latitude, and the sigma angle of the instrument varies with the angle of magnetic inclination when the setting of the instrument remains constant ($\Sigma = 90° - \Theta - T$). A curve of latitude variation is furnished with each instrument, generalized to cover the latitudes of the United States (Fig. 7, diagram *c*). For this curve the abscissa is distance measured in miles and the ordinate is change in instrument readings measured in scale divisions. The slope of the curve varies with different sigma settings, which are marked on the curve for reference.

To reduce all the readings of a given survey to the equivalent of a horizontal plane, the following procedure has been found practicable, in that it combines latitude and longitude corrections in one process. On the base map containing the locations of the stations, magnetic north is established. Through the base station a line is drawn at right angles to magnetic north. This is the locus of all points of zero latitude and longitude correction for the survey. Lines of equal latitude correction are then drawn (Fig. 8) parallel to the base line at intervals of ½ or

1 mile. On the latitude-correction curve furnished with the instrument (Fig. 7) is located the point that represents the value of the sigma setting for the survey. For each mile north and south of this point the correction value in instrument scale divisions can be read from the curve. These values are placed on the corresponding lines of equal latitude correction. For each station plotted on the map the latitude correction can then be taken either directly or by interpolation if the station is between two lines of equal latitude correction. (See station 9*B*, Fig. 8.) The correction is recorded on the line marked L in the notes (Fig. 6, column 6),

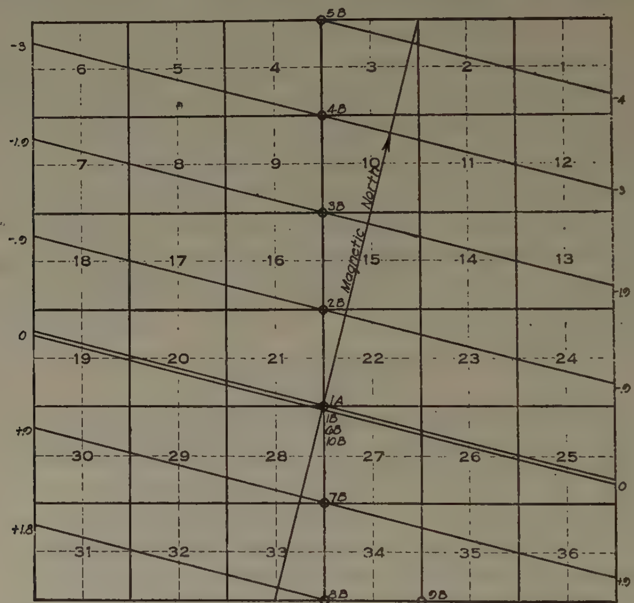

Fig. 8.—Combined latitude and longitude correction.

and is applied to the reading as previously corrected for temperature and daily variation.

It is worthy of re-emphasis that the desirability of these corrections, simple and facile though they be, should be judged by the contribution they make toward the interpretability of the results. An idea of the relative influence of the corrections in connection with a regional magnetic survey is given by Fig. 9, which shows a traverse from Sheridan, Ark., to the north line of Grant County with a sigma setting of 2° and a traverse from the south line of Gray County, Texas, to Pampa, with a sigma setting of 0.75°. Obviously, in each case the general geological features stand out in the original profile as clearly as in the last. Only in areas where the magnetic relief is of the same order of magnitude as the corrections or in problems such as those involved in depth-finding where quantitative values are essential, is there need for the corrections.

The compromise between absolute precision and speed, simplicity and flexibility, which any field instrument must embody, should be

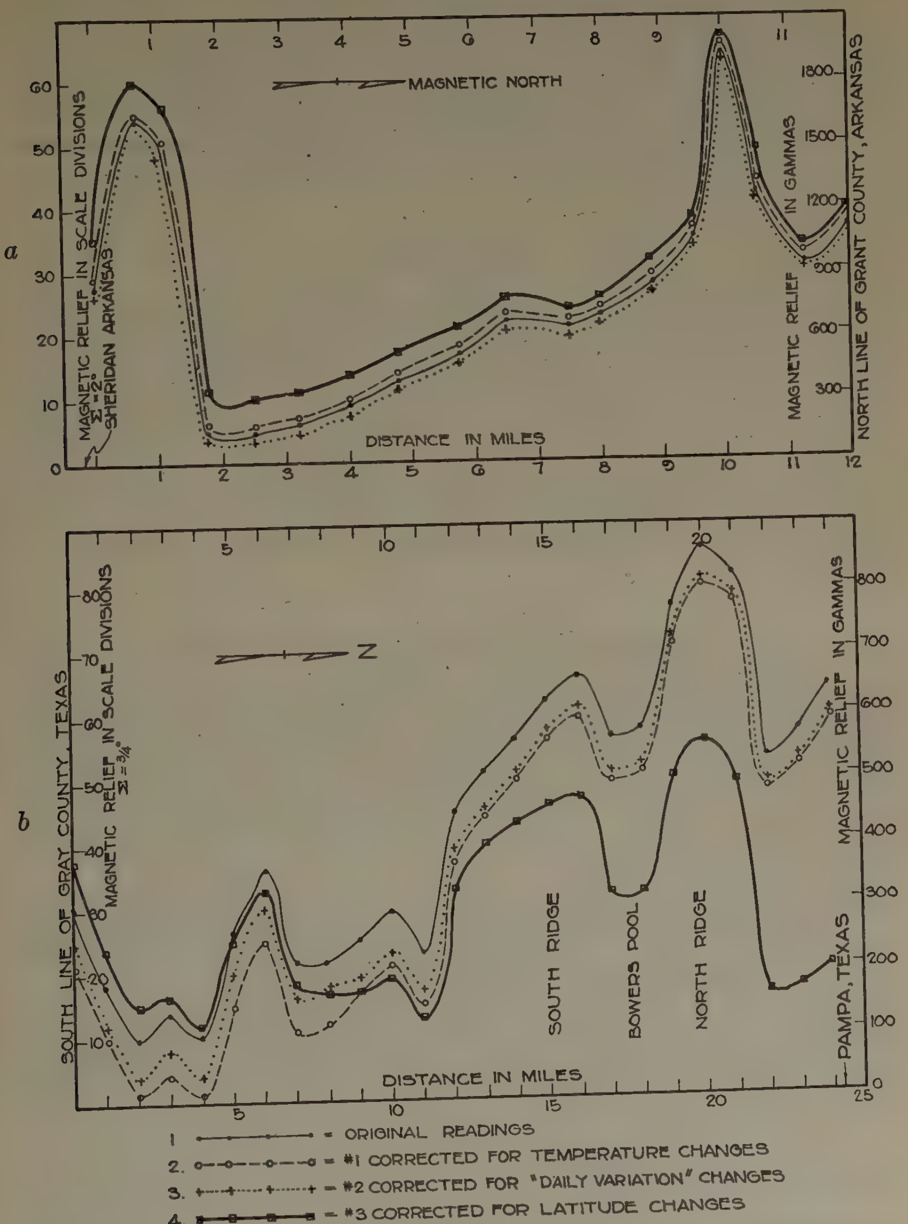

FIG. 9.—MAGNETIC PROFILES SHOWING INFLUENCE OF VARIOUS CORRECTIONS.

evaluated as closely as possible. A résumé of the sources of error connected with the use of the Hotchkiss Superdip and the degree of

influence which they may be expected to exert is published in the *Bulletin* of the American Association of Petroleum Geologists.[4]

It is, of course, obvious that every precaution should be taken to keep the agate edges and the pivot of the swinging assembly absolutely clean and dry. These are carefully examined with a high-power hand lens and cleaned with dry white pine. Static electrification of the glass face of the instrument on cold days produces unmistakably erratic readings. This can be eliminated simply by breathing on the glass.

Application

For a purview of some of the types of geological problems solvable with the help of the geomagnetic method, the following nine cases where the Superdip has been applied are taken from seven different states (Wisconsin, Minnesota, Texas, Oklahoma, Kansas, Arizona and Arkansas) in connection with exploration for eight different natural resources (copper, iron, oil, gas, lead, gold, bauxite and diamonds). A comparison between the working sensitivity of the Superdip and that of the ordinary dip needle from which it has evolved can be seen in the first three of these cases.

Copper-bearing Keweenawan Lava Flows

Fig. 10 shows magnetic profiles across a section of the Keweenawan amygdaloidal basaltic flows in Polk County, Wisconsin. Here the flows, striking about N. 50° E. and dipping 20° to 30° NW., are buried beneath 20 to 40 ft. of glacial drift. Along the northeastern extension of this Keweenawan series, native copper is to be found in the brecciated tops of certain of the flows.[5] Because the mineralization is concentrated in definite tabular horizons, it is of economic importance to trace these horizons as closely as possible. A magnetic survey[6] has been conducted by H. R. Aldrich, which has delimited the area in Wisconsin underlain by the flows and has traced certain horizons within that area.

The traceability of individual flows within the whole sequence is probably due to a variation in their content of ferromagnetic minerals such as magnetite and to the distribution of minerals within the separate

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[4] N. H. Stearn: The Hotchkiss Superdip: A New Magnetometer. *Bull.* Amer. Assn. Pet. Geol. (1929) **13**, 671.

[5] U. S. Grant: Preliminary Report on the Copper Bearing Rocks of Douglas County, Wisconsin. Wis. Geol. Survey *Bull.* (1901) 6.

[6] H. R. Aldrich: Magnetic Surveying on the Copper Bearing Rocks of Wisconsin. *Econ. Geol.* (1923) **18**, 662; A Demonstration of the Reflection of Geologic Conditions in Observed Magnetic Intensity. Geophysical Prospecting, A. I. M. E. (1929) 385.

flows brought about by segregation during cooling. A careful quantitative study of the magnetite and ilmenite content of one of these flows showed a systematic variation with depth in the flow, a horizon of concentration of magnetite occurring near the bottom.[7] Zones of magnetite concentration similar to this are said to cause the linear magnetic anomalies which are found over the lava series, and from these linear anomalies can be ascertained structure and areal distribution, as well as marker horizons within the flow sequence.

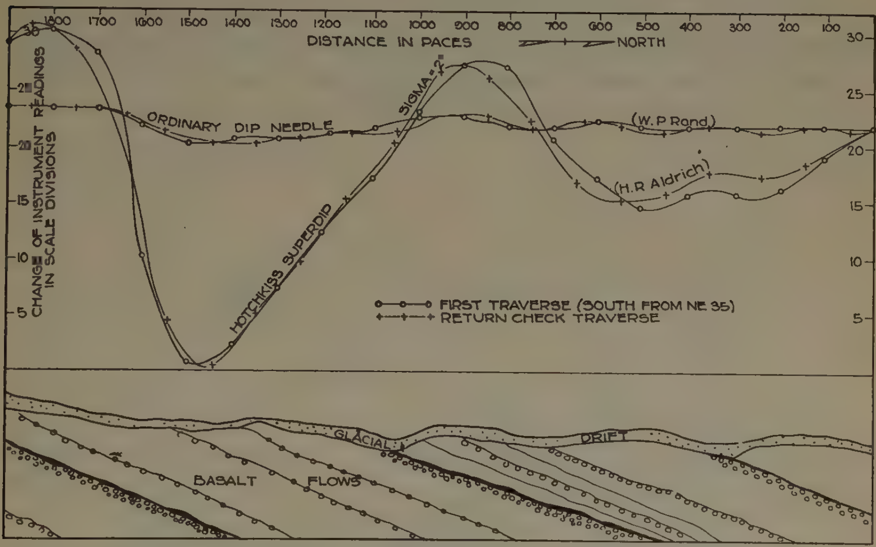

FIG. 10.—GEOLOGIC SECTION AND MAGNETIC PROFILE ALONG EAST LINE OF SEC. 35, T.36N., R.18W., POLK COUNTY, WISCONSIN.

The magnetic profiles show the operation of the Hotchkiss Superdip at a sensitivity approximately 10 times that of the ordinary dip needle. A comparison of the divergences between the two traverses made with each instrument shows the amount of discrepancy introduced by the absence of corrections. None of the corrections has been applied to these readings for either instrument. In this case it is obvious that the application of corrections is unnecessary and therefore uneconomical. The total instrument variation shown by the ordinary dip needle is 3°—very little above the limit of reliability of the instrument; whereas the magnetic relief shown by the instrument variation of the Superdip is totally unambiguous.

[7] S. Buckstaff: A Study of the Distribution of Magnetite in One of the Keweenawan Lava Flows. Unpublished Thesis, University of Wisconsin, 1923.

CUYUNA IRON FORMATION[8]

Fig. 11 shows magnetic profiles across a section of the iron formation of the Cuyuna Range in Crow Wing County, Minnesota. In the vicinity of this section the iron-bearing member strikes approximately N.65° E. and dips to the southeast at angles varying from 60° to 80°. It is banded cherty sedimentary rock, having local phases of sandy wash ores, with a slate horizon at its base. The total thickness of the iron formation varies from 100 to 160 ft. A quartzite formation about 300 ft. thick forms a presumably conformable footwall for the iron formation. The quartzite in turn is underlain by an extensive greenish schist. The hanging wall of the iron formation consists of an undetermined thickness of gray schist and slate. These formations are all concealed by an overburden of glacial drift ranging in thickness from 65 to 75 feet.

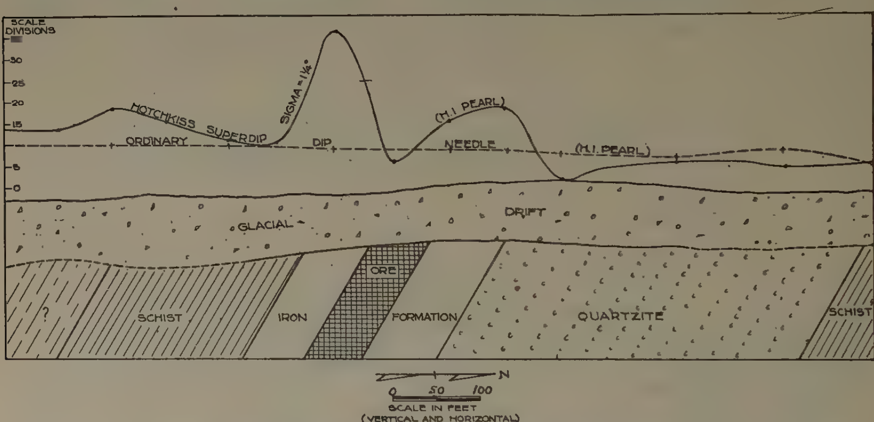

FIG. 11.—MAGNETIC PROFILE AND GEOLOGIC SECTION ACROSS SW.-SE. SEC. 6, T.46N., R.29W., CROW WING COUNTY, MINNESOTA.

The profiles shown in Fig. 11 are from a survey of the Bessemer mine property, made by Holman I. Pearl, of Crosby, Minn. Here the ordinary dip needle is of no avail; as shown by the featureless profile plotted from its readings. The Superdip profile, however, shows three horizons of increased magnetic intensity, one over the footwall and another over the hanging wall of the iron formation. Across the wide quartzite formation the magnetic intensity is almost uniform, and over the oxidized orebody (proved by drilling) there seems to be a degree of intensity similar to that over the quartzite. The possibility that detailed readings over the iron formation itself will show indications of the degree of oxidation which it has undergone, and thus point to the most likely places along the strike of the formation in which to seek for concentrated ore, is suggested by

[8] C. Zapffe: Geologic Structure of the Cuyuna Iron District, Minnesota. *Econ. Geol.* (1928) **23,** 612.

Mr. Pearl as a result of tests over several proved areas on the Cuyuna Range. Whether or not such detailed exploration is practicable, the possibility of delimiting the iron formation itself is indicated.

Buried Granite Ridges of the Texas Panhandle

Fig. 12 shows magnetic profiles across what seems to be the buried western extension of that complex igneous mass which forms the Wichita Mountains.[9] At the western extremity of these mountains isolated igneous peaks protrude through the flat plain of nearly horizontal Permian Red Beds, leaving the suggestion that further westward the igneous

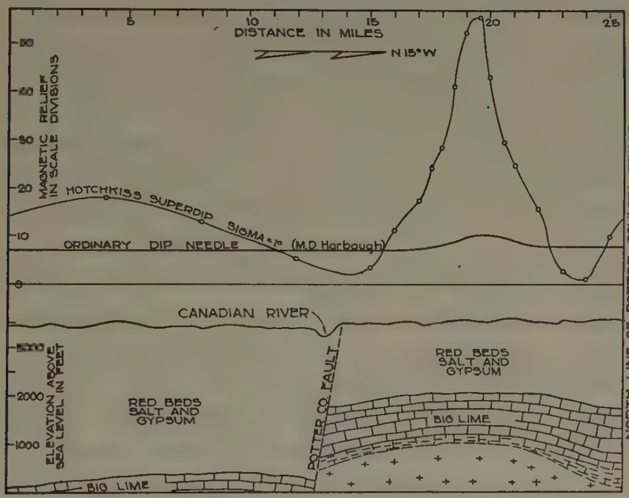

Fig. 12.—Magnetic profile and geologic section from two miles north of Amarillo to the north line of Potter County, Texas.

masses have been completely submerged beneath the plains. This suggestion has been corroborated by drilling and by magnetic evidence, which has traced the buried mountains over 160 miles northwestward from the last igneous outcrop. The buried extension has been named the Amarillo Mountains.[10]

The economic import of this westward extension of the igneous masses lies in the fact that their topography seems to control broad structures in the overlying beds as well as the localization of deposits of detrital material derived from the erosion of the masses themselves. Thus in the Texas Panhandle the topography of the buried mountains is an

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[9] J. A. Taff: Geology of the Arbuckle and Wichita Mountains in Indian Territory and Oklahoma. Okla. Geol. Survey *Bull.* (1928) 12 [Reprint of U. S. Geol. Survey *Prof. Paper* 31 (1904)].

[10] C. N. Gould and F. E. Lewis: The Permian of Western Oklahoma and the Panhandle of Texas. Okla. Geol. Survey *Cir.* (1926) 13.

important factor in oil and gas exploration, because the Big Lime, whose structure reflects that topography, and the granite wash whose deposition is controlled by it, are the producing formations.[11]

The profile shown in Fig. 12 lies about 140 miles west by northwest of the last igneous outcrop of the Wichita Mountains. Here the minimum thickness of the overburden approximates 2500 ft. The position of the top of the Big Lime as located in scattered wells indicates a fault with a downthrow to the south of between 1200 and 1500 ft., which occurs on the south side of the igneous mass. A broad gas-producing dome has been outlined south of the indicated fault. The anticline which reflects the igneous mass has yielded a gas field.

The ordinary dip needle shows no indication of the deep dome or the fault. It shows a variation of 1.5° over the igneous mass. This variation exceeds the operating error of the instrument by very little. The results are therefore on the verge of ambiguity. The Hotchkiss Superdip here shows a working sensitivity about 37 times that of the ordinary dip needle. With this sensitivity it shows without ambiguity a reflection of the deep dome, the location of the fault and the position of the igneous mass.

Western Extension of Beckham County Fault

Fig. 13 shows the reflection in magnetic intensity variation of a portion of the buried western extension of the Wichita Mountains in that part of Wheeler County, Texas, which lies adjacent to the Oklahoma state line. The dominant feature of this magnograph is the abrupt and pronounced magnetic "low" with a linear trend striking N. 75° W. This feature is considered to be definite evidence of the continuation of the Beckham County fault.

The Beckham County fault is normal, having a downthrow to the north of from 300 to 500 ft. It has been traced by water-well data from sec. 12, T.8N., R.23W. to sec. 28, T.9N., R.25W. in Beckham County, Oklahoma, by Frank Gouin,[12] who suggests the probability of its westward continuation.

A magnetic traverse across the known portion of the fault showed an abrupt drop in intensity over the fault amounting to about 400 gammas. This anomaly, continuous along the fault line but with variations in magnitude, is considered to be related to the igneous basement, visualized as having an abrupt north scarp with faceted spurs buried beneath about 2500 ft. of sediments. The magnetic saddle seen in sec. 37, 38, 43, 44, 57 and 58 of the magnograph may possibly reflect a sag in the igneous mass between two faceted spurs.

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[11] C. M. Bauer: Oil and Gas Fields of the Texas Panhandle. *Bull.* Amer. Assn. Petr. Geol. (1926) **10**, 733.

[12] F. Gouin: Geology of Beckham County. Okla. Geol. Survey *Bull.* 40-M.

Because the fault plane is considered to dip northward the surface expression of the fault would be expected to occur south of the indicated projection of the fault trace on the magnograph.

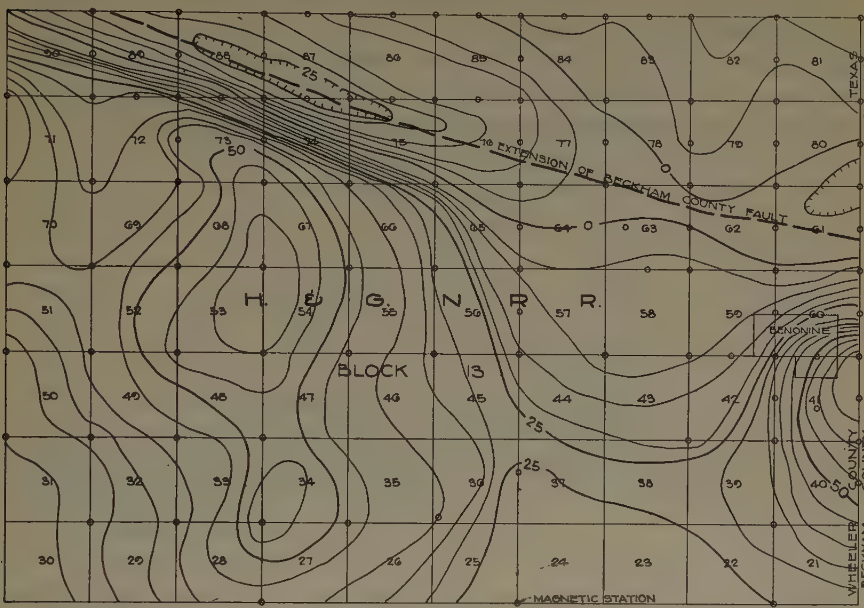

FIG. 13.—CONTOUR MAGNOGRAPH OF 70 SQUARE MILES IN SOUTHEAST WHEELER COUNTY, TEXAS.
Contour interval, 5 instrument scale divisions ($\Sigma = 0.75°$) or approximately 50 gammas.

SHOESTRING OIL POOLS OF EASTERN KANSAS

Fig. 14 shows the magnetic expression of the Bush City shoestring oil pool in Anderson County, Kansas.[13] The producing sands of Anderson County represent individual units of deposition, classified as resembling bars, beach deposits, or channel fillings, according to their shape and distribution. The Bush City shoestring is considered to have originated as a channel filling. It is found from 20 to 40 ft. below the top of the Cherokee shale at a depth of about 600 to 800 ft. The depth varies locally because the sand body passes over a variety of minor structural features. It is over 13 miles long and averages only about ¼ mile wide. The producing sand is lenticular in cross-section, with a maximum thickness of 55 ft., composed of fine-grained sand with silt laminae, and saturated with oil. This has been thoroughly outlined by drilling, the eastern end showing a dissipation of the sand body into a broad sandy shale and

[13] H. H. Charles: Oil and Gas Resources of Kansas, Part VII, Anderson County. Kans. Geol. Survey *Bull.* (1927) 6.

the western end showing a transition through a heavy oil phase into barren sand.

Fig. 14 shows that in this area there is a local magnetic anomaly of considerably greater magnitude than that which is associated with the shoestring pool itself. Thus it becomes necessary to look for an anomaly superimposed upon an anomaly, just as one would look for a minor structure superimposed upon a major structure. The possible significance of the larger anomaly has not been subjected to analysis.

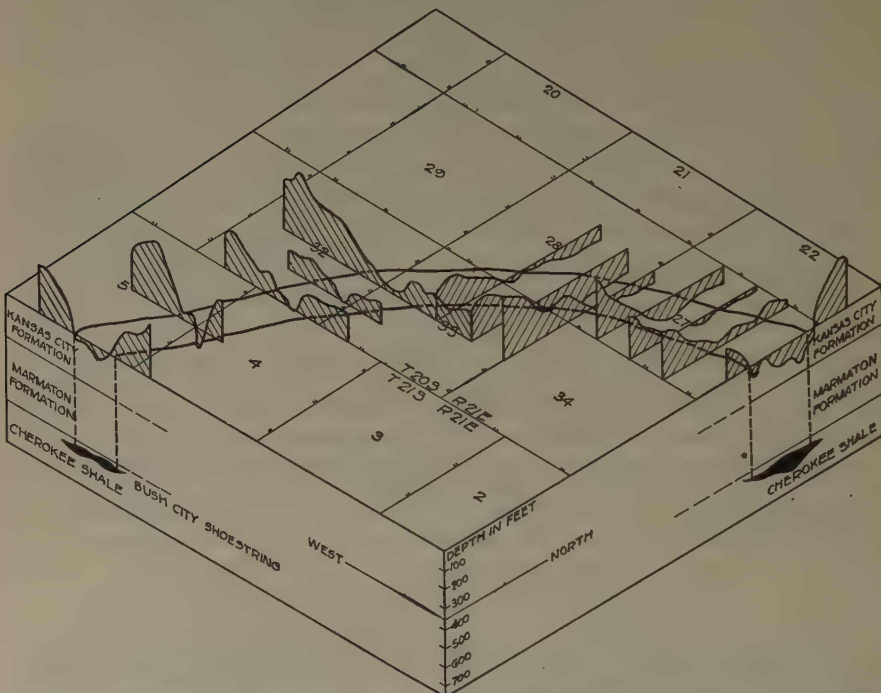

FIG. 14.—ISOMETRIC PROFILE MAGNOGRAPH OF BUSH CITY SHOESTRING IN ANDERSON COUNTY, KANSAS.

The magnitude of the anomaly that can be seen to be associated with the shoestring oil pool is between 20 and 40 gammas. The profiles in sec. 4, 5, 32 and 33 show magnetic variations which are clearly related to the producing area, and which would be recognizable if found in an undrilled area of the region. The profiles in sec. 27 and 28 show slight magnetic reflections of the producing area; but these would never be recognized in undrilled terrain. A profile through the east half of sec. 26 crosses the projected strike of the sand without showing any appreciable reflection of it. No drilling has been done in this area. This combination of circumstances might be interpreted as meaning that the wells themselves are responsible for the magnetic anomaly (especially

since the effect of their presence would be to induce negative readings except at points very near them) were it not for the fact that geological evidence indicates that the shoestring in this vicinity loses its distinct character. There remains the possibility, then, that the magnetic anomaly is truly portraying the sand itself. The broadening of the magnetic low in the profiles in sec. 27 corresponds to an indicated broadening of the sand. Nevertheless the actual *practicability* of finding other shoestrings by the magnetic method remains questionable on account of the intense detail necessary, the presence of other anomalies in the area and the unsettled question of whether an undrilled sand body would give the same reaction as that produced by the drilled sand body.

Mineralized Intrusives of the Santa Rita Mountains, Arizona[14]

Fig. 15 shows two magnetic profiles across geologic features encountered in the American Boy mine, near Patagonia, Arizona. In the vicinity of this mine an old and shattered monzonite has been intruded successively by dacite, aplite and rhyolite. Each intrusion was accompanied by severe shattering of the rocks existent at the time of intrusion. Consequently faulting has offset the older contacts and dikes, and later rhyolite extrusions have concealed some of the faulting.

Profile *a* shows the magnetic reaction connected with the intrusives of dacite into the monzonite. The magnetic permeability of the monzonite exceeds that of the dacite, so that the presence of the latter is registered by a magnetic low. Profile *b* shows the magnetic expression of the aplite and rhyolite dikes which are intruded into the monzonite. The aplite dike forms the hanging wall for a valuable silver-bearing galena seam, so that its traceability is of decided economic importance. Over a linear distance of 300 ft. it was offset along five different fault planes. The rhyolite dike

a

b

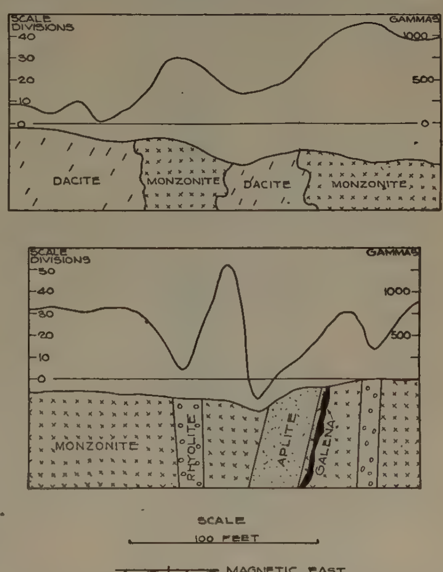

Fig. 15.—Magnetic profiles and geologic sections from the American Boy Mine, Patagonia, Arizona.
a. Location of intrusive contacts. *b*. Location of acid dikes.

[14] F. C. Schrader: The Mineral Deposits of the Santa Rita and Patagonia Mountains, Arizona. U. S. Geol. Survey *Bull.* (1915) 582.

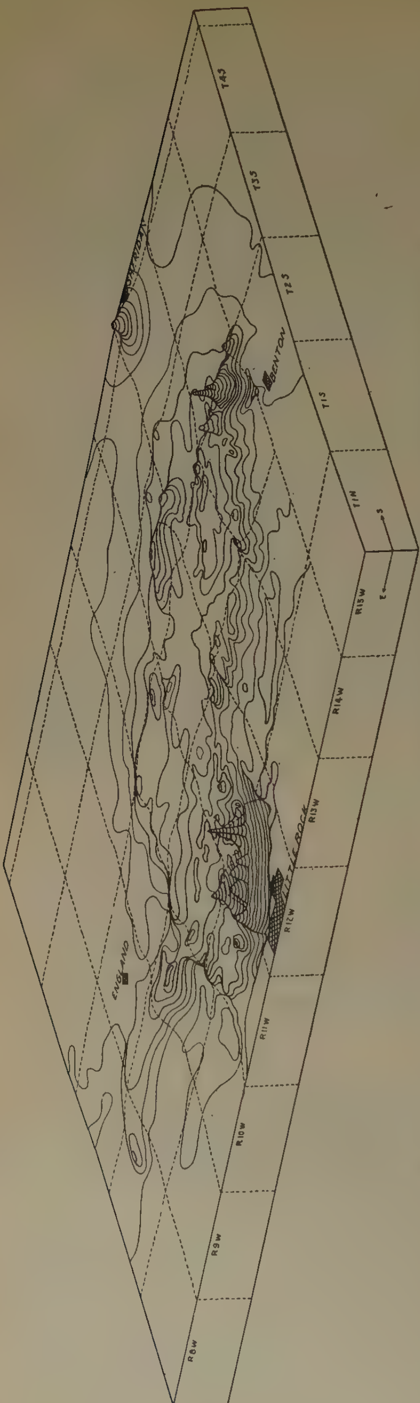

Fig. 16.—Isometric contour magnograph of bauxite region in Central Arkansas.

was associated with gold values, and the magnetic survey served to corroborate its eastward extension where it was concealed by surface material.

Bauxite Area of Central Arkansas

Fig. 16 shows an isometric contour magnograph over an area of 1488 square miles in central Arkansas, which embraces the bauxite-producing districts.[15] The bauxite ore deposits are thought to result from the weathering of the syenite rock masses which protrude through flat-lying Tertiary sediments immediately south of Little Rock and west of Benton. These masses have been exposed by post-Tertiary erosion. The possibility that post-Tertiary erosion has not been sufficiently extensive to bring about the exposure of other igneous masses that might still be buried in the area seemed to justify a magnetic survey for the purpose of indicating their presence. The fact that the bauxite is genetically and geographically related to the known igneous masses gives rise to the possibility of discovering other bauxite deposits associated with any igneous masses not yet exposed.

The survey discloses what seem to be indications of an

[15] N. H. Stearn: A Geomagnetic Survey of the Bauxite Region in Central Arkansas. Ark. Geol. Survey *Bull.* 5.

extensive igneous province lying beneath the Tertiary coastal plain sediments. It is nearly 400 square miles in extent, and appears to be fringed with outlying plugs and dikes. The magnetic reactions are of a high order of magnitude, coming from basic rocks as well as the nephelite syenite masses. The contour interval for the magnograph is 200 gammas. Anomalies as high as 3000 gammas were found in the area.

Diamond-bearing Peridotites of Arkansas

Fig. 17 shows a magnetic profile across one of the known occurrences of the diamond-bearing peridotites in Pike County, Arkansas. From three of the four known occurrences of the basic volcanic rocks found here several thousand diamonds ranging in weight up to 20¼ carats have been produced. The area lies close to the boundary between the folded Car-

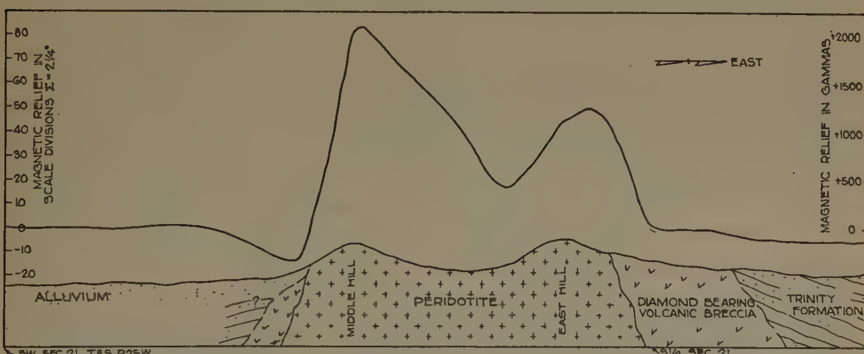

FIG. 17.—MAGNETIC PROFILE AND GEOLOGIC SECTION OF PRAIRIE CREEK PERIDOTITE NEAR MURFREESBORO, ARKANSAS.

boniferous rocks of the Ouchita uplift and the comparatively unfolded Cretaceous rocks of the coastal plain. According to Miser[16] the evidence points to the conclusion that the period of volcanic activity indicated by the peridotite intrusives occurred in the interval between the deposition of the Trinity formation and that of the Tokio formation; *i. e.*, between Lower and Upper Cretaceous time.

The principal exposure of the volcanic plugs shows a core of olivine porphyry associated with volcanic breccias and tuffs containing shale and sandstone fragments probably derived from deep-seated Carboniferous rocks. Miser has pointed out evidences of extensive igneous activity during which hitherto undiscovered igneous rocks may have been injected, which have not yet been exposed at the surface or which have been exposed and later concealed. That this condition actually exists in adjoining regions is established by the discovery of igneous rocks in deep

[16] H. D. Miser and E. S. Ross: Diamond-Bearing Peridotite in Pike County, Arkansas. U. S. Geol. Survey *Bull.* (1923) 735.

wells near Sheridan and Rison, Ark. But whether or not these rocks are diamond-bearing has not been established.

The magnetic profile shows an intensity over the plug of about 2000 gammas, and the shape of the profile indicates a sharp and steep boundary between the intrusive and the sediments, suggesting that it is an isolated injection from greater depth rather than a protrusion from a large basic igneous body near the surface. A detailed geomagnetic survey would unquestionably locate any concealed igneous bodies of the character and magnitude of the one shown in Fig. 17, or would definitely eliminate the possibility of the existence near the surface of such igneous bodies. Already in central Arkansas evidence of the presence of at least seven other basic igneous intrusives has been found. These are all buried under considerable thicknesses of coastal plain sediments.

Depth-finding by Vector Analysis[17]

Fig. 18 shows a magnetic depth-finding profile across a periodotite plug buried over 3000 ft. beneath the coastal plain sediments of Cleveland County, Arkansas. The ideal situation for the application of this method of depth-finding involves a single isolated point as the source of the magnetic anomaly, surrounded by a homogeneous medium. Although this situation is geologically impossible, in certain cases the approximation to it is sufficiently close to render the results usable.

This depth-finding method is based on the principle of vector analysis. In Fig. 18 the vectors I represent the actual recorded inclination by their direction and the actual magnetic intensity by their length. The vectors I have been resolved into the components In and Ia. The vectors In represent the component of normal magnetic intensity by their length and the normal inclination by their direction. Therefore, the vectors Ia represent the components of abnormality required to produce the resultant magnetic intensity and direction I. The locus of intersection of the extended components of anomaly Ia locates the position of the source of anomalous force.

Given the normal inclination and intensity, the anomalous inclination and the anomalous intensity at intervals over the anomaly, one can readily derive an approximation of the depth to the source of that anomaly by graphic methods. The anomalous inclination can be obtained by using the Hotchkiss Superdip as a dipping circle. The anomalous intensity can be obtained by converting the same instrument into a magnetometer, as previously explained. The facility with which the depth-finding operation may be accomplished is evidenced by the fact that the

[17] N. H. Stearn: Recent Experiments in Depth Finding by Magnetic Triangulation. *Eng. & Min. Jnl.* (1930) **129**, 396.

operation, the results of which appear in Fig. 18, was accomplished in about one-half day. In this case the maximum discrepancy between the computed depth and that shown by well data is about 3.1 per cent. This, however, is an exceptionally favorable geologic set-up for the application of this depth-finding method.

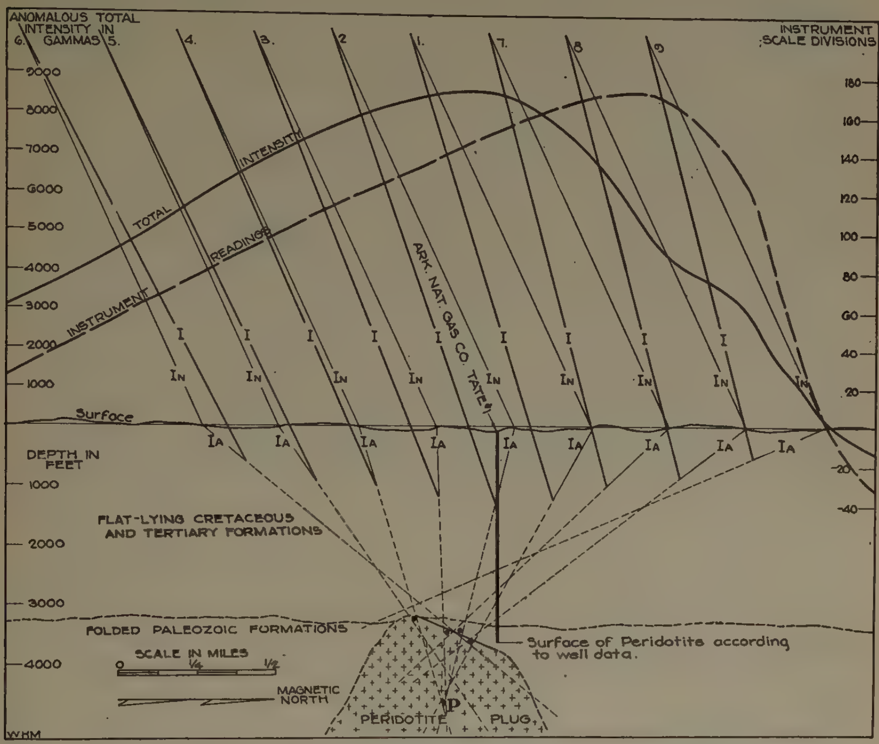

FIG. 18.—MAGNETIC DEPTH-FINDING PROFILE AND SECTION ACROSS PERIDOTITE PLUG NEAR RISON, ARKANSAS.

I. Recorded magnitude and direction of magnetic intensity.
IN. Magnitude and direction of normal component of magnetic intensity.
IA. Magnitude and direction of anomalous component of magnetic intensity.

SUMMARY

The foregoing examples of the application of the geomagnetic method to exploration by the use of the Hotchkiss Superdip are far from exhaustive even of the types of problems to which the method and the instrument may be applied. Nor is the implication intended that similar results may not be obtainable with other instruments. In categorizing the Superdip, however, it should be borne in mind that the instrument has been designed as a facile tool for the use of mining engineers and economic geologists, and that its use is recommended for magnetic anomalies of an order of magnitude greater than 10 gammas.

DISCUSSION

(*Donald H. McLaughlin presiding*)

W. O. Hotchkiss, Houghton, Mich.—Mr. Stearn, who has done the work in developing this needle, called it the Hotchkiss Superdip. I suggested that it should be called the Stearn Superdip, because he had done the work and deserved the credit for developing it. I had found the instrument too delicate in its readings for the purposes for which I had devised it. In the pre-Cambrian shield glacial covered areas, I found that it gave us data far beyond those which we were able to translate geologically.

A friend of mine made the remark that to be a real geophysicist a man should have a doctor's degree in mathematics, another one in physics, and another one in geology; merely having a doctor's degree in geology would not do. Many of us have only one degree, consequently the formulas and mathematical presentation necessary for much geophysical material goes over our heads, but I want to hold out a little ray of hope to all who listened to the papers today with interest. The development of this superdip principle was a result of mathematical computations. I had a man working for about three months on mathematical computations with regard to the ordinary dip needle, constructing a most elaborate series of curves to find in what position we could balance that ordinary dip needle to get the most sensitive readings. After I had paid this gentleman for all the work he had done, the very simple, elementary principle of the superdip came to me, a thing which any student of high school physics should have been able to work out but which came, as most simple things do, after one has gone into a lot of complicated investigation to surround the subject.

Now, geophysics is in the stage of complicated investigation in which we need the doctor's degrees in mathematics and in physics. I sometimes remark quietly to myself that I think doctors of mathematics and doctors of physics have run away with it to some extent and have gotten away from us geologists and engineers, who really must apply geophysics if it is going to be applied widely and commonly in the problems that come to us from week to week or month to month. I think out of this are going to come eventually certain relatively simple, usable principles in geophysics, which are going to be susceptible of application by those of us who are not so well versed in the mathematics and physics. After all, the thing that counts is proper interpretation. It is the interpretation which, as mining engineers and as economic geologists, we want reflected in the discovery of orebodies or materials of economic importance.

The first thing in looking over any property is to make a geological reconnaissance. We can eliminate then a certain percentage, and usually a large percentage, of any given area from further consideration. The next step is to make a *detailed* geological survey on the areas which still show evidence of promise. The next step is where our geophysics comes in. Before geophysics was available for use we went from that detailed geological survey directly to the most expensive methods of exploration, sinking test pits or shafts or drilling. Now geophysics comes in between the detailed geological survey and the still more expensive methods of actually getting underground and getting hold of a specimen of the material so that we can see with our eyes what is there. Geophysics is going to be most useful as it is applied in a logical series of events in the study of any given situation. That I think is a thing which all of us need to remember.

There is another thing for which I would plead on the part of the mining engineers. We are not going to know in our generation all about the electrical properties, the rigidity properties, or the magnetic properties, of any large portion of areas in which we may be interested, on which we would like to have the information. We know,

however, vastly more in a general way about the magnetic properties of the earth than we do about any other of the geophysical properties; consequently we have more to start from. Therefore, I want to make a plea to the average mining engineer for the use of the ordinary compass and the ordinary dip needle. They are usable vastly beyond any use that they are put to at present. Nobody can say whether a dip needle or a horizontal compass survey will tell anything about a given area or not. The only way to find out is to get out and try it. If you find any anomalies there, your problem is to interpret. It does not take long. It is not expensive to find the anomalies that you can get with the cheapest and most simply operated instruments, and I would urge every man who has a local problem of any sort to follow it up.

I was much interested the other day in reading a paper by Mr. Barton. I was interested because it indicated the possibility that I had overlooked an opportunity to make many millions of dollars, one of those cases we like to look back at and wonder what might have been the outcome if we had acted in a different manner. This superdip principle was developed about 1914 or 1915 first, and slowly, as time permitted, tested out until we found that it would give delicate readings. When I was in Washington in the very beginning of the war, I met Mr. Henry Doherty and suggested that we could survey magnetically, with adequate detail to detect salt domes, the whole Gulf coastal country for something like one-half million dollars. After I had explained the principle to him he said that it looked good, that he wished to see one of his assistants about it, and that I should consult the said assistant also. The assistant was very busy with war work, and so was I, and while we had several appointments to get together in 1917 we never did. It was seven years before Mr. Barton used the first magnetometer in the Gulf country. In that seven years by using magnetic surveys we could have made millions, maybe.

A Magnetic Method of Estimating the Height of Some Buried Magnetic Bodies*

By A. S. Eve,† Montreal, Que.

(New York Meeting, February, 1931)

In the spring of 1930, the question was raised as to the possibility of estimating the depth to which the pyrrhotite-nickel deposit at the Falconbridge mine extended in the earth. This body is 7500 ft. long, it dips at 80° to the north, and it is fairly straight with slight sinuosities.

Dr. F. W. Lee suggested that it might be possible to solve the problem by measuring the vertical magnetic intensity, with a good magnetometer, both near the surface of the earth and also at a level a few feet higher in the air. The writer regarded this suggestion with some scepticism, but it was decided to put the scheme to a practical test. In July, 1930, Dr. Lee, Dr. Keys and the writer, with Mr. Joyce and Mr. Belloc as assistants, made a firm, light wooden platform (Fig. 1) measuring 4 by 4 ft. on the top, which was 10 ft. above ground. This platform could be moved easily a few yards by four men, and carried across country in a wagon or by motor truck. Some iron nails were used in its construction, but we were not able to detect their influence on the Askania magnetometer. Nevertheless, in future work, the use of iron should be completely avoided.

Fig. 1.—Vertical Askania magnetometer on a platform 10 ft. high.

Readings were taken along a traverse at right angles to the orebody, at several stations, both on the ground and on the platform. The excess readings, over the dike, as compared with those about 800 ft. away, are shown in Table 1. The largest reading, 6.8, is open to suspicion and should be rejected. The depth of the top of the orebody is known by electrical survey, and by actual drilling, to be about 112 to 123 ft., and the covering is sandy glacial drift.

Similar readings were taken over a long straight magnetic diabase dike. These also are shown in Table 1. The magnetic vertical intensity is consistently less at the higher level, which is 10 ft. above the lower level.

* Published with permission of the Geological Survey of Canada.

† Director of the Department of Physics, McGill University.

In the early work, with these results, the orebody was regarded as a vertical bar magnet, but the results thus obtained were disconcerting, and it became apparent that the orebody was not like a bar magnet but rather like a vast number of vertical bar magnets side by side stretching for 1½ miles east and west. In fact, the orebody was a long sheet, about 60 to 70 ft. wide, with the south pole magnetism all along the top and the north pole along the bottom.

TABLE 1.—*Readings over Orebody and Dike*

Station	On Ground	On Platform	Differences
READINGS ALONG TRANSVERSE AT RIGHT ANGLES TO OREBODY			
200N	30.5	30.8	−0.3
150	28.9	27.2	1.7
125	32.8	30.5	2.3
100	35.8	29.0	6.8
75	33.2	30.6	2.6
50	29.6	27.0	2.6
0	17.4	16.5	0.9
READINGS OVER STRAIGHT MAGNETIC DIABASE DIKE			
200N	−5.0	−6.0	1.0
150	4.7	3.1	1.6
100	10.9	8.4	2.5
75	12.0	10.3	1.7
50	11.0	10.0	1.0
25	14.4	10.4	4.0
0	11.1	10.0	1.1

The mathematics of such a case is remarkably simple. Let the observer O (Fig. 2) be at a height D straight above the vertical magnetic body, of which the height is h and breadth is b.

Let $+m$ be the pole strength per square centimeter at the bottom, and $-m$ at the top. Consider an element dx long and b wide at a horizontal distance x from the observer at O. The vertical pull on unit pole at O due to this element is $\frac{mbdx}{D^2 + x^2} \frac{D}{\sqrt{D^2 + x^2}}$; therefore the whole vertical pull of the upper magnetism is

$$\int_{-\infty}^{+\infty} \frac{mbDdx}{(D^2 + x^2)^{3/2}}$$

Put $x = D \tan \theta$ so that $dx = D \sec^2 \theta \, d\theta$, and the integral becomes

$$2\int_0^{\frac{\pi}{2}} \frac{mb}{D} \cos \theta \, d\theta, \text{ or } \frac{2mb}{D}.$$

The magnetism at the bottom of the dike will contribute similarly to an amount $\frac{2mb}{D + h}$, but this will be a repulsive force, so that the whole vertical magnetic intensity, due only to the whole long vertical sheet, is

$$\frac{2mb}{D} - \frac{2mb}{D + h}$$

or

$$\frac{2mbh}{D(D + h)} \qquad [1]$$

If the observer goes to the top of his platform, at a height H above his former level, the vertical intensity in gauss is found by putting $D + H$

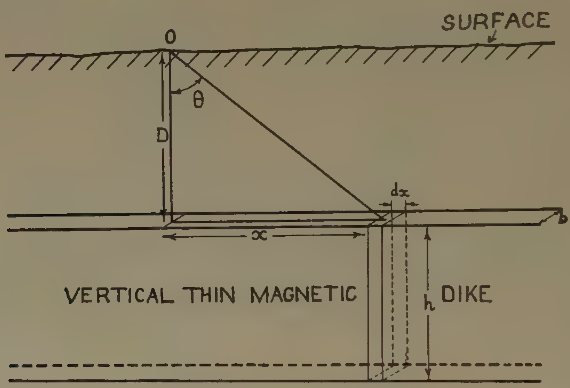

FIG. 2.—DIAGRAM FOR CALCULATING THE VERTICAL MAGNETIC INTENSITY DUE TO A LONG, THIN VERTICAL MAGNETIC DIKE.

in place of D in equation 1, and so our theory is complete. On the platform the pull is

$$\frac{2mbh}{(D + H)(D + H + h)} \qquad [2]$$

Suppose that the ratio of the two excess readings, above the normal, of the magnetometer on the ground and on the platform, at the same station, is denoted by r, then r is the ratio of the expressions 1 and 2, so that

$$(D + H)(D + H + h) = rD(D + h) \qquad [3]$$

Dr. D. A. Keys proceeds to reason as follows:

"Let us suppose that for a first approximation, h is very much greater than D. In such a case we obtain at once

$$\frac{D + H}{D} = r$$

$$\text{Hence } D = \frac{H}{r - 1}$$

"In our experiments, let us suppose that the excess readings over the orebody are 33.2 and 30.6, when the instrument is on the ground and on the platform respectively. Using these values, we first calculate the approximate thickness of the overburden D, obtaining

$$\frac{D + H}{D} = \frac{D + 10}{D} = \frac{33.2}{30.6} = 1.085$$

from which we obtain

$$D = 118 \text{ ft. approximately.}$$

This gives the distance from the surface to the south *pole* of the vein, and since the magnetic poles are always a small distance inside the ore, the depth to the top of the ore would be less than this. The above result is thus in fair agreement with the known depth of from 110 to 120 feet.

"We may now proceed to find the approximate thickness or extent of the vein downwards in a vertical direction. Using the value of $D = 118$ ft. found above, and inserting it in equation 1 we obtain

$$\begin{aligned} 128(128 + h) &= 1.085(118 + h)118 \\ 16{,}384 + 128h &= 15{,}108 + 128.03h \\ 1276 &= 0.03h \\ h &= 42{,}000 \text{ ft. approximately.} \end{aligned}$$

This value is large owing to the fact that we actually assumed that it was large in determining the value of D.

"If we take the value of $D = 120$ ft., which is about the value that is found from electrical surveys and diamond-drill records, we obtain for h a more reasonable depth. Thus

$$\begin{aligned} 130(130 + h) &= 1.085 \times 120(120 + h) \\ 1690 + 13h &= 1561 + 13.02h \\ 129 &= 0.02h \\ h &= 6000 \text{ ft.} \end{aligned}$$

"The reader must remember that these calculations are made in the case of a vein which has a vertical dip, while the vein we are considering in the field has a variable dip, which may amount to as much as 80° to 60°.

"Turning our attention to the diabase dike, we may make similar calculations. The actual reading on the ground was = 11.1 divisions, but the reading of the instrument at a distance was −16.3, which gives the value of the excess reading at the central station = 27.4 divisions. On the platform, the corresponding reading was 26.3 divisions, which is 1.1 divisions less. Using these values and supposing for a first approximation that H is much greater than D, we may find the value of D, which is the approximate thickness of overburden. Thus

$$D = \frac{10}{\frac{27.4}{26.3} - 1} = \frac{10}{1.033 - 1} = \frac{10}{0.033} = \frac{10,000}{33} = 333 \text{ ft.}$$

"We know by experience that this value is too small. Let us take, therefore, a larger value, say 340 ft. for D, and calculate h the thickness in a vertical direction of the diabase dike from equation 1.

$$\begin{aligned} 350(350 + h) &= 1.033 \times 340(340 + h) \\ 122,500 + 350h &= 119,415 + 351.22h \\ 3085 &= 1.22h \\ h &= 2500 \text{ ft. approximately} \end{aligned}$$

"These results confirm our view that the diabase dike is under a greater overburden than the pyrrhotite-nickel vein. The magnetometer curves also indicate that the diabase dike is much broader than the vein. We see in this way that the calculations do give some idea of the extent and overburden of a magnetic vein or dike, but unless the dip of the vein is practically vertical the method will be only approximate at best. But such approximate measurements may often be of service, and it is for that reason that they are given here."

Dr. L. V. King has extended this theory for the case of a *thick*, very long dike, and a summary of his results may be briefly stated.

1. The magnetic potential due to a long line, with pole-density ρ per unit length, at shortest distance R from the observer, is

$$2\rho \log R.$$

2. The vertical force in dynes on unit pole due to the horizontal top of a broad dike, with surface density m, is

$$2m\left(\tan^{-1}\frac{X + b}{Y} - \tan^{-1}\frac{X - b}{Y}\right)$$

where $2b$ is the breadth of the dike, Y the height of the observer above it and X the horizontal distance from the top center line.

This *vertical* intensity may also be written $2m\ (\varphi_A - \varphi_B)$, or $2m\varphi$, where φ_A, φ_B, φ are the angles APM, BPM, APB (Fig. 3). The *horizontal* force due to the magnetism on the top of the dike

$$2m \log \frac{\sin \varphi_A}{\sin \varphi_B} \text{ or } 2m \log \frac{PB}{PA}$$

3. Similar results may be written for the bottom of the dike. In the figure $ABCD$ is a vertical section of an inclined dike which extends to infinity both ways, running at right angles to the section $ABCD$.

4. Hence the vertical and horizontal intensities at any point P, due to the whole dike, are

$$2m(\text{angle } APB - \text{angle } CPD)$$

and

$$2m \log \frac{PA}{PB} \cdot \frac{PD}{PC} \text{ respectively.}$$

By measuring both vertical and horizontal intensities over the ground, and on a raised platform a few feet (say 10 or 20 ft.) high, and subtracting

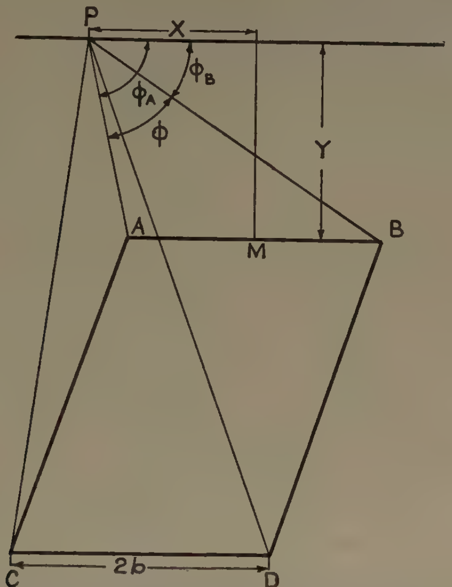

Fig. 3.—Diagram for calculating the horizontal and vertical intensities for a broad, very long, inclined magnetic dike.

these from the normal magnetic components at a distance from the dike, it is possible to deduce the breadth b, the vertical thickness h, the depth D and the surface density m.

Conclusions

It may be again pointed out that theory has far outstripped field practice. Moreover, the difference in the readings due to so small a height as 10 ft. makes the calculated vertical extent of the dike a highly uncertain quantity, and it is probable that in many cases it may be necessary to have a wooden tower with platforms at 10, 20 and 30 ft. above the ground. This seems an elaborate measure, but its cost is trifling compared with drilling, if accurate data can be obtained.

DISCUSSION

(Donald H. McLaughlin presiding)

D. A. Keys, Montreal, Que. (written discussion).—Dr. Eve has described a method of estimating the height of a very long, thin, magnetic dike by finding the vertical magnetic anomalies at points on a traverse line which crosses the strike of

the vein, readings being taken on the ground and on a raised platform over the center of the dike. Sufficient data on the heights of the dikes investigated in the Sudbury Basin were not available to check the values from the observations. An experimental verification of the principles involved in making such determinations is given in the present paper, a vertical sheet of mild steel artificially magnetized with south polarity on top serving as the thin magnetic dike. Inasmuch as no field results are available for the case of a buried thin magnetic dike of limited length, the following method might prove helpful should occasion arise.

A sheet of mild steel ½ in. thick, about 4 ft. high and 10 ft. long, supported on its edge vertically with the length perpendicular to the magnetic meridian (Fig. 4) was

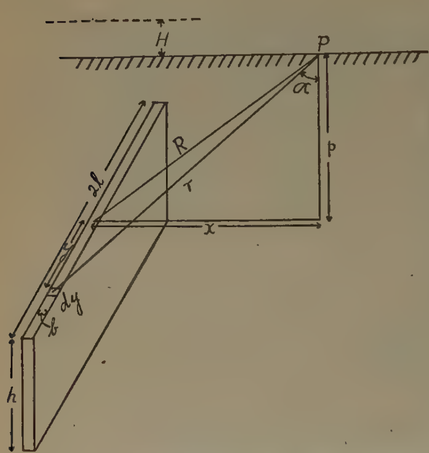

FIG. 4.—EXPERIMENTAL METHOD OF ESTIMATING HEIGHT OF NARROW MAGNETIC DIKE OF LIMITED EXTENT.

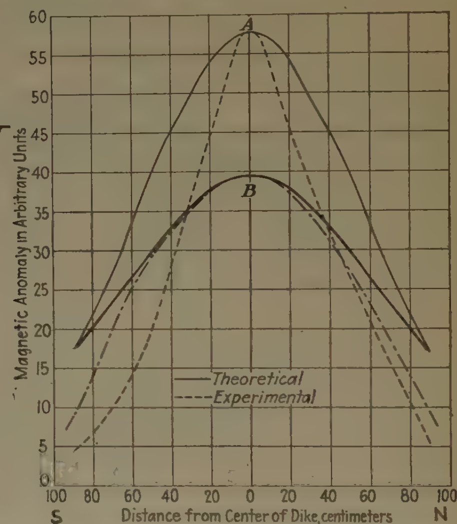

FIG. 5.—THEORETICAL AND EXPERIMENTAL VERTICAL MAGNETIC ANOMALIES AT TWO DIFFERENT LEVELS OVER A MODEL MAGNETIC DIKE OF LIMITED LENGTH.

magnetized by winding 10 turns of insulated copper wire lengthwise round the middle and passing a current of one ampere through the wire in a direction making the upper edge a south pole. This plate served as the model magnetic dike.

An Askania vertical variometer was placed on a platform so that the needle of the instrument was 88 cm. above the top of the iron plate, when vertically above its center. The vertical anomalies at points along the magnetic meridian which passes through the center of the plate were found for distances up to 90 cm. both to the north and to the south of the plate. A second set of readings was then taken along the same meridian, with the needle 111 cm. above the top of the iron plate. These two sets of readings correspond to taking observations on the ground and on a raised platform. The results of such a series of readings are shown in the broken curves of Fig. 5, the dotted line A showing the anomalies at a height of 88 cm. and the broken line B when the instrument is raised an additional 23 cm. above the top of the plate. The actual anomalies as read on the instrument were 38.2 divisions when over the center of the sheet on the lower level and 26.5 divisions on the level directly above. Hence the experimental ratio of the vertical intensities at the two levels due to the magnetic field of the iron plate is $38.2/26.5 = 1.44$. The readings taken were all multiplied by an arbitrary factor to compare the shape of the experimental curves with the theoretical curves shown as the full lines in Fig. 5.

Theory of the Experimental Verification

The magnetic intensity at a point P (Fig. 4), due to a thin magnetic dike of thickness b, length $2l$ and uniform pole strength $-m$ per unit area, is analogous to finding the illumination produced by a rectangular window, supposed uniformly bright. This latter problem is solved by Herman (Geometrical Optics, 210) but a simpler proof follows.

The magnetic intensity due to an element of area bdy at a distance r from P will be $\frac{mbdy}{r^2}$ and the vertical component will be $\frac{mbdy \cos \alpha}{r^2}$. But $\cos \alpha = \frac{p}{r}$. Hence the resultant vertical intensity at P, due to the upper magnetism, at a distance x cm. north or south of the dike is given by

$$
\begin{aligned}
V &= \int_{-l}^{+l} \frac{mbpdy}{r^3} \\
&= mbp \int_{-l}^{+l} \frac{dy}{(x^2 + p^2 + y^2)^{3/2}} \quad \text{since } r^2 = x^2 + p^2 + y^2 \\
&= 2mbp \int_0^l \frac{dy}{(R^2 + y^2)} \quad \text{where } R^2 = x^2 + p^2
\end{aligned}
$$

Let $y = R \tan \varphi$, then $dy = R \sec^2 \varphi \cdot d\varphi$ and we shall have

$$
\begin{aligned}
V &= 2mbp \int_0^{\tan^{-1}\frac{l}{R}} \frac{R \sec^2 \varphi d\varphi}{R^3 \sec^3 \varphi} \\
&= \frac{2mbp}{R^2} \int_0^{\tan^{-1}\frac{l}{R}} \cos \varphi \cdot d\varphi \\
&= \frac{2mbp}{R^2} \cdot \frac{l}{\sqrt{R^2 + l^2}} \\
&= \frac{2mbpl}{(x^2 + p^2)\sqrt{x^2 + p^2 + l^2}}
\end{aligned}
$$

A similar expression for the north poles on the bottom of the dike may be found by replacing p by $(p + h)$ where h is the vertical distance between the poles, the "height" of the dike. The numerical difference between these two values for a given x gives the resultant vertical field V at P.

In the above experiments, the magnetic poles were considered to be not on the surface but 10 cm. below; p will then be 98 cm. The half length $l = 152.4$ cm., $h = 102$ cm. and since $2mb$ of equation 1 is a constant, the variation of the vertical component of the magnetic intensity for different values of x may be computed. The full curve A, Fig. 5, indicates the theoretical variation in V when passing over the plate at the first level and the full curve B at the level 23 cm. above the previous position. In plotting these curves, the value of $2mb$ was taken as unity.

The experimental curves A and B fall off more rapidly than the theoretical values, especially when the magnetometer is nearer the dike. This may be due to magnetic leakage from the sides of the plate. When the magnetometer is at the upper level, shown at B, Fig. 5, the experimental and theoretical curves are in better agreement. The theoretical ratio of the vertical anomalies at the two heights is found by calculation from the above curves to be $57.67/39.34 = 1.46$, a value in excellent agreement with the experimental value of 1.44.

The deduction of the height h of the dike from field observations would be more difficult but possible. If r is the ratio of the readings at two levels over the center of the dike, at a vertical distance H apart (Fig. 1) then r is given by

$$\frac{1}{p\sqrt{p^2 + l^2}} - \frac{1}{(p + h)\sqrt{(p + h)^2 + l^2}}$$

divided by an exactly similar expression with $(p + h)$ replacing p in the denominator. In this expression everything is known but h, if we suppose the depth to the top of the dike has previously been found by an electrical resistivity method and the length $2l$ of the dike found by magnetic methods.

The results of the experiments and calculations presented in this paper confirm, in some measure, the general method of finding the approximate height of a buried dike, although the effects of discontinuities, consequent magnetic poles and other variations that may occur in nature make the method liable to large errors.

Historical Review

H. Lundberg and K. Sundberg, New York, N. Y. (written discussion).—Professor Eve's paper is an interesting addition to the knowledge of magnetic orebodies. In this country, very little has been printed about this type of magnetic methods and its use. Magnetic methods have been used to locate ore, especially in Sweden, since the beginning of the seventeenth century. The first instrument used was an ordinary compass and later the so-called Swedish mining compass was developed—probably around 1650. Later still, magnetometers were developed, by which the magnetic field is quantitatively investigated. The first Swedish magnetometer was designed by F. Wrede in 1843, but not until Professor Thalén designed his instrument for determination of the horizontal components in 1873, and E. Tiberg made the instrument for determination of vertical component in 1881, did the magnetometer come into practical and general use. Later contributions to the further development of magnetic methods in Sweden were made by Th. Dahlblom about 1895, and by Carlheim-Gyllenskiold in 1910, and recently by Karl Sundberg. Further extensive small-scale and field investigations of magnetic fields around ore models and at known orebodies have been carried out continuously since 1875 by students of the Royal Institute of Technology at Stockholm, whereby considerable material of great scientific and practical value has been collected.

The question of depth determinations was considered very early. E. Ask, in a paper written in Latin, on How to Find Iron Ores with Magnets, published in Upsala, Sweden, in 1723, described how to determine the depth to ore with a mining compass. It was not, however, until the magnetometer was introduced that more accurate methods were developed by (1) Thalén, (2, 4) Tiberg, (3) Dahlblom, (5) Carlheim-Gyllenskiold and (6) Sundberg.[1]

Depth determinations by magnetic methods comprise two problems: the determination of the *upper* and *lower* ends of the magnetic body. Generally the ore is considered equivalent to an ideal magnet, of which the poles are points and the position of these points is determined. The distance between the poles is considered by different investigators to be five-sixths to nine-tenths of the total length of the orebody, consequently the position of the upper and lower ends of the ore can be calculated approximately from the position of the poles. Thalén, Tiberg and Dahlblom

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[1] R. Thalén: *Kungliga Vetenskapsakademiens förhandlingar* (1874) No. 2.

E. Tiberg: *Järnkontorets Annaler* (1884) 29.

T. Dahlblom: Ueber magnetische Erzlagerstätten und deren Untersuchung durch magnetische Messungen. Verlag von Crag & Gerlach, Freiberg in Sachsen, 1899.

Carlheim-Gyllenskiold: A Brief Account of a Magnetic Survey of the Iron Ore Field of Kiirunavaara. Stockholm, 1910.

K. Sundberg: Några undersökningar rörande järnmalmers magnetiska fält. Rapport till Ingeniörsvetenskapsakademien, 1924.

methods are based on these principles. Carlheim-Gyllenskiold considered the ore equivalent to an elliptical cylinder, for which case he calculated the magnetic field and arrived at methods for depth determination which he applied to the famous Kiruna orebody in northern Sweden. Sundberg compared the magnetic field at Kiruna with the magnetic field around ore models of similar shape and found better agreement between the theoretical calculations of the field and observed values when assuming linear poles, one at the upper, the other at the lower end of the models, than when applying the Carlheim-Gyllenskiold theory. Methods for depth determination based upon the theory of linear poles were applied to the Kiruna ore, where an extensive and detailed survey was carried out, both at the surface and in a captive balloon above the orebody. It is interesting to note that the experience described by Professor Eve is very much in agreement with the results obtained at Kiruna.

Methods for Depth Determination

Some of the methods for depth determination referred to above are given below. The following symbols are used:

F = horizontal component of orebody's magnetic field.
R = horizontal resultant of earth's and ore's magnetic field.
G = vertical component of ore's magnetic field.
z = depth below surface to center of orebody.
$2l$ = distance between magnetic poles of orebody.
$t = Z - l$ = depth below surface of upper pole of orebody.
G max. = maximum of orebody's vertical component.
X = distance from G max. to observation point.

1. *Method by Thalén to Determine Depth Z to Center of Orebody*

The point A right above the orebody is determined (generally A is the intersection point between the magnetic meridian and the "neutral line," along which the horizontal component of the magnetic field from the ore is zero). If the distance between A and the point where F has its maximum value is X_1, then

$$Z = 2X_1$$

The method gives reliable results for steeply dipping bodies only and upon conditions that $\frac{l_2}{X_1^2 + Z^2}$ is small.

2. *Determination of Depth t to Upper Pole According to Tiberg*

If G max. = maximum value of vertical component of the ore's magnetic field and G_x = vertical component at distance X from G max., Tiberg gives the relation

$$t = X \times \cot \gamma$$

where

$$\cos^3 \gamma = \frac{G}{G \text{ max.}}$$

3. *Determination of t According to Dahlblom*

We choose any line through G max. and construct the component $F' = F \times \cos \beta$ along this line, β being the angle between the line and F. The resultant K between F' and G is inclined a varying angle ψ to the horizontal plane. For ψ 60° the direction of K is approximately toward the upper pole, which therefore is found by drawing a few lines in the direction of K. The intersection point is the upper pole (provided $\psi > 60°$).

4. *Determinations of Z After Tiberg*

$$Z = X \times \cot \varphi$$

where

$$tg\ \varphi = \sqrt{2 + \left(\frac{3}{2} \times \frac{G}{F}\right)^2} - \frac{3}{2} \cdot \frac{G}{F}$$

5. *Construction of Center of Orebody According to Carlheim-Gyllenskiold*

The resultant L between F and G is constructed and a point P located, where L is parallel to the orebody. A line drawn through P perpendicular to L should intersect the center of the orebody.

6. *Method by Sundberg*

We assume a linear pole of the length $2a$, this length being approximately the length of the vertical magnetic anomaly, and choose for our investigation the center line on the surface, which is perpendicular to the pole line and symmetrical to the ends of the pole. Along this center line the magnetic anomaly L, due to the linear pole in any point Q is

$L = \frac{2k}{\gamma} \times \sin \varphi_0$ the direction of L apparently being straight toward the pole line.

where

K = certain constant,
γ = distance from Q to pole line,
φ = angle OQA, where O = center point, A end point of pole line.

In the area of strongest total anomaly L the influence of the lower pole may be neglected and the approximate position of the upper pole is found by the method of Dahlblom described under 3. The position of the upper pole having been found, the values $L_c = \frac{2K}{\gamma} \sin \varphi_0$ due to the upper pole are calculated in the maximum value L max. as unit and geometrically subtracted from the observed L values L_0. For the difference $\overline{L_0 - L_c}$ the following formula applies:

$$\overline{L_0 - L_c} = \frac{L \text{ max.} \times \gamma \text{ max.}}{\gamma} \times \sin \varphi'_0$$

where γ max. = distance from L max. to upper pole, $\gamma_1'\varphi_0'$ refers to lower pole. This formula gives γ' and thus the position of the lower pole.

Application of Methods

In practice it is found that observation values within a small area right above the upper pole and toward the footwall must be used for methods 1 to 4.

To further illustrate the methods, some typical results of small-scale investigations over an ore model carried out at the Royal Institute of Technology at Stockholm may be referred to. The investigations were made by E. Rothelius in 1924 and the general arrangement is shown in Fig. 6, the ore model being an elliptic magnet 35 cm. long, 7 cm. wide at largest point and 8½ mm. thick. Table 2 gives the results obtained.

Referring to the table and the construction shown in the figure, the following conclusions may be drawn from the investigation as to the reliability in this case of the depth determination according to methods 2, 3, 4 and 5:

Method 2.—The values for $t = X$-cot γ obtained in points 10, 11, 12 are approximately correct, the actual t value being about 32.5 centimeters.

Method 3.—Construction, based upon points 7, 8, 9 and 10, gives approximately correct value.

Method 4.—The actual Z value is approximately 53.5 cm. and only points 10 and 11, therefore, give correct results.

Method 5.—Gives too low a value.

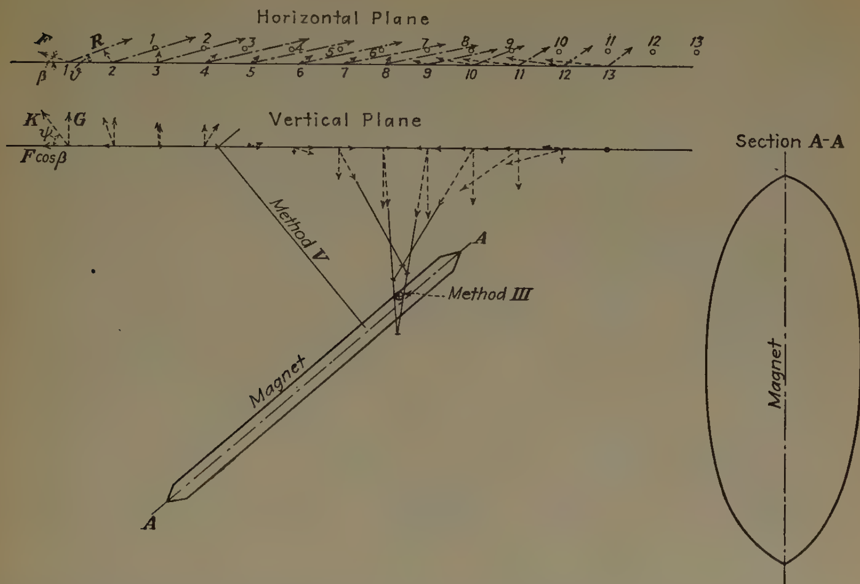

FIG. 6.—INVESTIGATION OF MAGNETIC FIELD OVER MAGNET TO TEST RELIABILITY OF MAGNETIC METHODS FOR DEPTH DETERMINATION. SCALE 1:2.

TABLE 2.—*Result of Small-scale Investigation of Magnetic Methods for Depth Determination*

Test	R	δ^a	F	G	$\frac{3G}{2F}$	$tg\varphi$	X	$\frac{X \cot \varphi}{z}$	$\frac{G}{G \max}$	cos γ	γ	cot φ	X	X cot γ
1	0.74H	20°	0.32H	−0.37H	$-\frac{3}{2}$	3.3	70	21.2	0.46	0.77	40° 40′	1.16	80	93.12
2	0.95H	17.5	0.16H	−0.32H	−3	6.32	60	9.5	0.43	0.76	40° 30′	1.17	70	81.97
3	1H	16.5	0.12H	−0.29H	−3.8	7.9	50	6.35	0.37	0.72	43° 50′	1.04	60	62.52
4	1.18H	13.5	0.19H	−0.25H	−2	4.45	40	9.0	0.31	0.68	42° 50′	1.08	50	53.95
5	1.18H	13.0	0.15H	−0.05H	−0.5	2.0	30	15.0	0.06	0.39	67° 00′	2.356	40	94.24
6	1.18H	12.5	0.20H	+0.08H	0.56	0.96	20	21.0	0.10	0.47	61° 50′	1.842	30	55.26
7	1.18H	11.5	0.20H	+0.35H	2.6	0.36	10	28.0	0.42	0.75	41° 20′	1.137	20	22.74
8	1.05H	12.5	0.07H	+0.66H	13.5	0.07	0	0	0.82	0.94	20° 0′	2.747	10	27.47
9	0.8H	13.5	0.12H	+0.80H	10	0.10	10	100	1.00	1.00	0	∞	0	undet.
10	0.63H	21.5	0.40H	+0.68H	2.6	0.36	20	55	0.99	0.98	11° 30′	3.22	10	32.15
11	0.45H	31.9	0.60H	+0.48H	1.20	0.56	30	54	0.61	0.85	31° 50′	1.61	20	32.22
12	0.41H	35.5	0.60H	+0.16H	0.26	1.18	40	36	0.20	0.58	54° 30′	1.202	20	36.06
13	0.35H	38.0	0.70H	−0.003H	0.005	1.41	50	35	0.01	0.22	77° 10′	4.33	40	175.6

δ = angle between observation line and direction of R.

D. H. McLAUGHLIN, Cambridge, Mass.—How far down is the lower pole of the Kiruna orebody?

H. LUNDBERG.—I cannot give you the exact figure, but it was somewhat around 2000 or 2200 meters.

METHODS OF INTERPRETATION

C. A. HEILAND, Golden, Colo. (written discussion).—Magnetic methods, like the gravitational methods of prospecting, are methods by which we determine the influence of all masses surrounding the instrument, and the interpretation of the results is often very difficult because we cannot apply a depth control such as is possible in certain types of electrical resistivity methods, and the seismic method of prospecting. Thus we generally do not apply direct methods of depth determination in magnetic methods as we do in resistivity and seismic prospecting, but in most cases we employ indirect methods of interpretation. The principle of such methods is to assume a certain configuration of the disturbing masses, to compute the sum of their effects on the surface, compare the field results with these theoretical results, and modify the assumed subsurface structure until a satisfactory agreement between field results and theoretical results is obtained.

Only when we have one disturbing formation of simple geometric dimensions, such as vertical or inclined dikes as often encountered in mining, is it sometimes possible to compute their depth. A factor that is helpful in determining the depth of such simple bodies is that in the theory of interpretation a vertical or an inclined magnetic sheet may be treated like a simple bar magnet, and, as usually such orebodies have some extent in depth which is fairly great as compared with their depth, it is in many cases even sufficient to consider only one pole.

During the last century, the methods of magnetic depth finding have been carried far in Sweden. In fact, the method applied by Dr. Eve is described by Dahlblom[2] as early as 1898, probably as an outgrowth of a method used in shafts to determine the depth to the orebody by measuring the vertical intensity in different depths.

On account of the difficulties in its practical application, this method does not seem to have been used very extensively, inasmuch as there are other methods available to determine the depth to the orebody with similar accuracy. These methods have been applied extensively in Sweden and are described in some detail in the work of Dahlblom referred to above, by R. Thalén[3] in 1879, and by E. Haanel[4] in 1904.

These methods of depth finding rest on the fact that the magnetic anomalies on the surface are determined as a function of horizontal distance from the orebody, and that they are, consequently, also a function of its depth. To take a concrete example, the vertical intensity curve measured at the surface above an orebody will show a rapid drop from the maximum when the orebody is very close to the surface; but, the curve is very flat, and the area occupied by the maximum very broad, if the orebody is at greater depth. Hence, the horizontal distance between the point of maximum vertical intensity and the point where the intensity has dropped to one-half of its maximum value has very early been used as a direct indicator of depth. The distance between the extreme maximum and minimum of the horizontal-intensity anomaly is likewise an indication of depth. In addition, the total anomaly may be represented as the resultant vector of the horizontal and vertical anomaly; and as these vectors are tangents to the lines of force, not only the depth to the orebody but also its extent (especially if it is inclined) may be determined by means of force line diagrams. Finally, the dip of an orebody may be determined from the ratio of the minimum divided by the maximum anomaly in horizontal intensity (C.A.H.).

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[2] T. Dahlblom: *Op. cit.*

[3] R. Thalén: Untersuchung von Eisenerzfeldern durch magnetische Messungen. Trans. by B. Turley. Leipzig, 1879.

[4] E. Haanel: On the Location and Examination of Magnetic Ore Deposits by Magnetometric Measurements. Dept. of the Interior, Ottawa, Canada, 1904.

Depth Rules

It has been shown in practice that the assumption mentioned above, that an orebody may be represented either by one or by two poles, leads to satisfactory results. Consequently, in the Swedish iron ore mining, very early an appreciable number of depth rules were derived. Table 3 gives the principal characteristics of 10 depth rules, some of them being taken from the most recent book on magnetic interpretation based on the theory of pole sequences.[5]

Table 3.—*Depth Rules*[a]

Rule No.	Depth Rule	Characteristics of e	Pole Distance	Author
1	$d = e$	$\Delta Z = \frac{1}{3}\Delta Z_{\text{max.}}$	1	Haanel, 1904
2	$d = \frac{4}{3}e$	$\Delta Z = \frac{1}{2}\Delta Z_{\text{max.}}$	1	Tiberg, 1899
3	$d = 0.7e$	$\Delta Z = 0$	1	Thalén, 1879
4	$d = \dfrac{\Delta d\sqrt{\Delta Z_1/\Delta Z_2}}{1 - \sqrt{\Delta Z_1/\Delta Z_2}}$	Obs. on scaffold or in shaft	1	Dahlblom, 1898
5	$d = e$	$\Delta Z = \Delta H$	1	Nippoldt, 1930
6	$d = \frac{4}{3}E$	E = dist. of ΔH max. and ΔH min.	1	Nippoldt, 1930
7	Intersection	Disturbance vectors ΔH and ΔZ	1	Thalén, 1879
8	Intersection	Disturbance vectors ΔT and ΔI	1	Stearn, 1930
9	Force line diagram	Disturbance vectors ΔH and ΔZ	2	Thalén, 1879
10	$\Delta \propto \sim 3\Delta \left(\dfrac{H_{\text{min.}}}{H_{\text{max.}}}\right)^b$ (dip)	Intensity ratios of H	2	Heiland, 1930

[a] d = depth; e = horizontal distance from a point vertical above the pole; ΔZ = vertical intensity anomaly and ΔH = horizontal intensity anomaly.

[b] Holds for dip angles between 30° and 90°, of an orebody in depth 1 and pole distance 2.

Airplane Work

There is no question that the method of observing magnetic intensities in different heights offers great possibilities. When seeking orebodies, the construction of a scaffold is sufficient, as the variation in the height of the observation point is comparable with the depth of the body. In applying the magnetic method to prospecting for oil-bearing structures, such a procedure would not lead to a satisfactory result, as for most effects the variation of the anomaly with altitude is small. Most of the magnetic anomalies observed in oil prospecting come from the crystalline basement, from a depth ranging, approximately, from 2000 to 10,000 ft., and even more.

In such case, the observation of magnetic intensity in an airplane flying the same traverse twice in different altitudes becomes practicable. Magnetic measurements from aircraft are not new; in fact, Edelmann, I believe in 1910, designed a vertical balance to be used in a balloon. The balance principle is not applicable in an airplane, of course, and another method must be used. The writer has designed a method for the purpose, and preliminary experiments have proved its applicability,

[5] A. Nippoldt: Verwertung magnetischer Messungen zur Mutung. Berlin, 1930.

although some difficulties are yet to be overcome. The apparatus consists essentially of an earth-inductor, rotating at a constant rate of speed about a horizontal axis which is oriented in the magnetic meridian. The inductor is designed essentially along the same lines as the well-known General Electric magneto-compass used in aircraft. The constancy of the speed of rotation is maintained by a synchronous motor driven by a tuning fork controlled by vacuum tube. The axis of the inductor is automatically oriented in the magnetic meridian by coupling it with a regular magneto compass revolving about a vertical axis and placed in the other wing of the craft. The current induced in the inductor is proportional to the vertical intensity and is recorded photographically with a galvanometer. The difficulties that have been experienced thus far with the method are due to the necessity of low flying in the majority of cases, the influence of the vibrations of the ship on the galvanometric recording, the necessity of correlating the magnetic with the topographic data, and errors introduced in the banking of the ship. However, it seems that most of these difficulties may be overcome. The advantage of such method would lie in the rapidity with which the magnetic data may be collected, and the possibility of surveying inaccessible areas.

This method of aerial magnetic mapping may seem impossible and fantastic at first glance, but it has already been tried in oceanic magnetic work, as a Bidlingmaier type of compass variometer has been installed in the Graf Zeppelin.

Application of Formula

I. Roman, Houghton, Mich. (written discussion).—Assuming the author's simplification and writing the forces in equations 1 and 2 as F_0 and F, respectively, we may equate the two values obtained for $(2mbh)$ and obtain the relation:

$$(F_0 - F)l - FHk = FH^2$$

where

$$k = 2D + h$$
$$l = D^2 + Dh.$$

The quantities F_0 and F are observed, while H is controlled by the experimental arrangements. The quantities k and l are simple combinations of the unknowns D and h. If two values of H are selected, there results a pair of equations linear in k and l, which may be solved for these unknowns. When k and l are known, we have the relation:

$$D^2 - kD + l = 0,$$

to determine the values of D, after eliminating the extraneous root. Finally, h is determined.

If more than two platforms are constructed, at different heights, a least square analysis will serve to reduce the errors of observation, but a closed solution will be obtained by using only two platforms, assuming the author's simplification to be sufficiently accurate. In this case, it is not necessary to assume that the value of h is large.

The method in its present state cannot be expected to furnish more than an approximation of a rough sort. Besides the objections noted by the author, and the errors due to the formal calculations, there seems to be a more fundamental error which is of frequent occurrence in the literature of magnetic exploration. The first error may be passed with but a simple comment. The quantities measured are so small that the results are likely to be the result of calculational manipulation rather than of observation. Thus, a change in the value of D from 118 to 120 ft. changes the value of h from 42,533 to 6450 feet.

The other error may be explained by analogy with light. The light from a distant source illuminates a three-dimensional object in a manner which is simple to explain

qualitatively, but difficult to evaluate quantitatively. The near side of a sphere if light and the far side is dark, the intensity of illumination on the bright side being calculated for a very small area by the usual cosine law, and the dark side having no illumination. For other bodies, the calculation is more complicated. Thus, for a cube, the line separating light and dark surfaces is determined by the orientation of the cube with respect to the rays of light.

With Induced Magnetism

With induced magnetism, the effect is even more complicated. The earth's field produces in a magnetizable mass a distribution of magnetic polarization which depends not only on the magnetic properties of the mass material but also on the way in which the mass lies in the ground and on the presence of other magnetic material. If we assume that all other magnetic material is infinitely remote, that the mass is magnetically homogeneous and that the earth's field is uniform, the problem is still far from simple. There are various manners of calculation, but one method will explain the complexity. We may consider the mass as divided into very thin filaments oriented along the direction of the earth's field. Since the filaments are considered as very thin, the magnetism of each may be represented as concentrated at its ends, the north end being south, magnetically. These filaments may be considered as having a pole strength proportional to their cross-sectional area. The total effect for the mass may be considered as the integral effect of the various filaments, the mass being replaced by two or more magnetic sheets, coinciding very nearly with the boundary of the actual mass. But the local intensity of magnetization is dependent on the shape and orientation of the mass and is usually difficult to calculate. The effect of the mass on a unit pole at an external point is also difficult to determine. But this difficulty of accurate determination does not justify neglect of the facts, such as often occurs. It seems to me that many of the difficulties in quantitative use of the magnetic method of exploration are to be found in the fact that theory is behind practice, and not ahead, as the author states. There is a need for extended theoretical calculations, to be checked against experimental results. While the engineer must simplify his problem, in the interest of speed of obtaining results, such simplifications should be justified in one of two ways. Either the general analysis must be shown to reduce to the simplified case or actual results must justify the means used in obtaining them. It seems to me that in magnetic exploration, neither method has been successful, quantitatively. This does not criticize the method for qualitative results.

To be specific, consider the case discussed in this paper. The author says that there is a dike, which is assumed as vertical, for simplicity. Assuming this to be the case, we still have the problem of the magnetization on the vertical sides, especially the north and south sides. While the intensity of magnetism may be low, the integral effect at an external point may be important. This has been neglected.

While the present paper may be considered as preliminary evidence of the value of the method, it seems that much more work is needed to make the method safe for the field observer, and much of this will need to be done on the theoretical side.

A. S. Eve (written discussion).—I entirely agree with Mr. Roman that much remains to be done. It is interesting, however, to find that the curves obtained near the center of the Falconbridge dike are reasonably close to theoretical curves (see p. 206). The horizontal component in the magnetic latitude of Sudbury is small compared with the vertical, so that the north-south magnetizations of the vertical sides is also small, and as the vein is not thick the resultant movement is perhaps almost negligible. Over the center of a uniform vertical dike running east and west the effect of the horizontal component on the vertical sides would in any case give a zero magnetic vertical intensity.

A Method for Determining Magnetic Susceptibility of Core Samples

By William M. Barret,* Shreveport, La.

(New York Meeting, February, 1931)

In order properly to evaluate the data related to geomagnetic surveys, it is highly desirable to have all the available information concerning the magnetic properties of the involved media. This is true of surface as well as subsurface formations and becomes of increasing importance as advancement in instrumental and interpretative technique permits the analysis of relatively slight magnetic reflections of buried masses.

Since the magnitude and areal distribution of the usually complex resultant magnetic field at the surface of the earth is a function of the susceptibility[1] of the involved matter, it necessarily follows that susceptibility determinations will afford valuable data for geophysical correlation.

The present method was developed to meet the need for routine analysis of specimens in the geomagnetic laboratory where it was felt that general utility, ruggedness, ease of manipulation and reproducibility of results were of greater importance than extreme sensitivity.

Electric Circuits

The electric circuits are illustrated diagrammatically in Fig. 1. Current from the battery, B, passes through the variable resistance R_1, reversing switch S_1, double-break key K_1 to switches S_2 and S_3. From S_2 it flows through variable resistance R_2, ammeter A_2, to the primary coils of the fixed mutual inductance M_f and variable mutual inductance M_v. From S_3 it passes through variable resistance R_3, ammeter A_3, to the primary winding of the fixed mutual inductance M_t, which is designed to receive the test sample. The ballistic galvanometer[2] or fluxmeter[3] G is connected in the circuit containing the double-break key

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[1] Throughout this paper the term "susceptibility" will refer to volume-susceptibility as distinguished from mass-susceptibility.

[2] A ballistic galvanometer is a moving magnet or d'Arsonval galvanometer having a long natural period. For a detailed discussion of the instrument the reader is referred to F. A. Laws: Electrical Measurements, Chap. II, Ed. 1. New York, 1917. McGraw-Hill Book Co.

[3] The fluxmeter is essentially a moving coil ballistic galvanometer in which the control torque and air damping are reduced to a minimum. The present paper will refer specifically to the ballistic galvanometer.

K_1, short-circuit key K_2, galvanometer shunt R_5, which is preferably of the Ayrton[4] pattern, variable resistance R_4, and the secondary coils of the mutual inductances M_f, M_v and M_t.

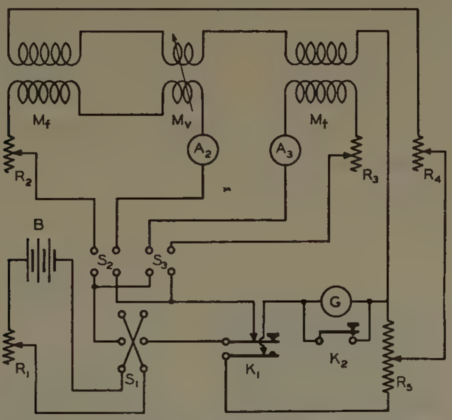

FIG. 1.—DIAGRAM OF ELECTRIC CIRCUITS.

FIG. 2.—COMPLETE APPARATUS ASSEMBLED FOR OPERATION.

Fixed mutual is on wall in background, while in extreme foreground is mutual inductance test coil. Vernier mutual is shown immediately behind telescope stand and ballistic galvanometer is on shelf. Optical system of galvanometer consists of telescope and scale, as illustrated. Ammeters, resistances, switches and keys are arranged for convenience of operator.

Fig. 2 shows the assembled apparatus. On the wall, in the background, may be seen the fixed mutual inductance M_f, while in the extreme

[4] The Ayrton or "Universal" shunt is distinguished by the fact that its multiplying values are independent of the galvanometer resistance.

foreground of the photograph is M_t. The variable mutual inductance M_v, used for vernier adjustment, is shown immediately behind the telescope stand and on the shelf is the ballistic galvanometer G, the deflections of which are read by means of the telescope and scale as illustrated. The ammeters, resistances, switches and keys are arranged for the convenience of the operator.

At the left of Fig. 3 is a detail view of the vernier mutual inductance M_v, the maximum value of which is approximately one millihenry. To the right is the mutual inductance M_t, which has a value of 9.86 millihenrys. The primary and secondary coils of M_t are layer-wound on a form bored to receive the sample container shown at the center of the

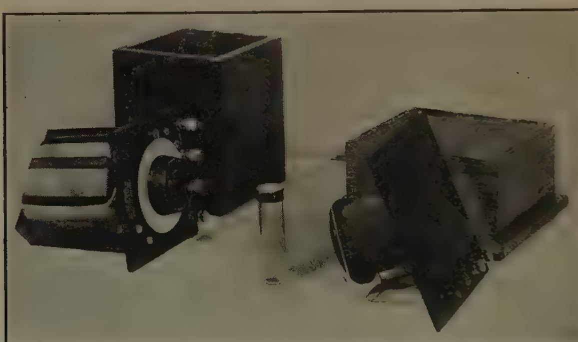

FIG. 3.—DETAIL VIEW OF VERNIER MUTUAL, SAMPLE CONTAINER AND MUTUAL INDUCTANCE TEST COIL.

Vernier consists of two coils, both of which are sections of cylinders. Inner coil may be rotated through 180°, corresponding to 100 divisions on silver-etched dial. Primary and secondary coil of test mutual are layer-wound on a form designed to receive sample container shown at center of photograph. Glass container has volume of 50 cu. cm. and is provided with aluminum friction cap.

photograph. This container, which is of glass, has a volume of 50 cu. cm. and is provided with an aluminum friction cap. The outside dimensions of the sample container are 2.7 cm. dia. by 10 cm. long.

MANIPULATION

The apparatus may be operated in accordance with the following available methods, the theoretical principles involved as well as the standardization of each arrangement being later discussed in detail.

1. *Null Method, Inductive Balance.*—Switches S_1, S_2 and S_3 are closed, energizing the primary coils of M_f, M_v and M_t. The value of the current I_3 is determined by the desired magnetizing force at M_t, while the magnitude of I_2 is varied until the galvanometer shows no deflection, with the vernier mutual inductance set at zero, when the primary circuits are opened or closed by the upper contacts of K_1. After obtaining this

balance with no sample at M_t the unknown specimen is introduced and brought to a cyclic magnetic condition by repeated reversals of S_1, after which the vernier M_v is adjusted until the galvanometer again indicates a balance. The reading of M_v is then proportional to the susceptibility of the unknown sample.

To avoid the galvanometer creep caused by induced secondary currents, it is well, when changing the setting of the vernier mutual, to short-circuit G by means of K_2. Also, when observing the initial behavior of the galvanometer with an unknown specimen, it is advisable to decrease the sensitivity by a proper adjustment of R_5. Further reference will be made to the regulation of the sensitivity as well as the appropriate damping resistance.

2. *Null Method, Current Balance.*—The arrangement is the same as for method 1. With the vernier mutual inductance remaining fixed, the value of I_2 is varied until the galvanometer indicates zero deflection when the primary currents are interrupted by K_1. When this condition is reached the magnitude of I_2 is a measure of the susceptibility of the unknown core material.

The value of I_2 may be determined with great accuracy by means of a potentiometer, though it is doubtful if this method possesses sufficient merit to warrant the additional complication.

3. *Seminull Method.*—Using the same arrangement as for the null methods, the galvanometer is adjusted for zero deflection, with no sample at M_t, by varying either the vernier mutual M_v or the current I_2. Then, with the unknown sample inserted in M_t, the deflection of G is noted when the primary currents are interrupted by K_1, the susceptibility being computed from the observed galvanometer throw.

The sensitivity of this method may be materially increased by fully depressing the double-break key when taking the deflection of G. With this operation the secondary circuit, containing the galvanometer, is opened immediately after the primary by means of the lower contacts of K_1, thus permitting the galvanometer coil to complete its swing practically undamped.

4. *Deflection Method, Closed Circuit.*—The primary circuit of M_f and M_v is opened by means of S_2. With S_1 and S_3 closed the field intensity of M_t is adjusted to the desired value and the unknown sample inserted. The exciting current I_3 is then interrupted by K_1 and the galvanometer deflection observed. The magnitude of the throw is proportional to the susceptibility of the specimen.

5. *Deflection Method, Open Circuit.*—The procedure is the same as for method 4, except that both upper and lower contacts of K_1 are opened, allowing the moving system of the galvanometer to function under undamped conditions. The susceptibility of the unknown sample is computed in the same manner as for the closed-circuit deflection method.

With the open-circuit deflection method, as well as the open-circuit operation of the seminull method, the double-break key should be held down until the galvanometer coil has completed its full deflection and returned to the zero position, at which point its motion is arrested by the short-circuit key K_2. In the closed-circuit deflection method and the closed-circuit operation of the seminull method the return of the galvanometer moving system to the equilibrium position may be hastened by opening the secondary contacts of K_1 after the throw has been obtained.

Theory

Null Methods

With reference to Fig. 1, switches S_1, S_2 and S_3 are closed, allowing currents I_2 and I_3 to flow through A_2 and A_3 respectively. Then, if the current I_3 is interrupted by means of the battery contacts of key K_1, the quantity of electricity discharged through the galvanometer by M_t will be

$$Q_t = \frac{KN_2I_3}{10^8R_t} \tag{1}$$

where Q_t is the quantity of electricity in coulombs;

K is a constant, equal to the total number of lines threading the secondary coil when the primary current is one ampere;
N_2 is the number of secondary turns on M_t;
I_3 is the primary current in amperes;
R_t is the total resistance of the secondary circuit in ohms.

When the current I_2, flowing through the primary circuit of M_f, is interrupted the quantity discharged is

$$Q_f = \frac{M_fI_2}{R_f} \tag{2}$$

where Q_f is the quantity of electricity in coulombs;

M_f is the mutual inductance in henrys;
I_2 is the primary current in amperes;
R_f is the total resistance of the secondary circuit in ohms.

In like manner, when the primary current through M_v is interrupted the discharge will be

$$Q_v = \frac{M_vI_2}{R_v} \tag{3}$$

where Q_v is the quantity of electricity in coulombs;

M_v is the mutual inductance in henrys;
I_2 is the primary current in amperes;
R_v is the total resistance of the secondary circuit in ohms.

By establishing the currents I_2 and I_3 in such directions that the algebraic sum of the discharges through M_f and M_v is opposed to the

discharge through M_t, and adjusting M_v for zero galvanometer deflection when I_2 and I_3 are simultaneously interrupted by opening the upper contacts of K_1, we have

$$Q_f \pm Q_v = Q_t$$

and

$$\frac{M_f I_2}{R_f} \pm \frac{M_v I_2}{R_v} = \frac{K N_2 I_3}{10^8 R_t}$$

or, since $R_f = R_v = R_t$, we may write

$$I_2(M_f \pm M_v) = 10^{-8} K N_2 I_3$$

or

$$M I_2 = 10^{-8} \phi N_2$$

where M is the algebraic sum of M_f and M_v in henrys;
ϕ is the total flux threading the secondary coil of M_t and is equal to KI_3.

The total flux ϕ may be conceived as the algebraic sum of the flux through the sample, the glass sample container, the aluminum sample-container cap and the air space between the sample container and inner coil winding of M_t. The susceptibility of glass differs from air by 0.174×10^{-6} while the variation between aluminum and air is 0.182×10^{-5}. As both of these values are below the order of sensitivity of the apparatus, we may consider the total flux as the resultant of the flux through the annular air space and the flux through the sample. Hence, for ferromagnetic and paramagnetic materials

$$\begin{aligned} M I_2 &= 10^{-8} N_2(\phi_a + \phi_s) \\ &= 10^{-8} N_2(H A_a + \mu H A_s) \end{aligned}$$

where H is the magnetizing force in gilberts per centimeter or gauss;
A_a is the cross-sectional area of the annular air space in square centimeters;
A_s is the cross-sectional area of the sample in square centimeters;
μ is the permeability of the sample.

Denoting the mean cross-sectional area of the inner[5] coil of M_t by A, and solving for the permeability, we have

$$\begin{aligned} M I_2 &= 10^{-8} N_2[H(A - A_s) + \mu H A_s] \\ \mu &= \frac{M I_2}{10^{-8} N_2 H A_s} - \left(\frac{A - A_s}{A_s}\right) \\ &= \frac{M I}{10^{-8} N_2 D I_3 A_s} - \left(\frac{A - A_s}{A_s}\right) \\ &= \frac{M I_2}{C I_3} - C_1 \qquad [4] \end{aligned}$$

[5] A is the mean cross-sectional area of the primary coil of M_t in square centimeters if the secondary is on the outside; if the secondary is wound inside the primary, A is the mean cross-sectional area of the secondary coil in square centimeters.

where D is a constant for the mutual M_t and is equal to the effective magnetizing force produced by an exciting current of one ampere;

I_3 is the magnetizing current in amperes;
C is a constant and is equal to $10^{-8}N_2DA_s$;
C_1 is a constant and is equal to $\left(\frac{A - A_s}{A_s}\right)$.

The susceptibility k may be obtained from the relation

$$k = \frac{\mu - 1}{4\pi} \quad [5]$$

Substituting the value of μ in terms of the susceptibility, we find

$$k = \frac{MI_2}{4\pi CI_3} - \frac{A}{4\pi A_s}$$
$$= \frac{MI_2}{C_2I_3} - C_3 \quad [6]$$

where C_2 is a constant and is equal to $4\pi C$;
C_3 is a constant and is equal to $A/4\pi A_s$.

Deflection Methods

Consulting Fig. 1, consider switches S_1 and S_3 closed and S_2 open. With no sample at M_t, if the current I_3 is interrupted, the total flux threading the air core will be

$$\phi_a = \frac{10^8RF}{N_2}d_a \quad [7]$$

where R is the total resistance of the secondary circuit in ohms;

F is the figure of merit[6] or ballistic constant of the galvanometer, expressed in centimeters;
d_a is the galvanometer deflection in centimeters.

Similarly, with the specimen in position, the total flux may be found from the following:

$$\phi_{a+s} = \frac{10^8RF}{N_2}d_s \quad [8]$$

The indicated extra flux due to the presence of the sample may be obtained by subtracting equation 7 from 8. Thus,

$$\phi_s = b(d_s - d_a)$$

where b is a constant and is equal to $10^8RF/N_2$.

---

[6] The figure of merit or ballistic constant is usually defined as the quantity of electricity in coulombs required to produce a deflection of 1 mm. on a scale 1 m. distant. The undamped ballistic sensitivity is frequently specified by instrument manufacturers. The sensitivity, when critically damped, is approximately one-third of the undamped sensitivity.

The increase in flux density due to the presence of the sample will be

$$B_s = \frac{b(d_s - d_a)}{A_s}$$
$$= b_2(d_s - d_a)$$

where b_2 is a constant and is equal to b/A_s.

The permeability may be found from the relation

$$\mu = \frac{B_s + H}{H}$$

or, since

$$H = \frac{10^8 RF}{N_2 A} d_a \qquad [9]$$

we have

$$\mu = \frac{b_2(d_s - d_a) + b_3 d_a}{b_3 d_a} \qquad [10]$$

where b_3 is a constant and is equal to $10^8 RF/N_2 A$.

In the above equation it should be noted that the term $b_3 d_a$ represents the effective magnetizing force and may be considered a constant for fixed values of H. Expressing equation 10 in terms of the susceptibility, we obtain

$$k = \frac{b_2}{4\pi b_3}\left(\frac{d_s - d_a}{d_a}\right)$$
$$= b_4\left(\frac{d_s - d_a}{d_a}\right) \qquad [11]$$

where b_4 is a constant and is equal to $b_2/4\pi b_3$.

The seminull method may be treated as a special case of the deflection method. In this instance the deflection d_a is balanced by adjusting either the vernier M_v or the current I_2 and equation 10 becomes

$$\mu = \frac{b_2 d_s + H}{H} \qquad [12]$$

and

$$k = \frac{b_2 d_s}{4\pi H}$$
$$= b_5 \frac{d_s}{H} \qquad [13]$$

where b_5 is a constant and is equal to $b_2/4\pi$.

Equation 13 indicates that for fixed values of H the susceptibility is equal to a constant times d_s.

Calibration

The most convenient method for calibrating the apparatus is to introduce into the mutual inductance M_t standard samples of known

susceptibility. With the null methods a single known specimen will suffice, since the relation between the susceptibility and the requisite inductive or current values is expressed by a straight line. Likewise, with the deflection methods, if the damping of the galvanometer is not excessive, a linear relation will also exist between the susceptibility and the deflection. If this method of calibration is adopted it is important to carefully reproduce the conditions of magnetization under which the samples were originally standardized, since the susceptibility is a function of the induction density for most materials.

As it is difficult to secure standard samples of unquestioned accuracy covering the desired range of susceptibilities, the following absolute methods of calibration may be utilized. For the sake of clarity the procedure to be pursued for each of the five methods of operation will be discussed in detail; however, it will be found expedient to apply these principles to but a single method, thereby obtaining known samples which may be used to experimentally determine the required constants of the other desired methods.

Instrument Characteristics

The resultant accuracy of each method is essentially related to an exact understanding of the behavior of the accessory equipment. For this reason it is believed advisable, before considering the composite apparatus, to treat briefly the calibration of the galvanometer, fixed mutual inductance M_f, variable mutual inductance M_v and test coil M_t.

Galvanometer

When operating the galvanometer on closed circuit it is usual to introduce sufficient resistance into the external circuit to make the instrument aperiodic. This condition of critical damping is reached when the moving system returns, after deflection, in the shortest time to the zero position but does not pass through it. The value of the external critical damping resistance is furnished by the manufacturer. For our purposes it will be found advisable to increase the value of R_4 if the resistance of the external secondary circuit is less than the external critical damping resistance of G, while if it is greater the galvanometer may usually be made aperiodic by a proper adjustment of R_5. If the shunt resistance is of the Ayrton type its resistance is chosen to conform to the requisite damping resistance of G.

The figure of merit or constant of the galvanometer may be conveniently determined with a standard mutual inductor or a condenser of known capacity, the latter method being particularly adapted to finding the constant for open-circuit measurements. The galvanometer constant is a function of the circuit resistance.

To determine the figure of merit on closed circuit the galvanometer is connected to the secondary of the standard mutual inductor and sufficient resistance added to duplicate the conditions of the circuit in which it will be employed later. When a known primary current through the mutual is interrupted we have the following relation:

$$F = \frac{MI}{Rd} \qquad [14]$$

where F is the figure of merit or the quantity of electricity that produces a deflection of 1 cm. on the scale,

M is the mutual inductance in henrys;
I is the primary current in amperes;
R is the total resistance of the secondary circuit in ohms;
d is the deflection in centimeters.

F is computed from the mean of several observations, each deflection being taken on the same side of the zero position to prevent reversal of magnetization of the magnetic impurities within the galvanometer coil when passing from one side of the zero point to the other.

Following the addition of a double-break key, the same procedure is repeated for the open-circuit value of the figure of merit, the secondary circuit being opened immediately after the primary.

The closed-circuit determination of F by means of a standard condenser is not as reliable as the mutual inductor method. It can be accomplished, however, if we make use of a Zeleny discharge key and connect the external secondary galvanometer circuit of Fig. 1 to the proper key terminals. With this arrangement only the "free charge" capacity[7] of the condenser is utilized, the "absorbed charge" effect being negligible. The ballistic constant will be

$$F = \frac{CE}{d} \qquad [15]$$

where F is the figure of merit,

C is the capacity of the condenser in farads,
E is the electromotive force applied to the condenser in volts;
d is the deflection in centimeters.

The induction apparatus is then removed from the circuit and the observations repeated to find the open-circuit value of F.

Due to imperfections in the arrangement of the galvanometer scale, the deflections usually are not exactly proportional to the discharged quantities. On closed circuit a further discrepancy is introduced by virtue of the fact that the galvanometer field is not perfectly uniform or

[7] A. Zeleny: Capacity of Mica Condensers. *Phys. Rev.* (1906) **22**, 65.

A. Zeleny and A. P. Andrews: Capacity of Paper Condensers and Telephone Cables, with Note on Determination of Specific Inductive Capacity of Dielectrics. *Ibid.* (1908) **27**, 65.

radial. For these reasons the scale should be calibrated for both open circuit and closed circuit, a correction curve being drawn for each case.

Fixed Mutual Inductance

The mutual inductance of M_f may be found from

$$M_f = \frac{FRd}{I} \tag{16}$$

the notations being the same as for equation 14.

A simplified arrangement results when the value of M_f is obtained by comparison with the standard mutual inductor. If the galvanometer is connected in series with the secondary coils of the two mutuals and equal primary currents are interrupted through each,

$$M_f = \frac{M_s d_f}{d_s} \tag{17}$$

where M_f is the mutual inductance in henrys of the fixed mutual;
M_s is the mutual inductance in henrys of the standard mutual;
d_f is the deflection in centimeters obtained with the fixed mutual;
d_s is the deflection in centimeters obtained with the standard mutual.

Variable Mutual Inductance

The mutual inductance of the vernier M_v is determined in the same manner as described for the fixed mutual. The deflections for various

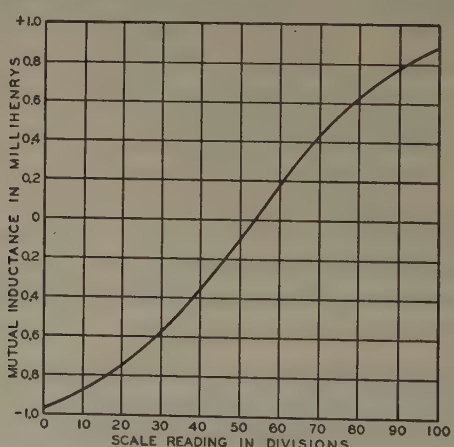

FIG. 4.—CALIBRATION CURVE OF VERNIER MUTUAL INDUCTANCE.

dial settings are observed and a calibration curve is drawn similar to Fig. 4.

Mutual Inductance Test Coil

In order that we may know the strength of the magnetizing field applied to the test specimen, it is necessary to find the relation between

the primary exciting current and the effective magnetizing force. For a short coil, having a form-factor approximating that of M_t, the mathematical computation of the effective magnetizing force is both complex and laborious, because of the appreciable end effects. The use of a long solenoid, substituted for M_t, would largely remove this difficulty though it would introduce a decided complication on account of the inconvenience of inserting a sample of desirable proportions into the center of a coil of such dimensions.

The experimental determination of the effective magnetizing force of M_t is more expeditious and precise than the mathematical analysis, and

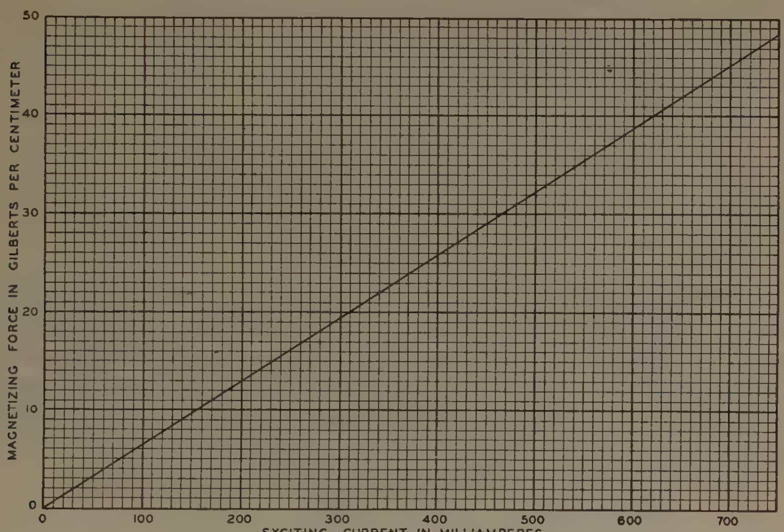

FIG. 5.—EXCITING CURRENT-FIELD INTENSITY CURVE OF MUTUAL INDUCTANCE TEST COIL.

may be carried out in the following manner. Referring to Fig. 1, switches S_1 and S_3 are closed, allowing the current I_3 to flow through the primary winding of the test coil. This current should be adjusted to a value that will produce a throw of approximately 10 cm. when the battery circuit is opened by K_1. After the mean of several observations has been recorded, the value of H may be found from equation 9.

A calibration curve for M_t is now plotted, expressing the relation between the effective magnetizing force and the primary current (Fig. 5). It will be a straight line passing through the origin and the point determined as above.

INSTRUMENTAL CONSTANTS

Null Method, Inductive Balance.—The absolute calibration of this method involves the computation of the constants C_2 and C_3 of equation

6. The value of I_3 is determined by the desired field intensity at M_t and may be found from the exciting current-magnetizing force curve. With the vernier mutual set at zero and, with no sample in M_t, the value of I_2 required to obtain a galvanometer balance is calculated from equation 6 or determined experimentally. Various dial settings of M_v

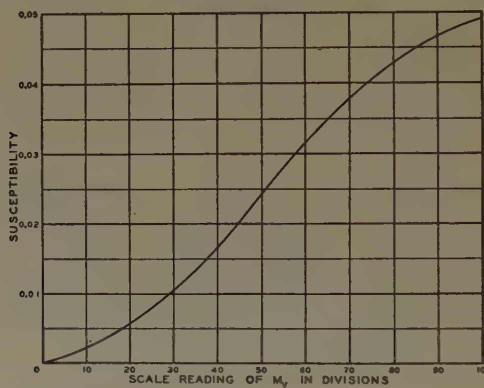

FIG. 6.—CALIBRATION CURVE OF INDUCTIVE BALANCE NULL METHOD.

are next assumed and the corresponding susceptibilities computed, after which a calibration curve similar to Fig. 6 is drawn. In constructing this curve the effect of the vernier mutual was considered additive over its entire range. Clearly, if the balance for no sample be made at the zero position of the vernier its effect will be both additive and subtractive.

FIG. 7.—CALIBRATION CURVE OF CURRENT BALANCE NULL METHOD WHEN MAGNETIZING FORCE IS 12.44 GILBERTS PER CENTIMETER.

In this manner, analyses may be performed upon diamagnetic as well as paramagnetic substances. To facilitate the measurements several curves may be plotted, covering the desired range, by assuming appropriate current ratios. It is important to note from equation 6 that a balance will be obtained for air with a constant current ratio regardless of the actual magnetizing force at M_t.

It will be evident that the use of a long-range mutual inductance for M_v would obviate the necessity for changing the current ratio. If its range were sufficient it would eliminate the fixed mutual M_f. However, it is believed that the greater accuracy obtained with the vernier will more than compensate for the increased complication.

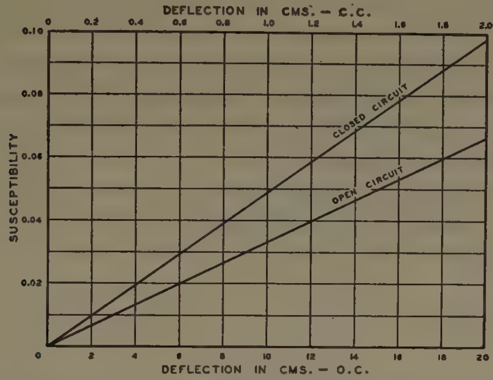

FIG. 8.—CALIBRATION CURVES OF SEMINULL METHOD WHEN MAGNETIZING FORCE IS 12.44 GILBERTS PER CENTIMETER.

Null Method, Current Balance.—The standardization of this method is analogous to the preceding case. With the vernier mutual remaining fixed the susceptibilities corresponding to various values of I_2 are computed from equation 6. A calibration curve, illustrating the linear relation between k and I_2, is shown on Fig. 7.

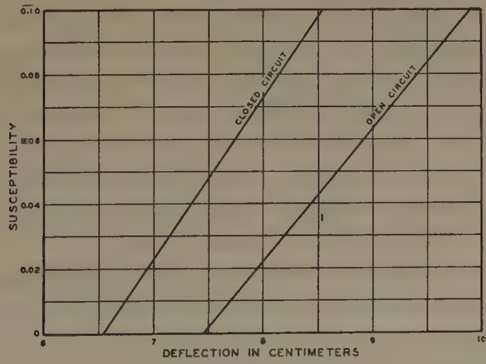

FIG. 9.—CALIBRATION CURVES OF CLOSED-CIRCUIT AND OPEN-CIRCUIT DEFLECTION METHODS WHEN MAGNETIZING FORCE IS 12.44 GILBERTS PER CENTIMETER.

Open-circuit data obtained with galvanometer sensitivity reduced from 1.0 to 0.0761.

Seminull Method.—The instrumental constant b_5 of equation 13 is calculated and the desired field intensity assumed, after which k is determined for various values of d_s. Two points are sufficient, since the relation is expressed by a straight line as indicated in Fig. 8.

The closed-circuit and open-circuit value of the instrumental constant will not be the same on account of the variation in F, which explains the difference in slope of the calibration curves when plotted to equal scales. Obviously, a given calibration curve will be true only for a definite value of H.

Deflection Method, Closed Circuit.—Using the closed-circuit value of the figure of merit, the constant b_4 of equation 11 is determined. The deflection d_a is constant for a definite magnetizing force and may be computed from equation 9 or found experimentally. By assuming values of d_s the related susceptibilities are calculated from equation 11 and plotted as a curve similar to Fig. 9.

For the closed-circuit and open-circuit deflection methods a different calibration curve must be drawn for each value of H, since these curves represent a first degree equation having the slope $k/(d_s - d_a)$ and x-intercept d_a.

Deflection Method, Open Circuit.—The procedure is the same as for the closed-circuit deflection method except that the open-circuit value of F is used in determining b_4 of equation 11. A typical calibration curve is shown on Fig. 9. In obtaining the data for this curve the galvanometer sensitivity was reduced, by means of R_5, from 1.0 to 0.0761, which explains the apparent discrepancy between the results in this instance and the open-circuit seminull method.

Discussion of Methods

Other things being equal, the sensitivity of the apparatus will be directly proportional to the figure of merit of the galvanometer employed. An undamped sensitivity of 3×10^{-4} microcoulombs per millimeter at 1 m. distance is entirely satisfactory, though for routine measurements a sensitivity of 2 by 10^{-3} will be found sufficient. In general, with any type of refined apparatus, extreme sensitivity is accompanied by a magnification of the disturbing influences. To avoid these undesirable effects it is well to reduce the sensitivity to a value commensurable with the desired accuracy.

A comparative analysis of the sensitivity of the various methods is shown in Table 1. These data were obtained experimentally, using a galvanometer having a ballistic constant of 0.02288 microcoulombs per millimeter under the existing closed-circuit conditions. Table 1 indicates that maximum sensitivity is obtained with the open-circuit operation of the deflection and seminull methods. The latter arrangement is to be preferred because it permits the use of larger magnetizing forces without requiring a decrease in galvanometer sensitivity. While the advantages and disadvantages of each method will be influenced by the magnetic properties of the materials under investigation, experience suggests the applicability of the inductive balance null method for routine purposes

where convenience of manipulation and reproducibility of results are of greater importance than increased sensitivity.

In the discussion of the null methods no reference was made to the double deflection of the galvanometer which occurs when obtaining a balance. This is due to the different time constants of the circuits involved and, in a lesser degree, to viscosity of the sample. If the galvanometer period is of the order of 25 sec. or larger, the effect is of little consequence.

For precise work with the deflection methods, it is essential that the galvanometer zero be stable and that internal thermal currents be minimized by the provision of a copper circuit, including the binding posts.

TABLE 1.—*Sensitivity of Methods*

Method	Possible Observational Accuracy	Minimum Variation of k Detectable	Relative Sensitivity
Open circuit:			
deflection	0.10 mm.	0.0000331	1.0000
seminull	0.10 mm.	0.0000331	1.0000
Null method:			
inductive balance	0.20 div.	0.0000970	0.3410
current balance	0.10 m-amp.	0.0001360	0.2430
Closed circuit:			
deflection	0.10 mm.	0.0004450	0.0744
seminull	0.10 mm.	0.0004450	0.0744

If the instrument is subject to zero shift the effect may be reduced by taking several deflections in the proper direction, greater than will be required in its subsequent use. To avoid eyestrain as far as possible, the galvanometer may be equipped with a lamp and scale instead of the customary telescope and scale.

Pulverizing the sample has a marked influence upon its susceptibility, which increases with decrease in voids, because of the change in reluctance of the magnetic path.[8] To duplicate conditions as nearly as possible, the specimens should be passed through a fine mesh screen and placed in the sample container without packing to avoid internal strains in the material. The relation of mechanical stress to the magnetic properties of the specimen, termed *magnetostriction*, has been investigated by Ewing.[9]

Substances may be compared at constant volume or constant mass. In the latter instance, to compensate for variations in density, it will be necessary to employ some magnetically inert material, throughout which

[8] L. B. Slichter: Certain Aspects of Magnetic Surveying. Geophysical Prospecting, A. I. M. E. (1929) 246.

[9] J. A. Ewing: Magnetic Induction in Iron and Other Metals, Ed 3., 196. New York, 1900. Van Nostrand.

the powdered sample is uniformly disseminated. The former procedure is more convenient and, it is believed, more reliable.

According to Glazebrook[10] the demagnetizing effect of the sample may be ignored for weak susceptibilities. For very high susceptibilities, however, a correction may be applied, taking into account the original magnetizing field and the demagnetizing action of the specimen, the latter effect being proportional to the intensity of magnetization of the core material, or to $(B - H)/4\pi$.

It is to be remembered that the field within the test coil M_t is not uniform. This condition, however, introduces no appreciable discrepancy if the average value of the magnetizing force is used for H, as derived

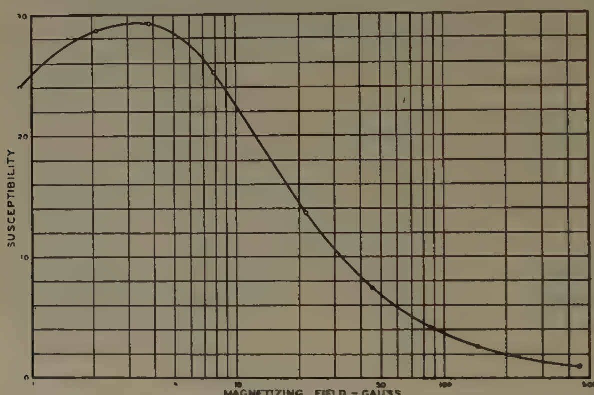

FIG. 10.—FUNCTIONAL RELATION BETWEEN SUSCEPTIBILITY AND MAGNETIZING FORCE FOR SAMPLE OF MAGNETITE. (AFTER P. WEISS.)

in the foregoing formulas. The validity of this assumption has been demonstrated by comparative tests with a long solenoid. Within limits, the sensitivity of the apparatus is related to the design of the test coil. For a comprehensive theoretical treatment of this subject the reader is referred to the work of Gray.[11]

It has been stated previously that, for most materials, the susceptibility is a function of the effective magnetizing force. From this it will be evident that for comparative data on various specimens the field intensity of M_t should be kept constant. Fig. 10 has been reproduced from the work of P. Weiss[12] and admirably illustrates this fact for a sample of magnetite from Traverselle, northwest Italy, which had been magnetized parallel to the binary axis. It will be observed that the

[10] R. T. Glazebrook: Dictionary of Applied Physics, **2**. London, 1922–23. Macmillan.

[11] A. Gray: Absolute Measurements in Electricity and Magnetism, Ed. 2 London, 1921. Macmillan.

[12] P. Weiss: Recherches sur l'aimantation de la magnétite cristallisée. *L'Éclairage Électrique* (1896) **7**, 487.

susceptibility at 0.9 gauss, which approximates the earth's field, is 48 times the value corresponding to 400 gauss, while the maximum value is obtained with a magnetizing force of about 3 gauss. From these data it would appear that the most useful measurements for interpretative analysis should be conducted with magnetizing forces approaching the normal field strength of the earth.

Conclusion

It is apparent that the utility and accuracy of the various methods outlined will depend upon the following:

1. Quality of the equipment.
2. Precise calibration of the component parts of the apparatus.
3. Exact determination of the instrumental constants of the composite arrangement.
4. Manipulative technique of the operator.

In the past, valuable data have been contributed concerning the magnetic properties of ferromagnetic minerals. The transition from the ferromagnetic to the paramagnetic or diamagnetic state, under the influence of thermal effects and mechanical stress, warrants further investigation by the geophysicist.

In the correlation of magnetic field data the geophysicist is primarily interested in the laws of energy radiation, and in order that he may arrive at authentic results, certain unknowns must be eliminated from his equations. Careful magnetic analyses of numerous well cores will afford an undisputed aid in clarifying the complex nature of his problem. The present method is suggested in the firm belief that it will furnish the geomagnetician a serviceable and scientific tool for obtaining these data.

Acknowledgment

The writer wishes to acknowledge his indebtedness to Mr. Randolph H. Mayer, whose criticisms have proved of material assistance in the preparation of this paper.

DISCUSSION

(Donald H. McLaughlin presiding)

D. M. Collingwood, Dallas, Tex. (written discussion).—The method described by Mr. Barret apparently covers the use of well cuttings, since the samples are used in a pulverized condition. Often, cuttings are the only samples of the subsurface rocks available, particularly in oil exploration. The susceptibility of the pulverized sample, however, often differs from that of the original rock in place, due principally to differences in orientation and alignment of mineral particles and their crystallographic axes, in the disseminated arrangement of the particles, and in compaction and amount of voids. Correction for the last mentioned can be made in accord with the ratio of the density of the solid rock to that in the pulverized form. For an accurate determination of this ratio, as well as for a better study of the distribution

of the ferromagnetic particles, it is desirable to have the original sample in the form of a solid core. The importance of taking cores, particularly in oil-exploration borings, should be stressed in this connection.

In view of the more direct susceptibility data obtainable from solid cores, I would like to ask Mr. Barret whether his apparatus could not be adapted for their use.

In the interpretation of magnetic anomalies due to deeply buried rocks, it does not seem necessary to determine the susceptibilities of the rocks involved with the degree of accuracy possible in the various methods of using Barret's apparatus. However, the sensitivities of these methods would allow such differentiation of susceptibilities as would permit the detection of minimum anomalies of 2 to 25 gammas due to contrasting rocks at the surface. This seems entirely adequate for any possible refinements of practical interpretation of geomagnetic surveys made by field instruments in general use at the present time.

J. G. Koenigsberger, Freiburg i. B., Germany (written discussion).—The method of induction balances, which Mr. Barret has developed for determining the magnetic susceptibility of core samples, was used in somewhat similar manner by Chevallier,[13] Grenet[14] and Puzicha,[15] to measure the susceptibility or the remanent magnetism of rocks, or both. The writer has used a magnetometric method.[16] Sediments do not have a noticeable remanent magnetism, but the remanent magnetism of intrusive rocks is large, and there would be scientific and practical interest in measuring it on cores, when their strike and dip is known. The decrease of the susceptibility of rocks—*i. e.*, of little grains of magnetite—is slower and begins at higher intensity of magnetic field as the susceptibility of a magnetite crystal parallel to a binary axis observed by P. Weiss.

Applicability of Method

C. A. Heiland, Golden, Colo. (written discussion).—The problem of determining the magnetic susceptibility of rocks is almost as old as magnetic prospecting itself; therefore the number of methods that have been suggested for this purpose is comparatively great. Their applicability depends a great deal on the accuracy desired, and often, perhaps, the accuracy has been carried much too far. It should always be remembered that both the true and apparent susceptibility of a formation are, in most cases, subject to appreciable variation, which is due to the fact that the magnetization of a rock does by no means always depend on simple induction in the earth magnetic field, but often on its geologic history. This is particularly true for igneous and metamorphic rocks, the magnetization of which is found to depend greatly on the process of cooling, and mechanical stresses undergone by the rock in the course of its geologic history. In other words, determinations of magnetic susceptibilities, particularly on igneous rocks, do not nearly have the significance for the interpretation of the results; as, for instance, the accurate determination of the density of rocks has for the interpretation of gravitational results, or the determination of the resistivity of rocks for electrical prospecting. Therefore, only in the case of sedimentary rocks and certain types of acidic and intrusive igneous rocks—in other words, for weakly magnetic rocks—may the determination of their susceptibilities be made with a fairly great degree of accuracy, as these are practically the only magnetic rocks that produce consistent magnetic anomalies.

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[13] R. Chevallier: *Ann. de phys.* [10] (1928) **4,** 5.
[14] M. Grenet: *Ann. de phys.* [10] (1930) **13,** 263.
[15] K. Puzicha: *Ztsch. f. prak. Geol.* (1930).
[16] J. G. Koenigsberger: *Terrestrial Magnetism* (1930) 145.

Important Methods for Determining Magnetic Susceptibility

Some of the most important methods which have been applied to date for the determination of magnetic susceptibilities are described briefly below:

1. The mineral specimen is pulverized and enclosed in a test tube and the deflection it produces on a magnetometer a definite distance away from the needle is observed. The relative susceptibility of various samples may thus be determined by using the same quantities in the same distance.

2. J. Koenigsberger uses solid specimens close to the needle of a magnetometer, and computes the susceptibility from the dimensions of the specimen, its distance and the deflection produced.

3. Wiedemann has used a torsion balance, the sample to be tested being attached in pulverized form in a glass tube to the end of the beam. The deflection of the beam produced by the action on the sample of an electromagnet of known strength is measured.

4. According to a method suggested by Lord Kelvin, the sample, in cylindrical form or pulverized and contained in a glass tube, is attached to one arm of a balance, and its increase in weight is determined when placed between the poles of an electromagnet.

5. Stutzer, Gross and Bornemann placed a glass tube containing the pulverized sample in two concentric coils, the inside being the secondary, the outside the primary coil. Two such coil pairs were used, the secondary windings of which are connected in opposition and to a ballistic galvanometer. Thus no deflection is produced on the galvanometer when pulsating direct current is passed through the primaries and both coil pairs are filled with air, but the balance is disturbed if the sample is placed in one of the coils.

6. Ruecker used a solenoid and determined its effect on a magnetometer with and without the sample inside.

7. Ambronn has employed two solenoids, placed on opposite sides of a magnetometer, so that their influence is balanced when they are both filled with air. The deflection produced when the sample is placed in one of the solenoids is compensated by passing current of known strength through an additional winding on the solenoid. The method recently applied by Stschodro is similar to Ambronn's method.

8. Stschodro has also employed a ballistic method. The sample of cylindrical shape (core or glass tube with mineral powder) is dropped in the inside of a pair of concentric coils inclined 45°. The outside coil is the magnetizing coil, supplied with direct current, the inside coil is the secondary coil connected to a ballistic galvanometer, the deflection of which is proportional to the susceptibility of the sample.

In most of the methods described above, the sample either must be in pulverized form or must be cut to fit the apparatus. This destruction of the natural condition of the rock has no great influence in weakly magnetic substances, but changes the results considerably when strongly magnetic rocks or minerals are tested, not only on account of the increase in air space but also because most of the remanent magnetism and, primarily, its chief direction, is destroyed.

The writer has for some time been working on the assumption that the magnetization of a rock, particularly the remanent magnetism, is influenced by mechanical stresses, and that it is, therefore, desirable, to disturb the natural condition of the rock as little as possible when determining its susceptibility. This seems to be nearly impossible, because, when comparing samples by the magnetometric method, their dimensions should be as nearly identical as possible, as the effect on the magnetometer depends greatly on the form of the sample, and the ratio of length to width in particular.

In order to be able to test rock samples in their natural condition, and secondly, in order to eliminate the influence of the shape of the sample, the writer has devised the following method:

A cast is made of the rock sample to be tested, and the cast is filled with a plastic substance in which fine iron powder is evenly distributed. A number of samples of the plastic substance with a different degree of concentration of magnetic material are kept on hand, and the permeabilities of these are determined by any of the standard methods. Thus an exact reproduction of the sample is obtained, in form of a model of known permeability. The model and the sample are placed an equal distance apart on the horizontal arms of an astatic magnetometer. The deflection obtained gives the permeability of the sample in terms of the known permeability of the model, and the influence of the shape is eliminated. Furthermore, the component of remanent magnetism is determined by reversing both sample and model. Finally, by applying the astatic principle, the influence of the magnetic diurnal variation on the magnetometer reading is eliminated.

Solid Samples

W. M. Barret (written discussion).—Mr. Collingwood has pointed out the desirability of employing solid samples for magnetic susceptibility determinations. Certainly, it is true that different values of k will be obtained when a solid specimen is subsequently analyzed in a pulverized condition. However, it is also true that various samples from the same formation will frequently exhibit marked dissimilarities of magnetic properties, and only by accepting mean values is it possible to obtain reliable criteria. Pulverizing leads, therefore, to the significant advantage of determining an average value for a given rock body by the simple expedient of selecting a specimen composed of representative portions of the substance.

In the recovery of underground samples, together with shaping or pulverizing for analysis, it is highly probable that the material undergoes certain changes in magnetic composition. Thus the ideal procedure for measuring these properties would be based upon determinations made *in situ*. For some time, the writer has devoted attention to the possibilities of such an arrangement, the theoretical principle of which is similar to that of the well-known inductance bridge. If successfully developed, the device will permit an intimate study of the magnetic susceptibility of subsurface formations. It is intended to explore, by means of a balanced coil assembly and appropriate energizing and measuring apparatus, wells that have not been provided with metallic casing. A record of the variation of k, from the surface to remote depth, could be obtained thereby.

With reference to Mr. Collingwood's question concerning the applicability of the ballistic method to work with solid samples, it may be said that the apparatus lends itself readily to such measurements. It is required, however, that the specimen conform to the cylindrical proportions of the container, or else that the test coil be designed to receive a core of standard size.

A New Geophone

By C. A. Heiland,* Golden, Colo.

(New York Meeting, February, 1930)

The new geophone described herein was developed by Charles H. Hull, instrumentmaker of the Colorado School of Mines, and the writer.

The first geophone was invented during the war for the purpose of detecting enemy sapping or underground mining operations and for locating enemy artillery. It is based on the principle of the seismograph—a heavy lead mass suspended by one or two diaphragms, which are clamped to an air-tight case. When the ground vibrates during the transmission of sound waves, the case moves with it while the so-called steady mass remains practically motionless. Attached to the top of the casing is a hose leading to an earpiece shaped like a stethoscope. When the mass and the diaphragm move up and down in the case, the air is compressed and rarified in the inside; thus the vibration is communicated as sound wave to the human ear.

The geophone combines a surprising simplicity of construction with an extreme sensitivity. Other listening instruments were developed during the war, adaptations of the telephone and the microphone—that is, instruments operating through the medium of electricity—of which the so-called seismicrophone, an adaptation of the carbon-granule microphone receiver, probably has found the greatest application, but such instruments apparently have not proved as useful as the simple geophone despite obvious advantages arising from the use of electricity. The carbon receivers were more subject to extraneous noises and to what is called "frying," and they did not offer the advantage possessed by the geophone; namely, the simultaneous use of two receivers, which makes it possible to determine the location of the source of sound.

During the war, the U. S. Bureau of Mines made a study of the geophone and of the propagation of sound waves through rock. These studies have been continued, as the earlier results proved the possibility of using the geophone in various non-military ways, principally in mining, and the results of these studies have been published.[1]

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* Professor of Geophysics, Colorado School of Mines.

[1] A. Leighton: Application of the Geophone to Mining Operations. U. S. Bur. Mines *Tech. Paper* 277 (1922). W. T. Ackley, Jr. and C. M. Ralph: Improvement of the Geophone by the Use of Electrical Sound Amplifiers. U. S. Bur. Mines *Rept. of Investigations*, Ser. No. 2629 (1924). L. C. Ilsley, H. B. Freeman, D. H. Zellers: Experiments in Underground Communication through Earth Strata. U. S. Bur. Mines *Tech. Paper* 433 (1928).

Applications of the Geophone

The principal applications of the geophone outside of geophysical prospecting are:

1. Rescue of entombed miners.
2. Detection of leaks in buried water mains.
3. Detection of fire underground.
4. Mine surveying (location of tunnel headings and bore holes; prevention of accidents in blasting through, etc.).
5. Location of knocks in automobile valves and cylinders.

As to geophysical prospecting, the use of geophones is frequently mentioned in connection with seismic prospecting. In reality, the name "geophones" for these instruments is a misnomer; the name "seismographs," or "electrodynamic seismographs" should be used instead. The name geophone in itself implies the application of the principles of acoustics; it is an instrument with which sound may be received. In seismic prospecting, the seismographs commonly used pick up vibrations of a frequency below the audible range. Qualitatively, of course, there is no difference between a geophone in the narrower sense and a seismograph, as both respond to vibrations; quantitatively there is a difference, as a geophone is tuned to a much higher (audible) frequency than a seismograph. The use of a seismograph with a natural frequency lower than audible is preferred by many for seismic prospecting, especially in long-distance shooting, as generally the frequency of the ground vibrations decreases with horizontal distance.

Not much has become known about the application of the geophone in the narrower sense in geophysical prospecting. The application, according to the above, is limited to comparatively shallow depth, as the intensity and frequency of sound sources conveniently usable with geophones (hammer blows, falling weights, etc.) does not travel through great horizontal distances.

The applications of the geophones to geophysical prospecting that have become known are as follows (continuing the numbers of the list above):

6. Location of faults, fissures and clay seams (in shallow depths and underground).
7. Determination of depth to water table.
8. Determination of thickness of alluvial or glacial covers, etc.

In application 6, use is made of the decrease of reception energy through absorption from faults, etc.; in 7 and 8, the direction of reflected rays (and thus depths) are determined with two geophones.

There is no doubt that the applications of the geophone in geophysical prospecting may be considerably extended, primarily in underground work for general geologic information, as the different geologic strata

show an extreme variability in their power to carry sound. In the course of our experiments to determine the usefulness of the geophone in geophysical prospecting, the instrument herein described was developed.

Sensitivity of the Geophone

In working with the Bureau of Mines type of geophone one is impressed with the sensitivity of a device of such simple construction. Four factors seem to account for its great sensitivity:

1. The section of the air tubes as compared with that of the case.
2. The sensitivity of the human ear.
3. The natural frequency of the diaphragm.
4. (a) The thickness of the diaphragm as compared with the weight of the lead mass; (b) the elastic properties of the diaphragm, *i. e.*, the material it is made of.

1. It is easy to see that factor 1 should play an important part in the function of the geophone. The intensity of sound sensation depends on the amplitude of the movement of the ear drum, and that in turn is proportional to the movement of the air column in the air tubes of the geophone. When its diaphragm moves, it forces a certain volume of air out of the case into the tube. Assuming that volumes are approximately preserved in the transfer of air from the case into the tube, it follows that on account of the much smaller section of the tube the movement of the air in it must be of considerably greater amplitude than in the case.

2. The ear is probably the most sensitive of all human organs, speaking in terms of detectable energy. While it is difficult to establish a physical measure for the sensitivity of the sense of smell (and taste), the sense of touch reacts only to comparatively great pressure energies. The eye, next to the ear, probably is the most sensitive organ; the minimum energy detectable is of the order of 10^{-10} ergs, but its ability to distinguish images of fairly fast sequence is not great. The ear can detect, within the most favorable frequency range, energies of sound fields as low as 10^{-18} watts centimeter^{-2}. The minimum audible amplitude of a telephone receiver diaphragm is about 6.10^{-10} centimeters.

3. The amplitude of the diaphragm with the mass attached will be the greater the better its natural frequency is in harmony with the frequency of the sound to be picked up; the impression of the movement of the diaphragm on the ear will in turn be greatest if its natural frequency equals that to which the human ear is most sensitive. The ear is most sensitive to sound frequencies between 1000 and 4000 cycles. (The natural period of the ear drum itself is about 1400.) The diaphragm of the geophone, therefore, is tuned to a frequency within these limits, to give maximum sensitivity.

4. It is clear that such a relation must exist between the thickness of the diaphragm and the weight of the attached lead mass as to secure

maximum deflection for any given impulse; also, its material must be selected so as to achieve the same purpose. According to experiments conducted by the Bureau of Mines, a geophone with a nickel diaphragm could pick up almost 100 times less energy than a geophone equipped with a mica diaphragm of the same thickness.

To verify the sensitivity of the geophone for which the factors just discussed are responsible, reference may be made to some estimates of A. Leighton about the range of geophones equipped with nickel diaphragms. Blows of a sledge hammer probably may be heard at a distance of 3000 ft. through rock, 2000 ft. through coal, 400 ft. through clay and 550 ft. through a mine cover. By actual experiments, it has been found that talking can be heard clearly through a coal pillar from 45 to 150 ft. thick; that a pick blow can be detected through 900 ft. of coal; that a sledge pounding can be heard 1150 ft. away with sufficient clearness to enable location of direction; that the explosion of 1 oz. of dynamite transmits sound through more than 2000 ft.; that from the surface a working miner may be heard through 250 to 500 ft. of cover.

Quantitative Tests with Geophone

In the commercial application of the geophone outside of the geophysical prospecting field, little or no quantitative testing is done. In locating a leak in a main, only the comparative intensity of the sound is determined at a number of locations by estimating the strength of the sound sensations. In mining, chiefly the direction of the source of sound is determined by two geophones. In applying the geophone to geophysical prospecting problems, the need for some device capable of indicating numerically the intensity of the sound impulses became immediately apparent.

Heretofore, two methods have been used to determine the intensity of sound sensation at any set-up of the geophone. First, if the ground characteristics did not change much, the geophone was left stationary and the ground was struck with a sledge hammer at increasing distances. The distance of the location where the blows could no longer be heard was an indicator for the strength of the sound intensity received, and thus for the absorption of the intervening strata. Second, with geophone and source of sound remaining stationary, the intensity of hammer blows was varied. Instead of varying the width of swing of hammer blows, weights have been dropped from varying heights or small charges of powder of varying quantities have been used.

All these methods called for a more or less appreciable number of observations for any particular geophone set-up for the determination of the intensity. It seemed, therefore, advisable to construct an instrument that could indicate the amplitude of the waves directly. This would also have the advantage that such a geophone with some recording device

attached to it would be capable of recording the time of arrival of sound waves and could thus serve to determine the velocity of sound waves through strata.

At first the writer had in mind to use the geophone with an electrodynamic amplification attachment such as that constructed by Ackley and Ralph. A slight modification of their design was tried.

The moving element of a phonograph pickup (like that used in electric victrolas or at broadcasting stations for transmitting phonograph records) was attached to the diaphragm of a standard geophone. It was a comparatively easy matter to design a circuit for indicating the volume of the sound intensity received from the amplifier by an adaptation of the tube amperemeter scheme. It was found, however, that the electrical amplification feature was not desirable, as the volume indication seemed to depend too much upon the constancy of the filament current, so that two volume indications on two set-ups were not exactly comparable. Besides, the electrical equipment involved was more difficult to transport than the apparatus herein described.

The New Geophone

The new geophone[2] makes use of factor 1 mentioned on page 5 fo magnification of the air displacement. The air-tight case contains a single diaphragm with a lead mass attached to it. The air can escape above and below the diaphragm through narrow air passages. Rubber tubing attached to these leads the air into two small round chambers, which are closed on one side by thin membranes. The two chambers are so fastened on a frame that they are parallel and facing each other with the diaphragms (see Fig. 1). A thin hair is attached to both centers of the membranes and wound tight around a mirror spindle set in jewels. When the diaphragm in the geophone moves up it compresses the air above it and expands the air below it in the case, thus forcing one membrane out and the other in. Both pulls being in the same direction will bring about a deflection of the mirror. This deflection is proportional to the displacement of the diaphragm in the geophone. The movements of the mirror may be observed by a short telescope with an autocollimational system (similar to the telescope used on magnetometers). This is an arrangement which is most suitable for field use. The indicator box with the telescope may be set on a low tripod with the geophone placed directly below on the ground. For underground work, where a niche can conveniently be made in the wall rock at the height of the observer's eyes, the rubber hoses are best completely eliminated. The indicator box with telescope is then mounted directly on the geophone with metal tubing providing for the air passage from geophone to indicator.

[2] Patent applied for.

For demonstration of the action of the geophone to an audience, as well as for photographic recording, the device is designed as shown in Fig. 2. A lens of fairly great focal length is attached to the case in front of the moving mirror. In the focus of this lens is a small electric light bulb and a frosted glass scale. The movements of the diaphragm then appear as the movement of a light beam on the scale.

In the preliminary experiments the geophone was tuned to a fairly low frequency. The purpose of this was to use the instrument also as a seismograph. The magnification of the first model was of the order of 5000, but can no doubt be appreciably increased.

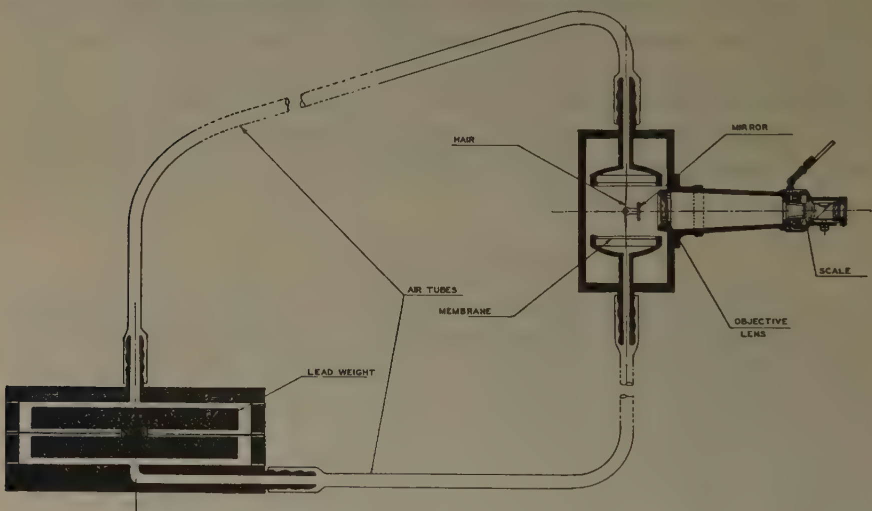

FIG. 1.—THE NEW (PUSH-PULL) GEOPHONE ARRANGED FOR VISUAL OBSERVATION.

Thus far, only one geophone has been used with one indicator. It seems possible, however, to use two geophones with one indicator, as described below.

The advantage of using two geophones is that the direction of the source of the sound can be determined. Two geophones work in a manner somewhat similar to the human ears. The ability of the ears to detect sound direction is based on two factors: (1) The ear can detect with a remarkable accuracy the difference in time of arrival of corresponding sound phases on either ear; (2) the head acts as a sound screen for the ear away from the source of sound. Using two geophones for determining sound direction, the second factor does not seem to become effective, as the ground acts as transmitting medium and there is no other medium interfering with transmission between two geophones. To make the first factor as effective in geophone sound reception as in hearing, it seems advisable to keep the two geophones a greater distance apart than the distance of the ears, as the velocity of sound through rocks is

much greater than through air. In order to determine sound directions with geophones, it is not necessary to place the two geophones with their connecting line at right angles to the sound direction to make the impulses strike the geophones in phase. A piece of tubing, telescope-jointed and thus adjustable in length, may be inserted in one rubber tube. The two

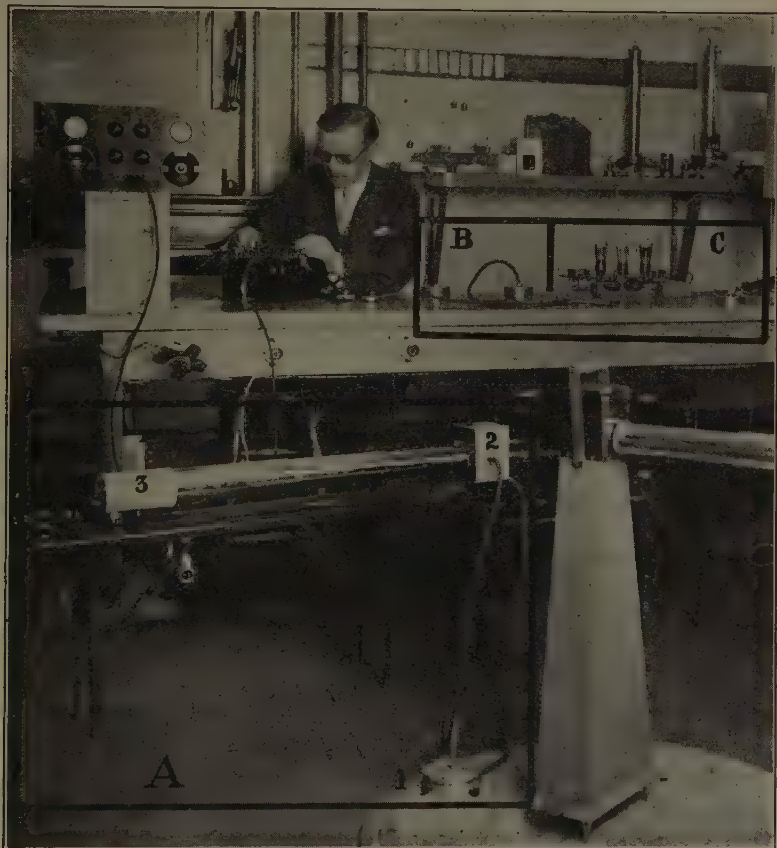

Fig. 2.—Geophones arranged for use.

A. The new geophone arranged for audience demonstration or photographic recording: (1) geophone, (2) indicator, (3) light tunnel.
B. Standard two-element geophone.
C. One-element geophone with electrical amplifier.

geophones then may be set at any angle to the sound direction and the length of one air tubing may be adjusted to cause the same intensity of sound sensation in both ears. Then the ratio of the lengths of both tubes indicates the angle of sound direction. This method, suggested by Prof. James Fisher,[3] is of great advantage in determining the angle of emergence of reflected sound waves at surface of the ground.

[3] Colorado School of Mines *Quarterly* (1929) **24,** 86.

The fact that directional hearing—the decrease of sound amplitude in one ear—results from a superposition of waves with different phases may be utilized for the determination of sound direction with two geophones and only one indicator. The connections of geophones and indicator may be so made as to give deflection zero when the waves striking both geophones are in phase. Where the line connecting both geophones cannot be set perpendicular to the sound direction, an adjustable tubing like that described could be used to equalize phases.

DISCUSSION

(*Allen H. Rogers presiding*)

L. B. Slichter, Madison, Wis.—I would like to speak of the determination of direction by geophones. A great deal of work was done at the Naval Experiment Station by Professor Bridgman, of Harvard, who developed an excellent geophone. He mounted a hundred or more of his geophones inside the shell of destroyers, in groups, and by means of an air compensator of the type described by Dr. Heiland obtained accurate direction.

In regard to the problem of determining the direction of a sound wave in rock by means of two geophones, under conditions where the geophones are not movable, I think Dr. Heiland's suggestion will prove workable under simple conditions; in fact, I can report upon an experiment in that connection which was made at the University of Wisconsin about eight years ago. A well was being drilled in sandstone with a churn drill; a shallow well, down about 125 ft. The impact of the drill was quite audible with geophones. A geophone was set up on a radial line through the drill, about 125 ft. away, and a second one about 50 ft. farther. Binaural readings of directions on the blow of the drill were taken with a compensator. We were able to check the depth of the well within a few feet.

The type of geophone developed by the U. S. Bureau of Mines and the type developed by Dr. Bridgman are excellent for use with air compensators.

C. A. Heiland.—Has anything been published regarding the experiments to which you have referred, and where?

L. B. Slichter.—The experiments in connection with the well have not been published. I believe that several accounts of the use of geophones and compensators on destroyers have been published. The only one I can recall is a little article in the *Wisconsin Engineer* some years ago.

F. W. Lee, Washington, D. C.—Dr. Heiland said that the use of his geophone is satisfactory for locating vibrations, especially leaks in mains. I do not know whether Dr. Heiland has been in contact with some of the water engineers in the various cities. I have been informed that about 25 per cent of the water is lost in the mains, which is not accounted for in distribution. This is not altogether ascribed to freezing, but to the action of heavy trucks which cause a vibration in the ground to crack the joints. So while the application may be very good for geophysics, it should not be forgotten that it is very good for modern city service for water surveys.

Seismic Propagation Paths

By Maurice Ewing,* Erie, Pa., and L. Don Leet,† Houston, Texas

(New York Meeting, February, 1930)

Assuming that wave velocities in seismic prospecting increase as a continuous linear function of the depth, the authors have derived formulas for computing, from two time-distance observations, the amount of velocity increase, depth of penetration and a graphical determination of the path of the vibrations, and have discussed the ground, reflected and refracted waves. The application of the formulas is illustrated numerically.

Earlier Investigations

Observations of the energy propagated from the center of an earthquake disturbance have been used to give information about the velocities and paths of elastic waves in the interior of the earth. One method of accomplishing this depends on the use of the times of transmission. Herglotz(7)§ and Bateman(3) obtained a solution of the integral equation involved, which determines the velocity of propagation at any point as a function of the distance from the center of the earth, with no assumption as to the manner in which the velocity of the disturbance varies with depth below the earth's surface.

The expressions on which the value of the time and the form of the ray of propagation depend contain an unintegrable definite integral, involving the velocity as an unknown function of the distance from the center of the earth. Wiechert and Zöppritz(15) obtained a manageable form for this function by assuming a law of variation which required a constant curvature form for the path within each of a succession of concentric spherical layers of the earth. C. G. Knott(9) worked out the problem for the first time by an absolutely rigorous method from the data of observation (Zöppritz-Turner tables), without making any assumptions as to the functional relationship connecting the speed of propagation and the distance from the earth's center. His method is a numerical solution of an integral equation, giving each velocity and the corresponding distance from the center of the earth in figures.

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* Department of Physics, Erie Center of the University of Pittsburgh.

† Geophysical Research Corporation.

§ Numbers in parentheses refer to bibliography at end of paper.

When the principles of seismology were first applied to a study of surface strata in seismic prospecting, empirical formulas were developed on the assumption that the elastic waves traveled essentially straight-line paths within the strata traversed. The time-integral solutions applied to studies of the earth's deep interior cannot be used because the small scale of the prospecting operations practically eliminates the earth's radius as a factor of the problem.

Other writers on seismic methods of surveying geologic structures[(2),(6),(13)] have stated the probability that paths along which the elastic vibrations travel are curved, concave to the surface, owing to variations of velocity with depth, though in general field practice thus far reported this path curvature has been neglected.

The expectation of an increase of velocity with depth, particularly in the sedimentary formations with which seismic prospecting is chiefly concerned, has experimental justification. Kusakabe[(10)] and Nagaoka[(12)] found a systematic tendency toward an increase of body-wave velocities with age of formation in specimens subjected to laboratory tests. L. H. Adams[(1)] reports experiments determining changes in compressibility with pressure.

Longitudinal-wave velocity, v, is expressed as

$$v = \sqrt{\frac{k + \frac{4}{3}\mu}{\rho}}$$

where k is the incompressibility of the medium, or resistance to change of volume, μ is the rigidity, or resistance to change of shape, and ρ is the density. Accordingly, an increase in v can result only from a relative decrease in ρ or increase in $k + \frac{4}{3}\mu$. Processes of lithification of sediments, aided by increased pressure,* in general seem to cause increases (to moderate depths) in k, μ, and ρ at the same time, though the term $k + \frac{4}{3}\mu$ gains relative to ρ and v increases. On the other hand, decreased density is one of the factors that enter into an explanation of the higher velocity of vibrations in salt domes than in surrounding sediments.

From the standpoint of field work, a more important indication of an increase of velocity with depth is curvature of the time-distance line.

* It should be borne in mind, however, that mere depth of burial of a medium and consequent increase of pressure do not of necessity result in higher velocities for vibrations which traverse it, as stated by C. A. Heiland,[(6)] who says, in a footnote on page 635 of his paper, "In reality, some waves will reach the receiver which penetrate the lower medium, because the seismic paths are curved (concave to the surface) on account of the increase of pressure at depth." A contributing cause in some cases—pressure—should not be confused with the results, changes in elastic constants and density, which do not inevitably lead to increased velocities. Observations of earthquake-wave velocities emphasize this, first by a flattening at depth of ray paths which are curved near the surface, and especially in the marked decrease of velocity in waves which penetrate the core below about 2900 km., where pressures obviously reach a maximum.

Rieber[13] specifically cites such observations in the San Joaquin Valley of California. He points out that where velocity is a discontinuous function of depth in a succession of strata (as commonly assumed), if the layers become thin enough, and the velocity changes small enough, a limiting condition is reached where the time-distance graph becomes a gradual curve denoting a velocity increasing as a continuous function of depth.

Fundamental Derivation of Formulas

The object of the present investigation is to attempt to derive practicable formulas for dealing with such a case. Velocity is assumed to be a continuous linear function of depth. Proof of the validity of the assumption can come only from comparison of computed with observed curves for a given region. It would almost certainly not hold rigorously for all cases. It is hoped, however, that formulas based upon the conditions assumed may be found helpful where those of rectilinear propagation cannot be regarded as even fair working approximations.

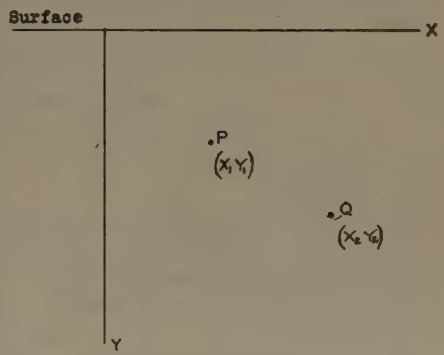

Fig. 1.—Diagram for finding path of elastic wave.

Consider a portion of the earth sufficiently small so that the curvature of the geoid may be neglected. Regard this surface as a plane, over which the velocity of elastic vibrations is a constant, b ft. per second. Following the assumption of a linear increase of velocity with depth, suppose that the velocity at y ft. below the surface at any point is $v = ay + b$ ft. per sec. The problem is to find the path by which an elastic wave would travel from a point P in this medium to another point Q, also in the medium.

It is clear from physical considerations that the path of the impulse will be in the vertical plane containing P and Q, so choose the line of intersection of this plane with the surface as the x axis. Let y be measured vertically downward. Let the coordinates of P be called x_1 and y_1 and those of Q, x_2 and y_2 (Fig. 1).

The time required for a wave or an impulse to travel along any path $y = f(x)$, from P to Q, is

$$T = \int_P^Q \frac{ds}{v} \qquad [1]$$

where ds is the element of the path. Using the relation $ds = \sqrt{dx^2 + dy^2} = \sqrt{1 + y'^2}dx$,

we have

$$T = \int_{x_1}^{x_2} \frac{\sqrt{1 + y'^2}}{ay + b} dx \qquad [2]$$

We know from the Principle of Least Action that the path the wave will actually travel is the one that will make the time T a minimum, so we want to find the function $y = f(x)$ that will accomplish this purpose. The Calculus of Variations affords a method. Denote the integrand of the integral in equation 2, by $F(x, y, y')$; that is,

$$F(x, y, y') = \frac{\sqrt{1 + y'^2}}{ay + b} \qquad [3]$$

According to E. B. Wilson (Advanced Calculus, 402), the proper function y is the one that is a solution of the equation

$$\frac{\partial F}{\partial y} - \frac{\partial^2 F}{\partial x \partial y'} - \frac{\partial^2 F}{\partial y \partial y'} y' - \frac{\partial^2 F}{\partial y'^2} y'^2 = 0 \qquad [4]$$

Proceed, now, to calculate these derivatives and substitute them in the equation:

$$\frac{\partial F}{\partial y} = -\frac{\sqrt{1 + y'^2}}{(ay + b)^2} a = -\frac{a}{ay + b} F$$

$$\frac{\partial^2 F}{\partial x \partial y'} = 0$$

$$\frac{\partial F}{\partial y'} = \frac{y'}{(ay + b)\sqrt{1 + y'^2}} = \frac{y'}{(1 + y'^2)} F$$

$$\frac{\partial^2 F}{\partial y \partial y'} = -\frac{a}{ay + b} \frac{\partial F}{\partial y'} = -\frac{ay'}{(ay + b)(1 + y'^2)} F$$

$$\frac{\partial^2 F}{\partial y'^2} = \frac{F}{(1 + y'^2)} + \frac{y'^2}{(1 + y'^2)^2} F - \frac{y'F}{(1 + y'^2)^2} 2y' =$$

$$F\left(\frac{1 + y'^2 + y'^2 - 2y'^2}{(1 + y'^2)^2}\right) = \frac{F}{(1 + y'^2)^2}$$

Substitution of these values in the differential equation gives:

$$-\frac{a}{ay + b} F + \frac{ay'^2}{(ay + b)(1 + y'^2)} F - \frac{y''}{(1 + y'^2)^2} F = 0$$

or

$$a(1 + 2y'^2 + y'^4 - y'^2 - y'^4) + y''(ay + b) = 0$$

or

$$y''(y + b/a) + y'^2 + 1 = 0$$

which is just a second order differential equation in y. Then the function y, which satisfies this equation and passes through P and Q, is the solution of the problem.

To integrate this equation, note that

$$\frac{d}{dx}(yy') = yy'' + y'^2$$

so that the equation may be written:

$$\frac{b}{a}y'' + \frac{d}{dx}(yy') + 1 = 0$$

The first integration gives

$$\frac{b}{a}y' + yy' + x = c$$

where c is an arbitrary constant.
Integrate again:

$$\frac{b}{a}y + \frac{y^2}{2} + \frac{x^2}{2} = cx + \frac{d}{2}$$

where d is an arbitrary constant.
This may be written:

$$\left(y + \frac{b}{a}\right)^2 + x^2 - 2cx = d + \frac{b^2}{a^2}$$

or

$$\left(y + \frac{b}{a}\right)^2 + (x - c)^2 = \left(d + \frac{b^2}{a^2} + c^2\right)$$

The arbitrary constants appear only in the x term and in the expression for the constant term, so we see that *all possible paths between any two points in the medium are circles with centers on the horizontal line at a height b/a ft. above the surface.*

The equation of the particular path which we set out to find is obtained by determining properly the arbitrary constants c and d, but the geometrical picture of the path as a circle with the y coordinate of its center at $-b/a$ leads in all cases to an easily remembered method for drawing or setting up the equation of the path. Another aid to the memory is the fact that if we consider that the surface of the ground is raised to a position well above its true position, but that at any given point beneath the present surface the velocity is not altered, while above the true surface the velocity decreases at the rate of a ft. per sec., the velocity at the line on which the centers of the circular paths lie is $b - a\,(b/a) = 0$. That is, we assume a surface at a distance b/a above the true surface and assume that the velocity at this hypothetical surface is zero.

Integration of the Time

The Ground Wave

It seems to make for greater simplicity to use polar coordinates in this integration. Consider Fig. 2. Choose the origin and the axis or initial line for φ as is shown. Then the element of path, $BC = r d\varphi$, is at a distance $AB = r \sin \varphi$ beneath the hypothetical surface. Hence

the velocity with which the impulse traverses this element is $ar \sin \varphi$. Then the time required* for the impulse to travel from P to Q is

$$T = \int_{\varphi_0}^{\varphi_1} \frac{r d\varphi}{ar \sin \varphi} = \frac{1}{a}\int_{\varphi_0}^{\varphi_1} \frac{d\varphi}{\sin \varphi} = \frac{1}{a}\left[\log \tan \frac{1}{2}\varphi\right]_{\varphi_0}^{\varphi_1}$$

$$T = \frac{1}{a}\left(\log \tan \frac{\varphi_1}{2} - \log \tan \frac{\varphi_0}{2}\right)^{\dagger} \qquad [5]$$

The solution in this form lends itself readily to graphical solutions. For the special case in which P and Q are at the same distance beneath the surface (this distance may easily be zero), the point E, directly beneath the center O, will be the midpoint of the path. The time, then,

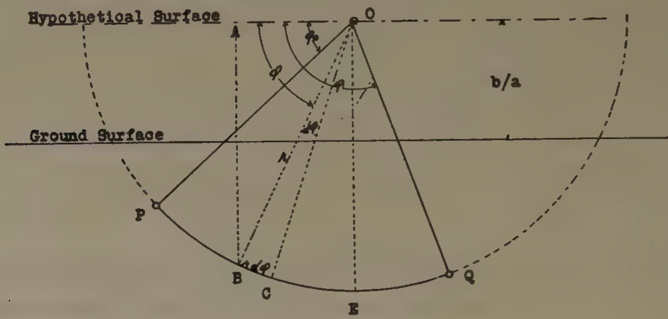

FIG. 2.—PATH OF GROUND WAVE.

required for the wave to travel from P to Q is just twice the time required for the wave to travel from P to E. Then, $\varphi_1 = \frac{\pi}{2}$, $\tan \frac{\varphi_1}{2} = 1$, $\log \tan \frac{\varphi_1}{2} = 0$, so

$$T = -\frac{2}{a} \log \tan \frac{\varphi_0}{2} \qquad [6]$$

This formula may be reduced to one which may in some cases be more convenient to use. From formula 578 in Peirce's Tables, we see that

$$-\frac{2}{a} \log \tan \frac{1}{2}\varphi_0 = -\frac{2}{a} \log \sqrt{\frac{1 - \cos \varphi_0}{1 + \cos \varphi_0}}$$

Replacing $\cos \varphi_0$ by d/r (see Fig. 3), and making use of a few of the elementary properties of the logarithm, we have:

$$T = \frac{1}{a} \log \frac{1 + \frac{d}{r}}{1 - \frac{d}{r}} = \frac{1}{a} \log \frac{r + d}{r - d} \qquad [7]$$

* See Peirce's Tables, Formula 291.

† Up to this point, the mathematical work follows, in principle at least, a lecture given several years ago by H. A. Wilson.

In a neighborhood in which the variation of velocity with depth is approximately linear, the constants a and b may be determined from the times T_1 and T_2 for the impulses to travel horizontal distances $2d_1$ and $2d_2$. If the shots and recorders are near the surface, $r = \sqrt{d^2 + \frac{b^2}{a^2}}$. Hence

$$\frac{T_2}{T_1} = \frac{\log \dfrac{\sqrt{d_2^2 + \frac{b^2}{a^2}} + d_2}{\sqrt{d_2^2 + \frac{b^2}{a^2}} - d_2}}{\log \dfrac{\sqrt{d_1^2 + \frac{b^2}{a^2}} + d_1}{\sqrt{d_1^2 + \frac{b^2}{a^2}} - d_1}}$$

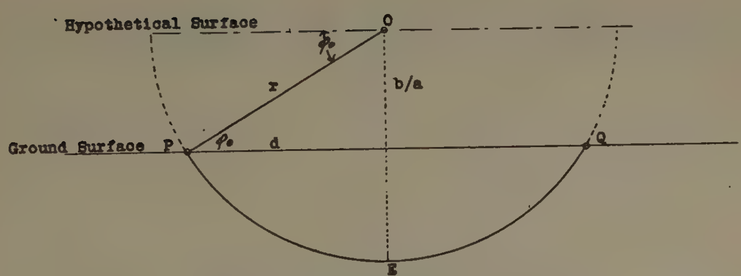

FIG. 3.—PATH OF GROUND WAVE.

Thus we have a numerical equation in the single variable b/a, for which we may solve by Hörner's method. When b/a is known, the value of a may be determined directly from either

$$a = \frac{1}{T_1} \log \frac{r_1 + d_1}{r_1 - d_1}$$

or

$$a = \frac{1}{T_2} \log \frac{r_2 + d_2}{r_2 - d_2}$$

These values should, of course, be the same. It is a good plan to compute both and take the mean value as the true value for a. An example of this computation is worked out later.

Another form for the expression for T in this case may be obtained from formula 681, Peirce's Tables, and equation 7:

$$T = \frac{2}{a} \tanh^{-1} \frac{d}{r} \qquad [8]$$

The Reflected Wave

Suppose, as before, that the velocity has the value b ft. per sec. at the surface and increases linearly with the depth at the rate of a ft. per sec. to a depth of h ft., where the velocity suffers a discontinuity. It is possible to get reflections from this discontinuity in the velocity. On the ideal assumption which we have made about a plane surface and a parallel reflecting plane, the point of reflection is the vertical projection of the midpoint of the line joining the shot point and the recorder, provided the shot and recorder are about the same distance below the surface. In this case, the path is composed of two circular arcs, instead of an arc of a single circle. Fig. 4 shows the outstanding features.

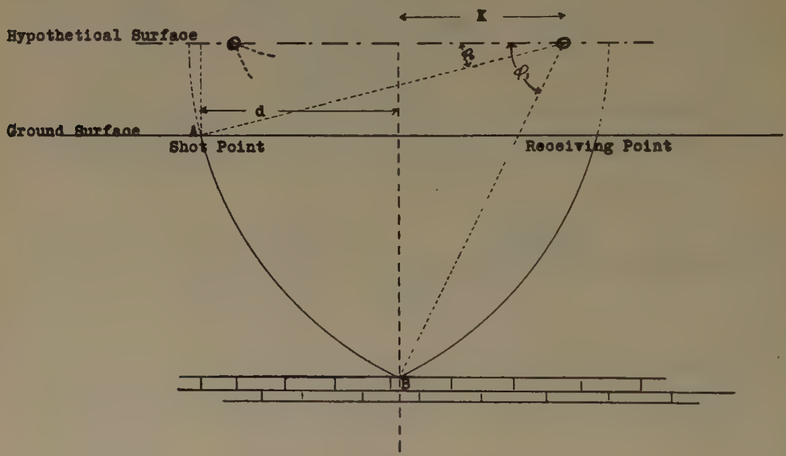

FIG. 4.—PATH OF REFLECTED WAVE.

Formula 5 shows that the total time required for the reflected wave to travel this path is

$$T = \frac{2}{a}\left(\log \tan \frac{\varphi_1}{2} - \log \tan \frac{\varphi_0}{2}\right) \qquad [9]$$

Formula 9 is suitable for graphical solutions but it can be converted into a form that is more convenient for numerical calculations. Following the procedure outlined, we have, for formula 9,

$$T = \frac{2}{a}\left(\log \tan \frac{\varphi_1}{2} - \log \tan \frac{\varphi_0}{2}\right) = \frac{1}{a}\left(\log \frac{1 - \cos \varphi_1}{1 + \cos \varphi_1} - \log \frac{1 - \cos \varphi_0}{1 + \cos \varphi_0}\right)$$

$$= \frac{1}{a}\left(\log \frac{1 - \dfrac{K}{r}}{1 + \dfrac{K}{r}} - \log \frac{1 - \dfrac{K+d}{r}}{1 + \dfrac{K+d}{r}}\right) = \frac{1}{a} \log \frac{(r - K)(r + K + d)}{(r + K)(r - K - d)}$$

Therefore
$$T = \frac{1}{a}\log\frac{1+\dfrac{d}{r+K}}{1-\dfrac{d}{r-K}} \qquad [10]$$

The value for K may be obtained by equating the expressions for the radii OA and OB; thus

$$\frac{b^2}{a^2} + K^2 + 2Kd + d^2 = \frac{b^2}{a^2} + 2\frac{b}{a}h + h^2 + K^2$$

whence
$$K = \frac{\left(h+\frac{b}{a}\right)^2 - \left(\frac{b^2}{a^2}+d^2\right)}{2d}$$

Once K is known, the value of r may be obtained from

$$r = \sqrt{K^2 + \left(h + \frac{b}{a}\right)^2}$$

When the distance from the shot to the observer ($2d$) is taken smaller and smaller, the paths become vertical straight lines (that is, circles with infinite radii). Thus the limiting value of expression 10 when d approaches zero will give the time required for a vertical reflection. The next step is to evaluate that limit. From the equations just given, it is clear that when d approaches zero, K and therefore also r approach infinity. It is seen at once that the limit of the numerator of the fraction in equation 10 is unity. But the value of lim. $\dfrac{d}{r-K}$, which occurs in the denominator, is indeterminate and requires further consideration. First,

$$\lim_{d\to 0} rd = \lim_{d=0} \sqrt{\left(h+\frac{b}{a}\right)^2 d^2 + K^2d^2} = \lim_{d\to 0} Kd = \frac{\left(h+\frac{b}{a}\right)^2 - \frac{b^2}{a^2}}{2}$$

also
$$\frac{\partial K}{\partial d} = -\frac{K}{d} - 1$$
$$\frac{\partial r}{\partial d} = \frac{K}{r}\frac{\partial K}{\partial d} = -\frac{K^2}{rd} - \frac{K}{r}$$

then
$$\lim_{d\to 0} \frac{d}{r-K} = \lim_{d\to 0} \frac{d^2}{rd - Kd} = \lim_{d\to 0} \frac{2d}{d\frac{\partial r}{\partial d} - d\frac{\partial K}{\partial d} + r - K}$$

$$= \lim_{d\to 0} \frac{2d}{-\frac{K^2}{r} - \frac{Kd}{r} + K + d + r - K} = \lim_{d\to 0} \frac{2rd}{(r^2 - K^2) + (rd - Kd)}$$

$$= \frac{\left(h+\frac{b}{a}\right)^2 - \frac{b^2}{a^2}}{\left(h+\frac{b}{a}\right)^2}$$

The justification for the last step is as follows:

$$\lim.\ (rd - Kd) = 0 \text{ because } \lim.\ rd = \lim.\ Kd$$

$$r^2 - K^2 = \left(h + \frac{b}{a}\right)^2$$

$$\lim.\ rd = \frac{\left(h + \frac{b}{a}\right)^2 - \frac{b^2}{a^2}}{2}$$

From this discussion, we see that

$$\lim_{d \to 0} \frac{1 + \frac{d}{r+K}}{1 - \frac{d}{r-K}} = \frac{1}{1 - \frac{\left(h + \frac{b}{a}\right)^2 - \frac{b^2}{a^2}}{\left(h + \frac{b}{a}\right)^2}} = \frac{\left(h + \frac{b}{a}\right)^2}{\frac{b^2}{a^2}}$$

Hence, for the wave reflected vertically, the time is

$$T = \frac{1}{a} \log \frac{\left(h + \frac{b}{a}\right)^2}{\frac{b^2}{a^2}} = \frac{2}{a} \log \left(\frac{h}{b/a} + 1\right) \qquad [11]$$

A check on this work on reflected waves is given by direct integration for the case of vertical reflection. In this case:

$$T = 2\int_o^h \frac{dy}{ay + b} = \frac{2}{a}\Big[\log\,(ay + b)\Big]_o^h = \frac{2}{a} \log \frac{ah + b}{b}$$

On the assumption that the velocity has a constant value v from the surface down to the reflecting layer, the relation between time and distance is

$$T = \frac{2}{v}\sqrt{h^2 + d^2} \qquad [12]$$

where d is half the distance from shot to recorder, and h the vertical depth to the reflecting horizon.

Since this is an assumption generally made in the field, a comparison between times computed by formula 10 and by formula 12 is given later in this paper.

The Refracted Wave

Assume that the velocity varies linearly with depth from b ft. per sec. at the surface to $ah + b$ ft. per sec. at h ft. below the surface. At h ft. below the surface there is a stratum at the top surface of which the

velocity jumps abruptly from $ah + b$ to B ft. per sec. Below h, the velocity increases linearly at A ft. per sec. From the previous work, we know that when d is great enough the path extends down into this second medium. If it is not so great that the path extends to the bottom of the second medium, the path will consist of three circular arcs, as shown in Fig. 5.

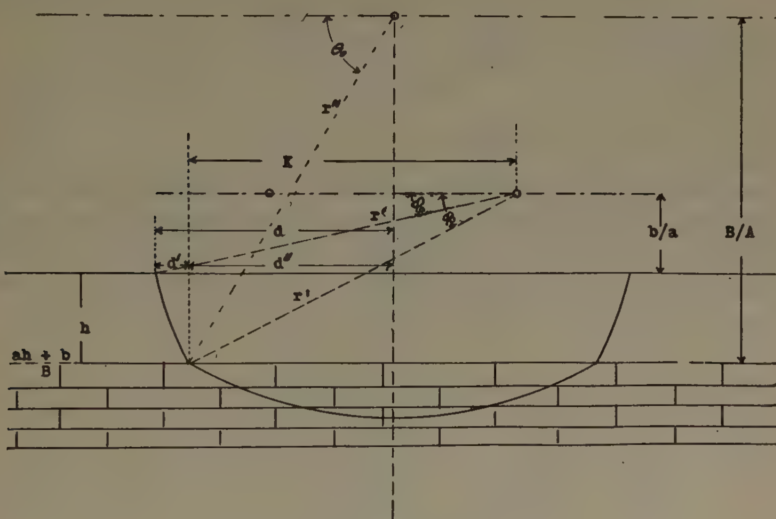

FIG. 5.—PATH OF REFRACTED WAVE.

The results already obtained in equation 10 indicate that the equation of the time-distance curve for this case is given by either of the two expressions:

$$T = \frac{1}{a} \log \frac{1 + \dfrac{d'}{r' + K}}{1 - \dfrac{d'}{r' - K}} + \frac{1}{A} \log \frac{r'' + d''}{r'' - d''} \qquad [13]$$

$$T = \frac{2}{a}\left(\log \tan \frac{\varphi_1}{2} - \log \tan \frac{\varphi_0}{2}\right) - \frac{2}{A} \log \tan \frac{\theta_0}{2} \qquad [14]$$

However, more must be said before these formulas can be used. As yet, nothing has been done to enable us, if given d, to determine d' and d''.

An explicit expression for d' has not been obtained. The relationship derived below, however, serves to make the time determinate.

Fig. 5 shows that φ_1 is the angle of incidence and θ_0 is the angle of refraction for the wave at the interface of the two media. Then, by the ordinary expression

$$\frac{\sin i}{\sin r} = \frac{V_1}{V_2}$$

we have

$$\frac{\sin i}{\sin r} = \frac{\dfrac{h + \dfrac{b}{a}}{r'}}{\dfrac{B/A}{r''}} = \frac{ah + b}{ar'} \cdot \frac{Ar''}{B}$$

and

$$\frac{V_1}{V_2} = \frac{ah + b}{B}$$

so we must have

$$\frac{ah + b}{ar'} \cdot \frac{Ar''}{B} = \frac{ah + b}{B}$$

or

$$ar' = Ar''$$

Numerical Examples

Consider two observations for which the distances from shot to recorder were $D_1 = 14{,}130$ m.; $D_2 = 13{,}440$ m. The times taken for the impulses to travel these distances were $T_1 = 6.01$ sec. and $T_2 = 5.80$ sec. The ratio of T_1/T_2 for these observations is 1.036. Now, using the formula

$$\frac{T_1}{T_2} = \frac{\log \dfrac{\sqrt{d_1^2 + \dfrac{b^2}{a^2}} + d_1}{\sqrt{d_1^2 + \dfrac{b^2}{a^2}} - d_1}}{\log \dfrac{\sqrt{d_2^2 + \dfrac{b^2}{a^2}} + d_2}{\sqrt{d_2^2 + \dfrac{b^2}{a^2}} - d_2}}$$

the procedure is to assume a series of values for b/a and substitute each of these together with the values of d_1 ($\frac{1}{2}D_1$) and d_2 ($\frac{1}{2}D_2$) in the right-hand member of this equation. The accuracy of the assumed value for b/a is then indicated by the quality of the agreement of the value for T_1/T_2 thus calculated with the value observed (1.036 in this case).

Values Assumed for b/a, Meter	Resulting Values for T_1/T_2
2500	1.025
5000	1.037
4800	1.036

Accepted value for b/a = 4800 m. (15,749 ft.).

$$a = \frac{1}{T_1} \log \frac{\sqrt{d_1^2 + \dfrac{b^2}{a^2}} + d_1}{\sqrt{d_1^2 + \dfrac{b^2}{a^2}} - d_1} = \frac{1}{6.01} \times 2.3590$$

$$= 0.3925 \text{ per sec.}$$

$$b = a \times 4800 \text{ m.} = 0.3925 \times 4800$$

$$= 1884 \text{ m. per sec. [6181 ft. per sec.]}$$

The Ground Wave

$$T = \frac{1}{a} \log \frac{r + d}{r - d} \qquad r = \sqrt{d^2 + \frac{b^2}{a^2}}$$

b/a 800 m.	$a = 0.3925$ per sec.	$b = 1884$ m. per sec.
$T_1 = 6.01$ sec.	$D_1 = 14{,}130$ m.	$d_1 = 7065$ m.
$T_2 = 5.80$ sec.	$D_2 = 13{,}440$ m.	$d_2 = 6720$ m.

TABLE 1.—*Time, Distance and Depth of Penetration of Ground Wave*

Distance Shot to Recorder, Meters	d^2, $m^2 \times 10^{-6}$	r^2, $m^2 \times 10^{-6}$	r, Meters	$\frac{r+d}{r-d}$	$\frac{r+d}{r-d}$	$\log \frac{r+d}{r-d}$	T, Seconds	$r - b/a$ Depth of Penetration, Meters
1,000	0.25	23.29	4,826	$\frac{5{,}326}{4{,}326}$	1.231	0.2078	0.530	26
2,000	1.00	24.04	4,903	$\frac{5{,}903}{3{,}903}$	1.512	0.4134	1.053	103
3,000	2.25	25.29	5,029	$\frac{6{,}529}{3{,}529}$	1.850	0.6152	1.568	229
4,000	4.00	27.04	5,200	$\frac{7{,}200}{3{,}200}$	2.250	0.8109	2.066	400
5,000	6.25	29.29	5,412	$\frac{7{,}912}{2{,}912}$	2.717	0.9994	2.546	612
10,000	25.00	48.04	6,931	$\frac{11{,}931}{1{,}931}$	6.179	1.8211	4.640	2,131
12,000	36.00	59.04	7,684	$\frac{13{,}684}{1{,}684}$	8.126	2.0950	5.338	2,884
14,000	49.00	72.04	8,487	$\frac{15{,}487}{1{,}487}$	10.415	2.3432	5.970	3,687
15,000	56.25	79.29	8,904	$\frac{16{,}404}{1{,}404}$	11.684	2.4582	6.263	4,104
16,000	64.00	87.04	9,329	$\frac{17{,}329}{1{,}329}$	13.039	2.5680	6.543	4,529

It is worth noting, in passing, that by dividing the distance between two stations by the difference between their times a velocity is obtained which represents the arithmetical mean of the maximum velocities attained by each ray $(ay + b)$. In other words, the apparent absolute surface velocity (surface distance/time) with which the ray traverses the distance between two positions is definitely related to the maximum velocity along its actual path. This, of course, is to be expected if it is following the shortest time path.

Comparison of computed time-distance curves with actual observations in this case and in several others available gave encouraging confirmation of the theory.

The Reflected Wave

Assuming a horizon at a depth of 3188 ft., from which reflections are obtained, Table 2 compares the transit time computed by means of

a single average velocity over the entire depth with the curved path value. This shows that, using the calibration velocity just obtained, the results of the two types of calculation are in sufficiently good agreement for shots of 5000 ft. or less. Thus, for work on a horizon of depth h the use of an experimentally determined value for a constant calibration velocity would introduce no large error due to variation in the length of the shot up to that order of distance. However, if the depth of the horizon should change, to continue to use the same calibration value would tend to make an increase in depth appear too small, and a decrease in depth too large.

TABLE 2.—*Comparison of Transit Time with Curved-path Value*

Distance D, Feet	$T = \frac{2\sqrt{h^2 + d^2}}{v}$, Seconds	Curved-path Value of T, Seconds
1,000	0.784	0.786
4,000	0.915	0.912
5,000	0.985	0.980
10,000	1.441	1.417
14,000	1.870	1.811

The Refracted Wave

Assume a high-speed horizon of sufficient extent to refract waves which encounter it. Beyond the point at which the ground wave penetrates just deeply enough to graze the surface of this horizon, the first wave to reach a recorder will have been refracted through the high-speed medium for part of its path. When first impulses are plotted, the time-distance curve would exhibit a decided break at this point, since the ground wave and refracted wave curves are not tangent. It also seems reasonable to expect that in this neighborhood the amplitude of the initial impulse would be considerably greater than for shots just shorter or just longer, since it is produced here by the combined action of two waves.

In cases where the velocity of propagation increases with depth, but not linearly, so that the equations derived herein do not hold, and where there is a high-speed horizon beneath the surface, the curves for the reflected, refracted and ground waves will bear relations to each other similar to those indicated above. The break in the curve for the first impulse and the accompanying increase in amplitude should be exhibited and should make the location of the important point at which the ground and refracted waves come in together a matter of comparative certainty.

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DISCUSSION

(*Allen H. Rogers presiding*)

D. C. Barton, Houston, Tex. (written discussion).—Decreased density is stated by the authors to be one of the factors which enter into an explanation of the high velocity of the salt. Density differences play an unimportant part in explaining the different speeds of transmission of seismic waves by different formations met in seismic prospecting, although the velocity is proportional to the reciprocal of the square root of the density. The range of density in the common rocks encountered in the applied seismic method is from 1.9 to 2.6; and the range of $1/\sqrt{D}$ is from 0.676 to 0.620; or a range of about ± 5 per cent from the mean value. The corresponding range in velocities is from 1.5 to 6.0 km. per second, or a range of about ± 60 per cent. The range of densities in the coastal salt-dome area is: salt, 2.15 to 2.22—mean, 2.19; sediments from surface to 3000 ft., 1.9 to 2.3—mean, 2.15; 3000 to 6000 ft., probably 2.20 to 2.35; in great depth probably not over 2.4. The corresponding range of the factor $1/\sqrt{D}$ is: salt, 0.676; sediments, 0 to 3000 ft., 0.682; 3000 to 6000 ft., 0.659; in great depth not less than 0.646. The comparative difference between the factor for the salt and the factor for the sediments above 6000 ft. ranges from $+1$ to -2.5 per cent. The corresponding velocities are: salt, 4.5 km. per second; sediments above 6000 ft., 2.0 to 4.0 km. per second. Specific gravity, therefore, plays a minor role in explaining the higher velocity of the salt or the variation of velocity in the other beds.

Compactness seems geologically to be a most important factor in determining the velocity of the rocks encountered in seismic prospecting. Rocks and the formations encountered in seismic prospecting are not continuous elastic solids, but aggregates of small particles of solids. Clastic sediments in general consist of small particles only more or less in contact with each other and of interstitial water which probably carries part of the burden of the overlying rocks. The individual grains have a certain degree of freedom of movement and intuitively it would seem that delay in transmission of the wave impulse from particle to particle should therefore ensue, roughly in proportion to the degree of freedom of movement of the constituent particles of the rock. With increasing age and depth of burial of clastic sediments, there tends to be reduction of pore space and increase of cementation; the constituent particles are therefore brought closer in contact with each other and to some extent are rigidly cemented together. Cementation and crystallization tend to take place easily in calcareous sediments, which, therefore, tend to be more compact than noncalcareous sediments. In such unshattered crystalline rocks, as crystalline limestones, rock salt, igneous and metamorphic rocks, the constituent particles in general tend to be interlocked and in rigid contact. Alluvium and the surficial weathered zone, therefore, have very low velocities; the Plio-Pleistocene of the Gulf Coast of Texas, low velocities, and the deeper sediments successively higher velocities; the sediments in East Texas, slightly higher velocities than sediments at the same depth in the Gulf Coast; the Pecan Gap and Austin chalks, high velocities; salt, very high velocity. With the increasing compactness, there is increase of specific gravity, but its effect is slight. The "lime" cap of the salt domes characteristically is badly shattered and is permeated by a ramifying network of solution channels, and on some domes is a friable granular rock with poor cohesion between the constituent grains. The anhydrite shows varying degrees of friability and, therefore, of cohesion between the constituent grains. The anhydrite of many domes is distinctly friable. Although crystalline, the "lime" cap and the anhydrite, therefore, should show distinctly lower velocities than normal for crystalline rocks. Physically the role of compactness is explainable in terms of incompressibility and rigidity.

E. E. Rosaire, Houston, Tex. (written discussion).—In the next to the last paragraph, the authors anticipate a considerable increase in amplitude due to the simultaneous arrival of, say, first and second layer refractions. Such a marked increase in amplitude has not been observed in the field. Further, our experience suggests that no such great increase in amplitude will take place, since at distances just greater than the (intersection) simultaneous arrival, the second layer refraction impulse is weak, and apparently is more readily defined at somewhat greater arrival distances.

Such a result is predicted by an examination of the effect of spreading or dispersion of the sound rays upon the arrival energy. This dispersion or spreading effect can be studied by differentiating the arrival distance with respect to the angle of departure of the ray from the surface. When calculated for reasonable conditions and with the same type of velocity increase (linear with depth) as used by the authors, a dispersion minimum is found for the second layer refractions. This minimum value occurred at a distance greater than the intersection (simultaneous arrival) point. At this point the dispersion for the second layer refractions was found to be about one hundred times as great as for the first layer refractions.

Although reasonable corrections were made for transmission and reflection at this assumed interface, no attempt was made to calculate the energy losses due to frequency attenuation, etc. It is probable that these effects, when properly applied, might nearly mask the effect of dispersion, so that as yet conclusions are unjustified. In my opinion, however, the arrival amplitude of the second layer refractions at the inter-

section will be only a fraction of the arrival amplitude of the first layer refractions at this point. I agree that simultaneous arrival may cause some but I hardly expect a great increase in amplitude.

This discussion concerns a very minor point and is not intended to detract from the merits of a paper that is a worthy contribution to the all too limited literature of applied seismology.

Comparison of Two Methods for Interpretation of Seismic Time-distance Graphs Which Are Smooth Curves

By Maurice Ewing,* Bethlehem, Pa. and L. Don Leet,† Cambridge, Mass.

(New York Meeting, February, 1931)

The most important quantitative method in seismic prospecting by refraction shooting is the method of profiles. A profile is established by firing a series of charges at various points along a straight line which terminates at the seismograph. The same data are obtained by repeatedly firing charges at one place and recording at a series of stations along the line of the profile.

The quantities which are measured are the distance x from each firing position to the seismograph and the time t required for the first disturbance from each explosion to travel to the seismograph. These data are used in plotting the familiar time-distance graph.

The only time-distance graphs that will be considered here are those obtained in localities where the velocity of propagation depends solely on the depth beneath the surface; that is, where there is no slope in the subsurface.

The essential calculation to be made from such a time-distance graph is the determination of $P(x)$, the maximum depth of penetration of a disturbance coming to the seismograph from a firing point at the distance x from the instrument. If the time-distance graph consists of a series of straight lines, as is often the case, the penetration $P(x)$ can be calculated by means of formulas which appear in almost every description of seismic prospecting.[1] Until the present, however, the only formula that has been available for calculating $P(x)$ for time-distance graphs which are smooth curves is one which requires an arbitrary assumption regarding the law of velocity increase with depth.[2] The practice in these cases has been to approximate the curve by means of a number of straight lines and then to apply the ordinary straight-line formulas. This method clearly can yield accurate results only when a sufficiently large number of straight lines is used. The labor of computation increases rapidly when the number of lines is increased, so too few lines often are used. Further than that, there is a strong tendency to overlook the approximate nature

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[1] See, for example, papers in Geophysical Prospecting, A. I. M. E. (1929).

[2] M. Ewing and L. D. Leet: Seismic Propagation Paths. See page 245.

of the method, though this was stressed as early as 1928 by Barton,[3] as follows:

> They (straight-line formulas) are based on the assumption that each formation is homogeneous and isotropic and therefore on the assumption that within each formation the wave path is a straight line. The actual situation practically is not so simple. In a region such as the Texas-Louisiana Gulf Coast, where there is an enormously thick section of clays and sands, the successively deeper and therefore older beds should tend to show an increasing degree of compaction and therefore an increasing speed of transmission of the seismic earth waves. A wave therefore should undergo continuous refraction and its path should be an arc and not a straight line. The rate of increase downward of the speed of transmission in some areas may be rather high and the resulting continuous refraction may be sufficient seriously to affect the applicability of the formulas and the time-distance curves of this paper.

When the points on a profile are plotted, one of the first problems of interpretation involves the decision as to whether they fall on straight lines or one or more curves. For years, straight lines have been fitted to time-distance points in the field under all circumstances. Converging lines of evidence, however, have convinced the authors that profiles over the average Gulf coastal plain sediments, in the absence of salt domes or similar foreign structures, yield definite, uniformly curved time-distance graphs. The data here presented compare two methods for treating profile observations in this region.

As a matter of fact, the unavoidable presence of accidental errors in the determination of the points of a graph sometimes renders it impossible to decide whether a smooth curve or a set of straight lines which approximate the curve very closely is best suited to represent a given set of data. In all branches of science the procedure in such cases is to select the curve unless there is independent evidence in favor of the straight lines.

The decision in handling Gulf coastal plain data, however, is of far more than mere academic importance. If the true graph is a smooth curve, it signifies subsurface conditions changing practically uniformly with depth, a condition expectable geologically and substantiated in drilling. The fitting of arbitrary straight lines to data from such a region implies a series of discontinuities, which do not exist. If the purely arbitrary nature of such straight lines is not recognized, serious mistakes easily may be made. Attempts have even been made to contour "horizons" represented by certain of these lines and to go so far as to interpret, in terms of sloping subsurface interfaces, minor fluctuations of personal judgment in fitting straight lines to the curve on different profiles. Similarly, this situation would account for statements that "geophysical horizons do not necessarily coincide with geological horizons," and the curious uniformity of interval between straight-line veloci-

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[3] D. C. Barton: The Seismic Method of Mapping Geologic Structure. Geophysical Prospecting, A. I. M. E. (1929) 588.

ties represented by series which run 5500 ft. per sec., 6500 ft. per sec., 7500 ft. per sec., 8500 ft. per sec., and so on.

Where a uniform curve will fit even moderately well; where there is the slightest uncertainty as to whether straight lines or curves fit the observed points best, the possibility of straight lines having the physical significance they imply should be discarded. The suggestion is offered that they would never have been drawn if a formula for treating curved graphs had been available.

Curved Path Formula for Penetration

The formula for treating curved time-distance graphs was developed by the senior author in June, 1929. This result has been kept secret until the present time on account of an agreement which has expired only recently.

The formula is applicable to time-distance graphs which are smooth curves concave toward the x-axis. It gives for $P(X)$, the maximum depth of penetration of a shot of length X, the expression

$$P(X) = \frac{1}{\pi}\int_0^X \operatorname{arc\,cosh} \frac{T'(x)}{T'(X)}\, dx \qquad [1]$$

where $T'(x)$ is the slope of the time-distance curve at the point having abscissa x. The derivation of this formula is given in the Appendix.

We may apply this result to a given time-distance curve by either an analytical or a graphic method.

The first step in the analytical method is to determine the equation of a curve which will fit the given points. Equation 1 may then be integrated to give an expression for $P(X)$ in terms of the constants of the time-distance curve. In the special case which seems to apply readily to much of the Gulf coastal plain data, the time-distance curve is of the form[4]

$$X = aT^2 + bT \qquad [2]$$

and equation 1 becomes

$$P(X) = \frac{b^2}{8\pi a}[\sinh q - q] \qquad [3]$$

where

$$q = 2 \operatorname{arc\,cosh} \frac{2aT + b}{b}$$

In the graphic method of using equation 1, we start by drawing tangents to the given $T(x)$ curve. From these we construct graphs for $T'(x)$, $\frac{T'(x)}{T'(X)}$, and arc cosh $\frac{T'(x)}{T'(X)}$ in the order in which they are named. A graphic integration then gives the penetration according to equation 1.

[4] The constants a and b in equation 2 can be determined so that the parabola represented by the equation will fit time-distance curves for normal Gulf Coast sediments.

Treatment of an Observed Curve

The observed points in Figs. 1 and 3 represent the average times and average distances for a considerable number of profile observations within a small area in the Gulf coastal plain east of Beaumont.

By the use of the method of least squares, the values of the constants a and b, for which equation 2 satisfactorily represents the data (see footnote 4), are found to be

$$a = 550 \text{ ft. per sec.}^2$$
$$b = 4500 \text{ ft. per sec.}$$

The curve constructed from these constants is shown in Fig. 1. The circles representing observed average points have an inside radius equal

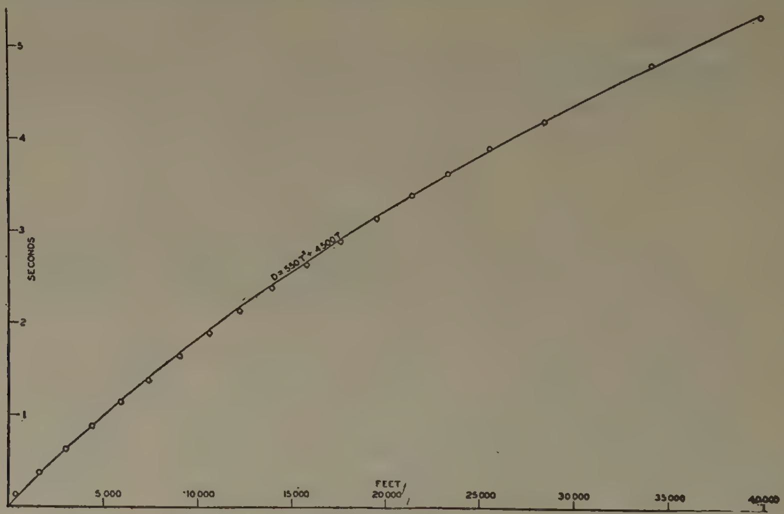

Fig. 1.—Curve fitted to Gulf Coast time-distance data.

on the time-scale to 0.02 sec., or on the distance-scale to 100 feet. The smooth curve of Fig. 2 shows the values for the penetration which are obtained by using these constants in equation 3. In Fig. 3, straight lines have been fitted to the same set of observed points. The penetrations calculated from these lines by the ordinary formulas are shown by the step-series of straight lines in Fig. 2.

In Fig. 2 it can be seen that for three points, at distances of 600, 8500 and 16,600 ft., the penetrations computed by each method are in fair agreement. The lack of complete agreement may be attributed in part to the fact that no unusual amount of care was taken with the finer details of the calculation in either case, and to the use of an insufficient number of straight lines. Between these points, and beyond 16,600 feet, there is a discrepancy which increases with the distance.

An important point to which attention should be directed, however, is well illustrated. The function $P(x)$ is found by the curve method to

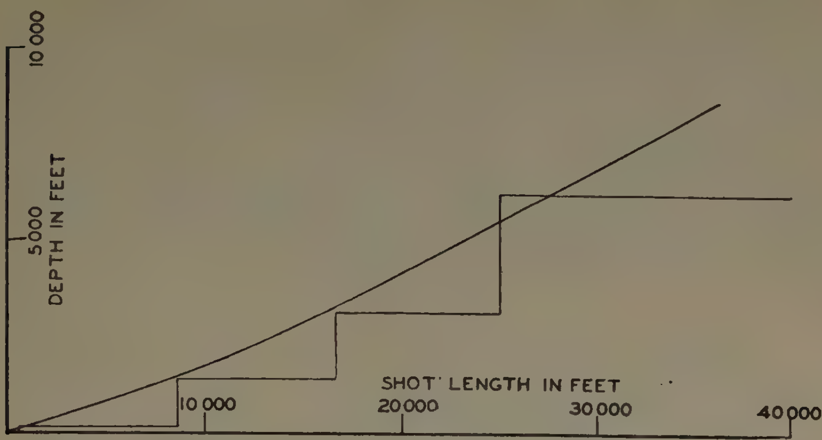

Fig. 2.—Values for penetration obtained by using the two methods.

be a continuous, increasing function, and by the straight-line method to be a discontinuous function which has a constant value between any pair

Fig. 3.—Straight lines fitted to set of observed points used in Fig. 1.

of successive discontinuities. It is thus obvious that the errors in the results of the straight-line method must be large at certain points unless many approximating lines are used.

Another significant fact is that the straight-line calculations indicate that the maximum penetration represented by the observed points out to 40,000 ft. was 6300 ft., whereas the curved-path interpretation shows that it was actually over 8700 ft. In yielding information as to depths within which desired structures do not exist, this is vital, and would seem of itself enough to more than justify discarding the straight-line interpretations in such an area.

From tests made, the authors have found that a computer can treat a set of data much more quickly and rigorously by the smooth-curve method than by the straight-line method, if he draws enough lines in the latter case to get reasonably accurate results.

Summary

1. Smoothly curved time-distance graphs actually are obtained, notably in the Gulf coastal plain sediments.
2. Although it is legitimate to approximate these smooth curves by a number of straight lines for the purposes of calculation of depth at a given point or points, it is a serious error to attribute physical significance to the arbitrary set of lines used.
3. The formula, given in this report, for the interpretation of curved time-distance graphs without assuming a law of velocity increase with depth, is easy to apply, and gives far more useful and satisfactory results.

Appendix.—Derivation of Formulas 1 and 3

By Maurice Ewing

The Herglotz-Bateman equation,[5] which is used in pure seismology to determine the velocity-depth relation from time-distance data, may be written

$$\ln\frac{R}{r_m} = \frac{1}{\pi R}\int_0^X \text{arc cosh}\,\frac{T'(x)}{T'(X)}\,dx \qquad [4]$$

Here $T(x)$ represents the time-distance curve, T being the travel-time for the epicentral distance x. The derivative, $T'(x)$, represents the slope of this curve. X is the epicentral distance for which the penetration is to be computed, and it can be seen from the equation that the values of x to be considered are those lying between zero and X. R is the radius of the earth and r_m is the distance from the center of the earth to the mid-point of the path by which the energy travels from the epicenter to X.

This equation is used to calculate r_m from the time-distance curve and thus to find the penetration, $R - r_m$. It is not valid unless the

[5] V. Conrad: *Enzyklopädie der mathematischen Wissenschaften,* 6, Pt. I, 444. E. Wiechert & L. Geiger: *Physikal. Ztsch.* (1910) 11, 394.

earth can be considered as a sphere in which the velocity of seismic waves depends only on the distance from the center.

To obtain a form of the Herglotz-Bateman equation suitable for use in seismic prospecting, we first note that the epicentral distances involved are negligible in comparison with the radius of the earth or the radius to the midpoint of a trajectory. Accordingly, it is assumed that the surface of the earth is a plane rather than a sphere, and that the velocity of seismic waves depends only on the depth beneath the surface.

This alteration of the assumptions produces no change in the integral which appears in equation 4. To evaluate the change which is produced in the rest of the equation, we set

$$P(X) = R - r_m \qquad [5]$$

and write

$$\begin{aligned}\lim_{R\to\infty} R \ln \frac{R}{r_m} &= \lim_{R\to\infty} \ln \left(\frac{1}{1 - \frac{P(X)}{R}} \right)^R \\ &= \lim_{R\to\infty} P(X) \ln \left(1 + \frac{P(X)}{R} \right)^{\frac{R}{P(X)}} \qquad [6] \\ &= P(X)\end{aligned}$$

Making use of this result, we write instead of equation 4

$$P(X) = \frac{1}{\pi} \int_0^X \text{arc cosh} \frac{T'(x)}{T'(X)} dx \qquad [1]$$

From this equation we may obtain $P(X)$, the depth to the midpoint of the trajectory corresponding to the distance X.

In the special case where

$$X = aT^2 + bT \qquad [2]$$

it is possible to perform the integration indicated in equation 1. Since

$$T'(X) = \frac{dT}{dX} = \frac{1}{2aT + b} \qquad [7]$$

equation 1 becomes

$$P(X) = \frac{1}{\pi} \int_0^X \text{arc cosh} \frac{2aT + b}{2at + b} dx$$

where t is defined by the relation

$$x = at^2 + bt$$

Thus

$$dx = (2at + b)dt$$

and

$$P(X) = \frac{1}{\pi} \int^T (2at + b) \text{ arc cosh} \frac{2aT + b}{2at + b} dt$$

By use of the substitution

$$z = \frac{2at + b}{2aT + b}$$

we may write for the above expression

$$P(X) = \frac{(2aT + b)^2}{2\pi a} \int_{\frac{b}{2aT+b}}^{1} z \operatorname{arc\,cosh} \frac{1}{z}\, dx$$

Integrating by parts, we obtain

$$P(X) = \frac{(2aT + b)^2}{2\pi a} \left[\frac{-b^2}{2(2aT + b)^2} \operatorname{arc\,cosh} \frac{2aT + b}{b} + \frac{1}{2}\sqrt{1 - \frac{b^2}{(2aT + b)^2}} \right]$$

or

$$P(X) = \frac{1}{\pi}\left[\frac{(2aT + b)b}{4a} \sqrt{\frac{(2aT + b)^2}{b^2} - 1} - \frac{b^2}{4a} \operatorname{arc\,cosh} \frac{2aT + b}{b} \right]$$

After the substitution

$$q = 2 \operatorname{arc\,cosh} \frac{2aT + b}{b}$$

is made, we have

$$P(X) = \frac{b^2}{8\pi a} (\sinh q - q). \qquad [3]$$

Interpretation of Gravitational Anomalies, I

By H. Shaw,* London, England

(New York Meeting, February, 1929)

Gravitational measurements made by means of the Eötvös torsion balance over any area enable a representation to be obtained of the total gravitational effects over the surface of that area arising from the combination of all sources of gravitational disturbance. The investigation of gravitational anomalies occurring in the upper layers of the earth's crust, in this way, has been attended with considerable success, and, especially in areas in which no undesirable disturbing factors were present, it has been possible to locate geological structures and mineral deposits with remarkable accuracy and also to delineate their confines and to indicate their depth below the surface with a certain amount of precision. Before this can be done successfully, however, it is necessary to determine the gravitational effect arising from all other extraneous anomalies in order that these may be satisfactorily eliminated. Disturbing features giving rise to these gravitational anomalies may be divided into two classes, according to whether they are situated at or below the surface. The former are determined readily by a detailed leveling of the ground surface, and may be satisfactorily eliminated provided the specific gravity of the terrain material is known, but the latter are generally unknown, and can only be ascertained with difficulty.

When, therefore, the observations obtained by means of the Eötvös torsion balance have been corrected for all surface irregularities, known as terrain and topographical effects, there remain residual gravitational values which are due entirely to subterranean conditions. The task of interpreting these results consists in disentangling the various subsurface anomalies, and subsequently in computing and recombining the effects of these various masses in order to verify the interpretation.

When the gravitational anomalies are due to large and comparatively simple structures it is a matter of little difficulty to arrive at a satisfactory interpretation of the subterranean conditions, but in more complex areas, where the geological conditions are more complicated and several irregular features exist, the process of disentangling the various constituents of the complex mass arrangement is more difficult and requires the expenditure of considerable time and labor.

When the subterranean effects have been obtained, they are subjected to a critical examination, and an effort is made to recognize and

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separate the various features which conjointly produce the resultant effects. In order to render this process possible, it is necessary that the gravitational effects resulting from each of the various constituents should be known, and their characteristics investigated. When this information is available it becomes possible to compute the effects of any combination of these constituents so as to obtain results which approximate closely to those obtained by measurement on the surface. The accuracy of the gravitational map resulting from field observations will depend on the density of the network of stations, which will control the reliability of the interpretation. Until recently it has been the general practice to assume, for convenience and simplicity, that the subterranean features under investigation approximate to infinite ones, or to certain regular and easily calculated figures, but this method is frequently erroneous and likely to lead to unreliable conclusions, as in practice most geological structures are quite definitely finite.

This difficulty has recently been overcome by the introduction of methods whereby the effect of a mass of any known form, however irregular, may be calculated, but there appears as yet to have been no systematic attempt to calculate, compare, and tabulate the effects of gravitational anomalies, so as to render this information generally available for purposes of interpretation. This investigation was undertaken with the view of studying these interpretation problems in a systematic and comprehensive manner, commencing with the simplest formations and progressing through more complicated anomalies, until eventually, it is hoped, the effects of most complex structures will be examined, tabulated, and the results correlated and made available for purposes of interpretation, so that the various irregularities may be readily recognized.

In each case it is desirable to submit the gravitational effects to a detailed and critical analysis, so as to ascertain the principal features and characteristics of the various anomalies and to investigate the properties of the resulting curves.

Deductions will be made from an examination of the actual curves of gradient and curvature values such as are obtained in practical field surveys and in some instances those of total gravity also, so that the methods adopted and the conclusions reached will be generally applicable to practical examples in which these curves are available as a result of field observations.

Analytical methods are omitted as far as possible, but in certain cases the results obtained by these methods are given, while geometrical constructions based on them are also included.

In the process of interpreting field observations it is frequently necessary to determine the effect (at stations scattered throughout the area) of definite hypothetical anomalies, and the computation of the

effects of these disturbing bodies often requires considerable time and effort. Much of this labor, it is hoped, will now be eliminated, as a reference to some of the figures given herewith will enable the necessary calculations to be made with rapidity and accuracy, while a critical study of these curves will greatly facilitate the work of interpretation.

Preliminary Assumptions

In order to render the calculations and consequent deductions universally applicable, and entirely independent of the particular scale employed, certain fundamental assumptions are made, and, except in special cases, are retained throughout.

In the first place only homogeneous deposits will be considered, so that there is but one density factor, while the subterranean disturbing body is in every case considered to be of greater density than the surrounding formations, the difference in specific gravity being unity in each case. In any particular case in practice, in which a different effective density exists, the corresponding gravity values may be obtained at once by multiplying by the appropriate density factor.

The quantities determined by the Eötvös torsion balance are the second derivatives of the potential of gravity, and, as will be seen later, these second derivatives give rise to two quantities known as the gradient and the differential curvature. The gradient indicates the increase of gravity as one moves along in the horizontal plane, and may be defined as the increase in the intensity of gravity per centimeter in the horizontal plane, while the maximum gradient, which is usually determined in practice, indicates the magnitude, and also the direction on the horizontal plane of the greatest variation of the intensity.

The quantity referred to as the differential curvature value is not equal to the actual curvatures of the level surface, but is proportional to the difference of the reciprocals of the maximum and minimum radii of curvature of that surface, the multiplying factor being g, the value of gravity. By convention this curvature value is regarded as oriented in the direction of minimum curvature, or maximum radius of curvature, and is represented on maps by a line oriented in this direction and of length given by curvature value $R = g\left(\frac{1}{r_{\text{min.}}} - \frac{1}{r_{\text{max.}}}\right)$.

Throughout the present investigation the variations of the gradient are represented by a continuous unbroken curve, while those of the curvature are shown by broken lines when occurring on the same diagram as the gradient.

The curves are in every case drawn to scale, and on the assumption of a unit density difference, the vertical scale is expressed directly in Eötvös units (E = 1×10^{-9} c.g.s. units).

Portions of the gradient curve above the axis (positive gradients) indicate that the gradient is directed towards the right, while gradients directed towards the left are represented by portions of the gradient curve below the axis. Similarly, the portions of the curvature curve lying above the axis correspond to concavities of the level surface in the upward direction or to synclinal formations, and those below the axis correspond to concavities in the downward direction or to anticlinal formations.

In addition to the quantities mentioned, the total variation of gravity, known sometimes as the gravity anomaly or the Δg curve, is sometimes given. This curve is obtained by integrating the gradient curve, therefore it is impracticable to give curves here which shall be universally applicable for the gradient and curvature.

In the case of the gradient and curvature, the factor controlling the numerical value of the gravitational effects is the ratio "Depth below Surface" to the "Thickness of Block," while the actual values of the depth and thickness are of little import. In the case of the total gravity curve, however, the actual dimensions are of the first importance, and for every different scale employed a different gravity anomaly curve will be obtained.

The shape of the curve will remain constant whatever the scale employed, and the numerical value alone will require adjustment for scale effect.

Quantities to Be Considered

In Fig. 1 let O be the center of gravity of the suspended system of the torsion balance, and select three perpendicular axes of reference OX, OY, and OZ, the last named being directed vertically downwards along the plumb line—perpendicular to the level surface—while OX and OY are tangential to this surface. If we denote by U the gravitational potential at O, the following four magnitudes can be determined by the Eötvös torsion balance:

$$\left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right), \qquad \frac{\partial^2 U}{\partial x \partial y}, \qquad \frac{\partial^2 U}{\partial x \partial z}, \qquad \frac{\partial^2 U}{\partial y \partial z},$$

which may also be written

$$U_{\Delta}, \qquad U_{xy}, \qquad U_{xz}, \qquad U_{yz}.$$

The first two give the curvature value, which is measured by the difference of the reciprocals of the principal radii of curvature $\frac{1}{\rho_1}$ and $\frac{1}{\rho_2}$ where ρ_1 and ρ_2 are the minimum and maximum radii of curvature respectively of the level surface.

$$\frac{1}{\rho_1} - \frac{1}{\rho_2} = -\frac{U_{\Delta}}{g} \text{ Sec. } 2\lambda$$

where λ is obtained from $\tan 2\lambda = -\frac{2U_{xy}}{U_\Delta}$ and gives the direction of ρ_2. The other two magnitudes $\frac{\partial^2 U}{\partial x \partial z}$ and $\frac{\partial^2 U}{\partial y \partial z}$ indicate the increase in the vertical component of gravity per unit distance along the horizontal plane in the direction of the x and y axes respectively.

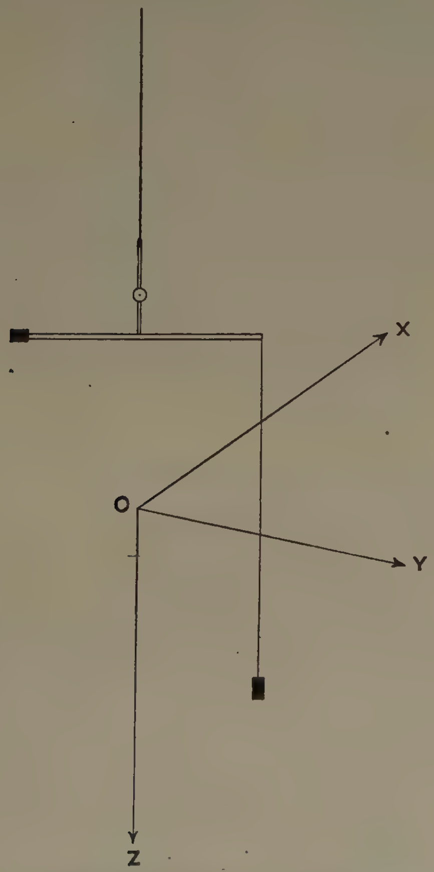

FIG. 1.—SUSPENDED SYSTEM OF TORSION BALANCE.

The maximum gradient of gravity is the resultant of these components and is equal to $\sqrt{(U_{xz})^2 + (U_{yz})^2}$ while its inclination θ to the X axis is given by $\tan \theta = \frac{U_{yz}}{U_{xz}}$.

The gravitational results obtained by instrumental observation include values due to normal earth effects and other known disturbing factors, and if corrections are made for these disturbing features, the values remaining are due to the subterranean conditions. In cases where

the conditions are known the effects may be calculated and compared with the field results but in practice the subterranean conditions are generally unknown and we are faced with the problem of elucidating the various characteristics from a consideration of the curvature values and gradients.

In certain cases the indirect method has been employed of calculating the curvature values and gradients for a number of mass distributions and comparing these with the actual field measurements. In some instances good results have been obtained by this method, but the theoretical mass distributions considered are usually of interest only in that particular instance and have not been extended so as to be available for general application.

Direct methods have also been employed by Meisser,[1] Nikiforov,[2] Koenigsberger,[3] and Jung,[4] in which details of the subterranean anomaly are calculated from the observed curvature values and gradients.

In the present paper the indirect method is mainly employed but the direct or analytical method is used also in certain cases, partly for the purpose of corroboration but chiefly to extend the usefulness of the investigation, as each method is capable of furnishing information that is not readily obtainable from the other. Graphical constructions are also given in one or two instances.

PART I.—THE INFINITE BLOCK

If we consider a heavy horizontal plate, bounded above and below by horizontal surfaces and extending to infinity in all directions, it is evident that the only gravitational effect of such a layer will be to increase the magnitude of the gravitational force throughout the area. No local irregularities will be introduced by such an infinite body, so that no local effects will be produced and there will be no gravitational "gradient" or "curvature" effects resulting from it. It is evident, therefore, that an infinite horizontal layer of this form may be added to or subtracted from a subterranean mass without producing any disturbance, so that bodies which differ only by such an infinite layer may be regarded for our purpose as equivalent to each other.

If, however, we consider the effect near the edge of a block bounded by two horizontal planes, which extends to infinity on one side of a

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[1] O. Meisser: Ermittlung der Tiefe von schwerestörenden Massen mittels Drehwage *Ztsch. f. Geophysik* (1925) **1**, 32.

[2] P. Nikiforov: Physical Principles of the Gravitational Method of Prospecting *Bull.* Inst. Practical Geophysics (Leningrad—1925) **1**, 153.

[3] J. Koenigsberger: Zur geophysikalischen gravimetrischen Landesuntersuchung und über die Tiefenlage der störenden Massen. *Ztsch. f. prak. Geol.* (1927) **35**, 65.

[4] K. Jung: Die Bestimmung von Lage und Ausdehnung einfacher Massenformen unter Verwendung von Gradient und Krümmungs grösse. *Ztsch. f. Geophysik* (1927) **3**, 257.

vertical plane containing the y axis in the three directions, $+y$, $-y$ and $+x$, we obtain the simplest possible case of a gravitational anomaly, which corresponds to a vertical step or to a simple geological fault. There is practically no limit to the possible arrangements even in such a simple case as this, and it is obviously impracticable to calculate the gravitational effects for a large number of different layers of various thicknesses and occurring at different depths, but a number of particular cases have been selected at frequent intervals throughout the range that is likely to be of practical interest, and their gravitational effects upon the gradient and curvature have been computed.

For purposes of convenience in this investigation, the thickness of the block has been maintained constant throughout, and the depth of the layer below the surface suitably varied. In this way it is hoped to cover the useful range sufficiently well to enable interpolation to be made between any two adjacent cases with a fair degree of accuracy, on any occasion when this should be considered necessary.

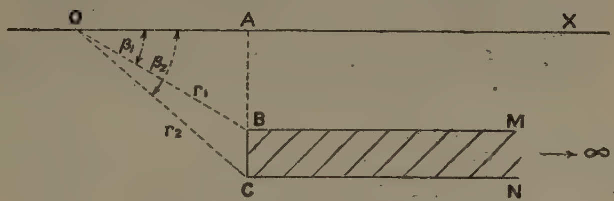


FIG. 2.—AN INFINITE BLOCK.

This vertical thickness of the block is taken as the unit of length and is used for the measurement of all distances; for example, the depth (d) of the upper surface of the block below the ground level is expressed in terms of the thickness, while distances along the surface of the ground are also measured in the same unit.

In this way the results are rendered independent of the actual dimensions of the body and are universally applicable to any system of units without the introduction of a factor for scale effect.

As we are dealing with a block which extends to infinity in three directions, we need only consider the effects encountered along a profile across and perpendicular to the edge, the block being regarded as bounded by a vertical face extending to infinity in both directions.

Referring now to Fig. 2, the block or step is regarded as bounded by the two horizontal surfaces BM and CN, and on the left by the vertical surface BC, while it extends to infinity in the direction of AX, which is along the x axis, and also to infinity along the y axis in both directions. Simple conditions are thus obtained, and the differentials containing y become zero, so that the gradient and curvature quantities are each represented by one magnitude.

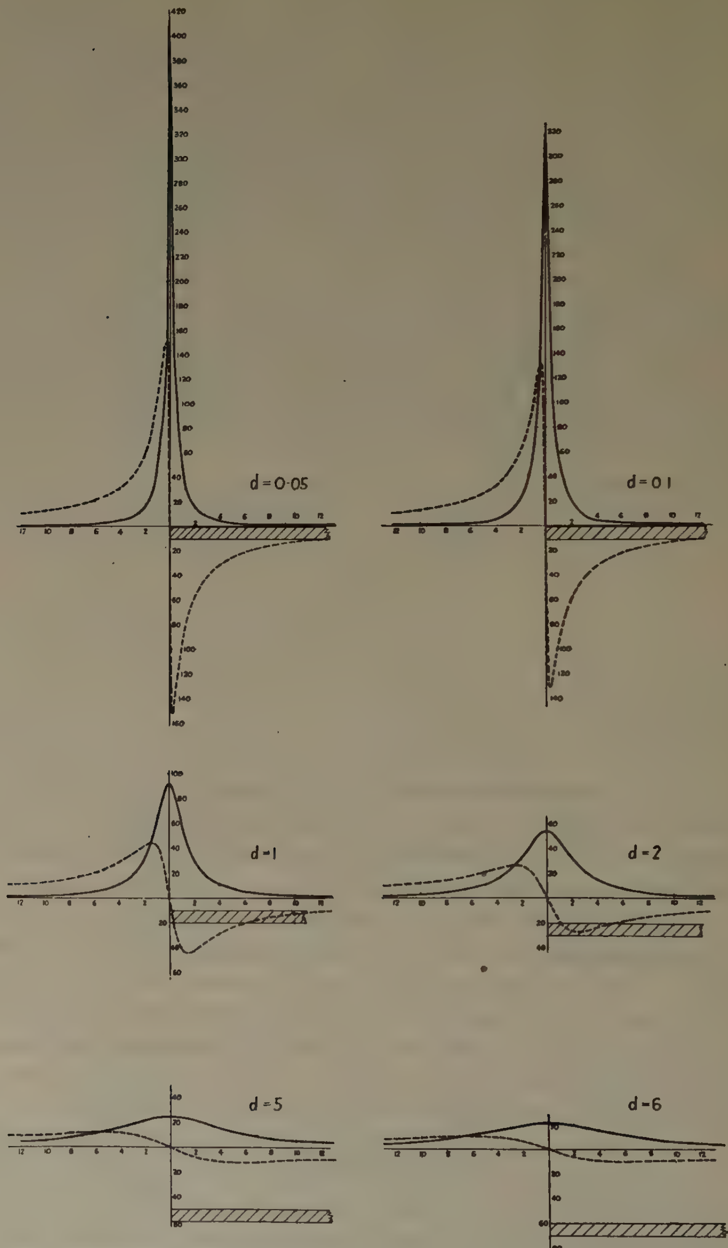
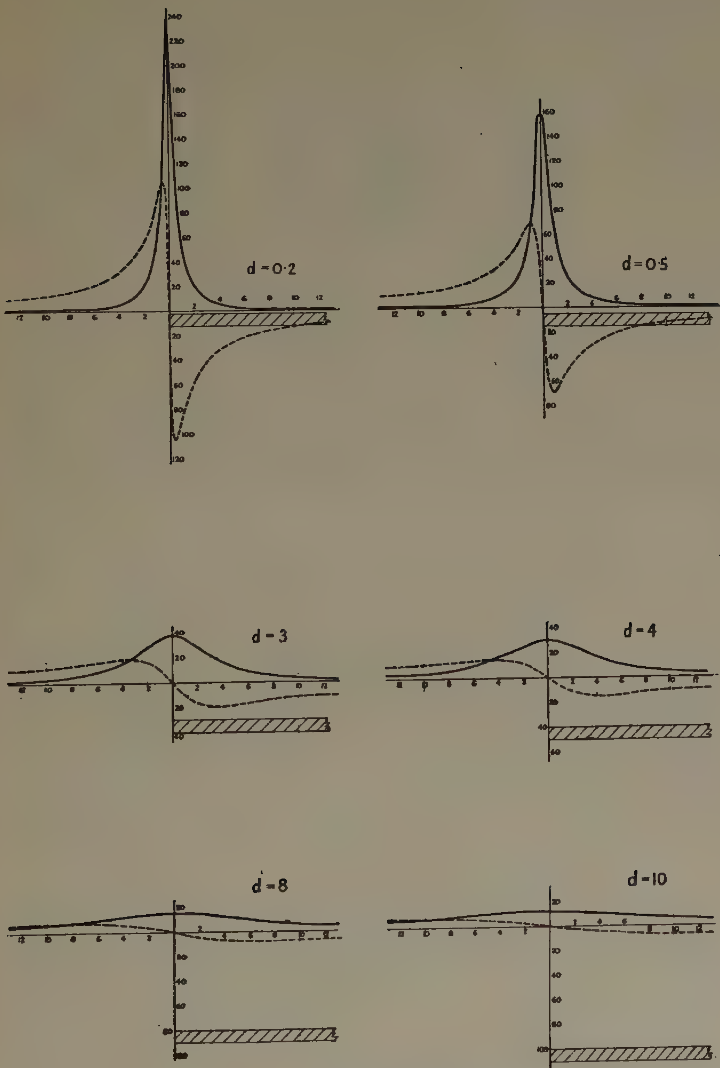


FIG. 3.—EFFECT OF INFINITE BLOCK AT VARIOUS DEPTHS FROM 0.05 TO 10.



GRADIENTS ARE SHOWS BY SOLID LINES; CURVATURES, BY DOTTED LINES

It has been shown that under these conditions the second differentials become

$$\frac{\partial^2 U}{\partial x^2} = 2\gamma\sigma(\beta_2 - \beta_1) \qquad \frac{\partial^2 U}{\partial x \partial y} = 0$$

$$\frac{\partial^2 U}{\partial y^2} = 0 \qquad \frac{\partial^2 U}{\partial x \partial z} = 2\gamma\sigma \log_e \frac{r_2}{r_1}$$

$$\frac{\partial^2 U}{\partial y \partial z} = 0$$

$$\therefore \text{Gradient} = \sqrt{\left(\frac{\partial^2 U}{\partial x \partial z}\right)^2 + \left(\frac{\partial^2 U}{\partial y \partial z}\right)^2} = \frac{\partial^2 U}{\partial x \partial z} = 2\gamma\sigma \log_e \frac{r_2}{r_1}$$

and

$$\text{Curvature} = \frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2} = -2\gamma\sigma(\beta_2 - \beta_1)$$

where the angles β_1 and β_2 are expressed in radians.

These values are calculated and plotted along a traverse across the edge of a number of blocks at varying depths below the surface throughout the range likely to be encountered in practice. The limiting values, between which this series of intermediate values has been taken, are $d = 0.05$ times the vertical thickness of the block and $d = 10$ times this thickness.

From an inspection of the series of curves shown in Fig. 3, certain interesting features are at once evident, but it should be remembered that these results, as well as the conclusions deduced therefrom, are applicable only to the particular case of an infinite layer and are not necessarily applicable in any way to bodies of finite dimensions. This fact must always be borne in mind when efforts are being made to deduce general conclusions from these observations, and although there can be little doubt that valuable deductions can be made from an examination of results due to infinite structures, such an operation should always be performed in practice with the strictest caution.

It is evident that two gravitational values may be available for the purpose of interpretation; namely, the gradient and the differential curvature, which give rise to three distinct methods of interpretation, according to whether we employ the gradient alone, the curvature alone, or the two quantities together.

Gradient Characteristics

In this method complete reliance is placed on observations of the gravitational gradient, and the reliability with which the gradient variation along a profile can be ascertained in practice is of the utmost importance, as the interpretation is based entirely upon the features and characteristics of the resulting gradient curve, without any reference to the curvature. It is necessary, therefore, that the gradient should be

determined at a considerable number of stations, sufficient to enable the gradient curve to be drawn smoothly, while it is desirable for the stations to be sited still closer together near certain critical positions; as, for example, in the vicinity of the position of maximum gradient, in order that the main characteristics of the curve may be determined with greater precision.

An important feature of this set of gradient curves is the symmetry about a vertical plane through the edge of the block, a feature which is due to the rectangular section of the block and which will disappear when the vertical face of the block is inclined.

From the curves of Fig. 3 it will be seen that:

Gradient at a point $+x$ = gradient at the point $-x$

Curvature at a point $+x$ = $-$ (curvature at the point $-x$)

An important fact is that the gradient shows no reversal of direction on such a traverse, the only gradient change being one of magnitude.

Variation of Maximum Gradient with Depth

It is obvious that for a block of constant thickness, the gravitational effects at points on the surface will be dependent to some extent on the

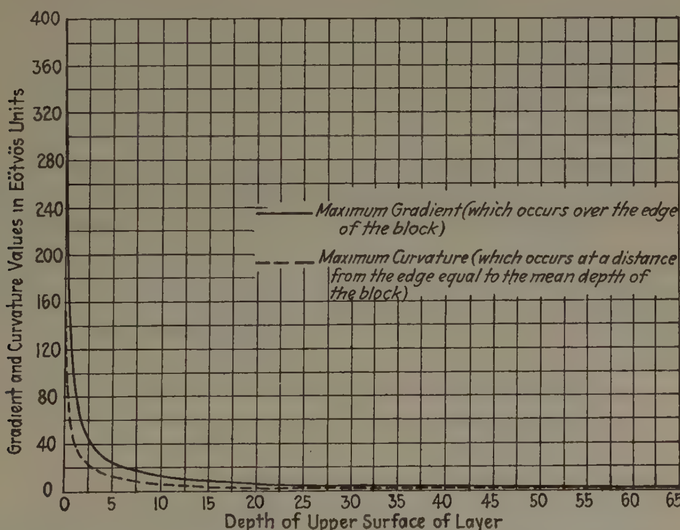


FIG. 4.—MAXIMUM GRADIENT AND MAXIMUM CURVATURE VALUES GIVEN BY INFINITE BLOCKS AT VARIOUS DEPTHS.

depth at which the block lies below the surface. For each position of the block, the gravitational gradient varies continuously over the surface and at a certain point or series of points reaches its maximum value.

If this maximum gradient value is plotted against the depth of the upper surface of the block, we obtain the upper curve of Fig. 4, which is asymptotic to both the vertical and horizontal axes.

The rapid falling off of the maximum gradient with increasing depth is at once apparent, especially for depths up to about four or five times the thickness, after which the maximum gradient decreases less rapidly.

If one assumes that under practical field conditions the minimum value of the gradient that can be detected with certainty is 5 units (5E), it follows from Fig. 4 that the limiting depth to which an infinite block of this type can be located is approximately 25 times its vertical thickness, while if one assumes a minimum gradient value of 10E, the corresponding limiting depth of the block will be reduced to 14 times the thickness.

Accepting this assumption, it follows that, with an effective density difference of unity and under favorable conditions, it is impossible by means of the Eötvös torsion balance to locate an infinite block step, or fault, of the type shown in Fig. 2, if the depth of the layer below the surface exceeds 25 times its vertical thickness. In the event of the effective density differing from unity, this figure will need to be amended to a corresponding extent.

Position of Maximum Gradient

It is easily shown that the gravitational gradient, which, as we have already seen, is given by the equation

$$\frac{\partial^2 U}{\partial x \partial z} = 2\gamma\sigma \log_e \frac{r_2}{r_1} = 2\gamma \log_e \frac{r_2}{r_1} \text{ [when } \sigma = 1.0]$$

reaches its maximum value when $x = 0$, which corresponds to a position vertically over the edge of the block. This fact is clearly demonstrated by the curves of Fig. 3. This figure shows that for an infinite block with a vertical face the value of the gradient curve is symmetrical about a vertical line through the vertical edge or face of the block, falling off equally on both sides. The numerical value of the maximum gradient is given by

$$\frac{\partial^2 U}{\partial x \partial z} = 2\gamma\sigma \log_e \frac{d+1}{d} = 2\gamma\sigma \log_e \left(1 + \frac{1}{d}\right)$$

The depth of the block below the surface is evidently the controlling factor, the magnitude of the maximum gradient value decreasing greatly as the depth of the block is increased. For a shallow layer the gradient curve shows large and rapid fluctuations in value in the vicinity of the point of maximum gradient, but for deeper deposits, not only are the fluctuations considerably smaller in value but they also change less rapidly, and extend laterally to much greater distances. In the case of a shallow block of this type, therefore, a very large gradient is obtained in the immediate vicinity of the vertical edge of the block, but falls off so rapidly that it is noticeable only within a very short distance; on the

other hand, the effect of a deep deposit is to give rise to small and gradually changing gradients, which are evident even at a considerable distance from the edge. One deduces therefore that any gravitational anomaly that persists over an extensive area of the ground surface is due to a body at considerable depth or of great vertical thickness, while one that is limited to a relatively small area of the surface is necessarily caused by a body at a correspondingly shallow depth.

The general shape of the curve obtained in any particular case in practice may be compared with Fig. 3, from which the ratio of depth to thickness may be ascertained to a fairly close approximation, when a direct comparison of the numerical values will indicate the effective density of the block.

Variation of Gradient with Distance from Face of Block

Fig. 5 indicates the distance from the edge or face of the block at which the gradient has fallen to certain definite proportions of its maximum value, and it is of interest to note that in each case the distance from the edge bears a definite relation to the depth of the block.

One curve in particular, that of one-half maximum gradient, is worth consideration, for it presents a very simple means of ascertaining the depth of such a layer. If this curve is examined, it will be found that the ordinate of any point of it is approximately 0.5 units greater than its abscissa, or the distance from the edge is approximately 0.5 greater than the depth of the upper surface of the block. As the vertical thickness of the block is regarded as unity, it is evident that at any point where the gradient has fallen to one-half of its maximum value, the distance of this point from the edge of the block is approximately equal to the mean depth of the block. If two positions at which the gradient equals one-half of its maximum value have been located, one on either side of the maximum gradient position, not only will the edge of the deposit be known, but its mean depth also.

The position at which the gradient is equal to one-half its maximum value is given by

$$2\gamma\sigma \log_e \frac{r_2}{r_1} = \tfrac{1}{2} \cdot 2\gamma\sigma \cdot \log_e \cdot \frac{d+1}{d}$$

or

$$2 \log_e \cdot \frac{r_2}{r_1} = \log_e \cdot \frac{d+1}{d}$$

$$\therefore \left(\frac{r_2}{r_1}\right)^2 = \frac{d+1}{d}$$

which gives

$$x = \pm\sqrt{d(d+1)}$$

where x is the horizontal distance from the edge of the block.

It is evident, therefore, that the position of one-half maximum gradient is not exactly in agreement with the foregoing approximation, but in

cases in which d has a large value, the discrepancy is negligible for all practical purposes.

In the case of a block lying near to the surface, the approximation does not hold closely, but when the depth of the block is not less than its thickness—*i. e.*, $d \nless 1$—this method of estimating the depth is accurate to within 5 per cent., the accuracy increasing with the value of d, while for values of d which are less than unity, it would seem desirable to abandon the approximation and to employ the correct formula.

Fig. 6 may be used to correct any discrepancy that may result from the use of the approximate formula in cases where the block is located at a shallow depth. The curve is seen to bend upwards with decreasing depth, so that the depth as determined by the approximation method is less than the true depth, the error increasing progressively as the depth

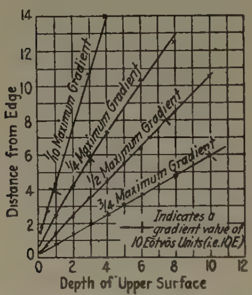


FIG. 5.—INFINITE BLOCK. DISTANCE FROM EDGE AT WHICH GRADIENT ASSUMES DEFINITE FRACTIONS OF ITS MAXIMUM VALUES.

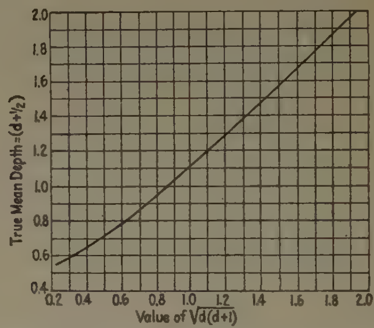


FIG. 6.—INFINITE BLOCK. CURVE CONNECTING TRUE MEAN DEPTH WITH MEAN DEPTH AS DETERMINED FROM POINT OF ½ MAXIMUM GRADIENT, FOR SMALL DEPTHS.

decreases, so that although the approximate formula may be regarded as sufficiently accurate for all practical purposes for any depth not less than the thickness of the block, yet if this depth is decreased to 0.1 times the thickness of the block the estimated depth of the block will be only 55 per cent. of the true mean depth.

Another relation, which may sometimes serve as an auxiliary verification, although not so useful as the one previously mentioned, arises from a comparison between the two curves representing one-half maximum gradient and three-quarters maximum gradient respectively. For any particular abscissa the ordinate difference between these two curves is approximately one-half of the corresponding depth of the block. The accuracy obtainable in this case is not so good as in the previous one, but the method may sometimes be of use in providing a rough check in practical cases where it is possible to take observations on one side only of the position of maximum gradient.

Positions at Which Gradient Attains Definite Values

Fig. 7 indicates the distance from the edge of the block at which the gradient falls to certain definite numerical values, and the variation of this distance with the depth of the block. This serves in practice to show the distance from the edge at which any given block will exert a definite effect. When the density of the block is known, the value approximating most nearly to the one obtained in practice can be chosen or interpolated from these curves so as to give the required sensitivity.

It should be noticed that, in all the cases considered, the curve rises as the abscissa increases, which indicates that as the depth of the block increases, the distance from the edge at which a gradient effect of a definite numerical value occurs also increases. This is more striking as the maximum gradient obtainable in such cases decreases with increase

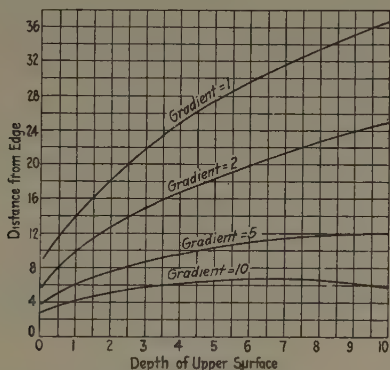


FIG. 7.—INFINITE BLOCK. DISTANCE FROM EDGE AT WHICH GRADIENT BECOMES EQUAL TO DEFINITE VALUES.

of depth, and so we find that as the depth of the block below the surface increases, the maximum value of the resulting gradient decreases, but that the distance from the edge at which the gradient value reaches certain fixed numerical values increases.

This is in agreement with the deduction previously made, "that the effect of a deep deposit is to give rise to small and gradually changing gradients, which are evident even at a considerable distance from the edge."

Gradient Value at Definite Distance from Edge

It is frequently desirable to know the variation of the gradient with depth, for definite points at a fixed distance from the edge of the block, as this is of value for the purpose of interpretation.

In Fig. 8 is given a series of curves, each one of which indicates the gradient value at a definite distance from the edge, this distance (in terms of the block thickness), being given on each line.

Apart from the 0 curve, which gives the gradient value vertically over the edge and is the same curve as was considered in Fig. 4, all the curves are seen to commence with an increasing gradient, which eventually reaches a maximum value, and afterwards diminishes. This maximum value occurs in each case at a depth approximately 0.5 less than the distance from the edge to which the curve corresponds; or in other words, at the point of maximum gradient, the distance from the edge, and the depth of the mid-point of the block are approximately equal, as is to be expected from previous considerations.

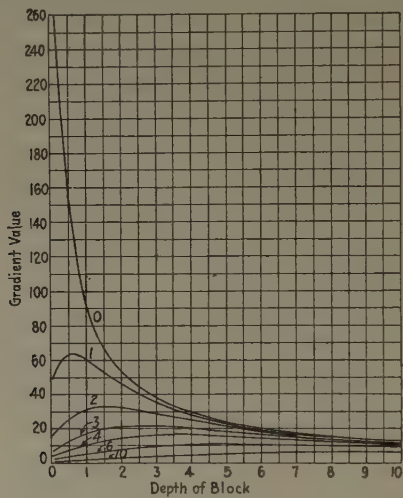


FIG. 8.—INFINITE BLOCK. VARIATION OF GRADIENT WITH DEPTH OF BLOCK, AT FIXED POINT.

Horizontal distance of point from edge of block is indicated on each curve.

Gradient Interpretation

The foregoing examination of the gravitational gradient arising from an infinite block which terminates in a vertical face reveals certain characteristics which make possible the detection and recognition of such a gravitational anomaly and also provide a valuable means of computing its actual position, depth and thickness. In some cases, it may also be possible to estimate the density difference between the two materials concerned.

From a consideration of the gradient alone, therefore, an interpretation of the anomaly may be made from the following considerations:

1. The symmetry of the gradient curve indicates the verticality of the step.

2. The maximum gradient occurs directly over the vertical edge, step, or fault, which makes possible the determination of its position with a considerable degree of accuracy.

3. The mean depth of the block ($d + \frac{1}{2}$) is equal approximately to the horizontal distance along the ground between the point of maximum gradient and that of one-half maximum gradient, except in the case of very shallow blocks.

4. The horizontal distance along the surface, between the two points at which the gradient value is approximately equal to one-half maximum gradient, is equal to twice the mean depth of the block, the point midway between these two positions being vertically over the edge of the block.

5. The horizontal distance along the surface between points of one-half maximum gradient and three-fourths maximum gradient is equal approximately to one-half the depth of the block below the surface.

6. A comparison of the shape of the gradient curve with the curves of Fig. 3 enables the ratio of depth to thickness to be ascertained and this, together with a knowledge of the mean depth, gives an indication of both the depth and thickness.

7. The effective density of the block or step may be ascertained by a direct comparison of the numerical values with those of the corresponding example of Fig. 3.

It will be seen, therefore, that under these conditions the disturbing body may be determined completely from a consideration of the gradient results alone, without any reference whatever to the curvature values. It would appear, however, in this connection, that the reliability of the interpretation is dependent on the accuracy with which the maximum gradient value may be evaluated and its position located. The gradient value varies rapidly in the neighborhood of this point and the correctness of the determination at this point is a factor controlling the correctness of the interpretation. In these circumstances, it is often desirable to site the balance stations very close together in the vicinity of the point of maximum gradient, in order to obtain more detailed information of the gravity variations at this point and also to locate the position of maximum gradient more closely.

Another point of great importance is the position at which the gradient attains one-half of its maximum value. Here also the gradient value is in general changing fairly rapidly, and the closeness with which the position of one-half maximum gradient is located limits to a great extent the accuracy of the interpretation.

Characteristics of Differential Curvature

Analysis of Differential Curvature Effects

Referring again to Fig. 3, it will be seen that in every case the curvature quantity changes sign when a traverse is taken across the edge of an infinite rectangular block, at which point it assumes a value equal to zero. In the case of the gradient profiles, the resulting curve is in every

case symmetrical about a vertical line through the edge of the block but in the case of the differential curvature profiles, which are indicated by dotted curves in Fig. 3, the symmetry is about the point of intersection of this line and the ground line, and as one proceeds from this point in either direction, the curvature rapidly attains a maximum numerical value, after which it falls away again somewhat more gradually. The steepness of the central portion of the curve decreases rapidly as the depth of the block increases.

The vertical through the edge of the block divides the differential curvature profile into two parts which are exactly equal and opposite to each other. The left-hand portion, which is beyond the edge of the block, is entirely above the horizontal axis, which indicates that beyond the edge of an infinite block of this type the maximum curvature vector is parallel to the traverse line.

The other portion of the curve, which is actually obtained on the surface above the block, is entirely below the horizontal axis, from which we see that under these conditions the curvature vector over the block is parallel to the edge of the block, or perpendicular to the line of traverse.

Variation of Maximum Curvature with Depth

The maximum numerical value also of the curvature varies greatly with the depth of the block, and this variation is shown by the lower curve of Fig. 4. The maximum value of the curvature is seen to diminish rapidly with an increase of depth up to about twice the thickness of the block, beyond which point the maximum curvature decreases at a much slower rate, and is asymptotic to the "depth" axis.

If it is assumed that under the practical conditions of a field survey, the minimum differential curvature value that can be detected with certainty is 5 units (5E), it is evident that the limiting depth at which an infinite block of this type may be located is approximately 12.5 times its vertical thickness, while if the minimum curvature value required for purposes of location is 10 units, the limiting depth of the block is reduced to 7 times the thickness.

It is evident, therefore, that an infinite block or step of this type can be located at a greater depth by means of the gradient than is possible by the curvature value, the gradient being capable of operating to a depth of from 2 to 2.5 times that which can be usefully employed by the curvature.

Position of Maximum Curvature

Fig. 9 shows the relation of the horizontal location of the maximum curvature position with the depth of the block. From this curve it is apparent that with an increasing depth of block the position of maximum curvature recedes from the vicinity of the edge of the block and that this

variation depends directly on the depth of the block and the horizontal displacement of the point from the edge of the block is approximately equal to the mean depth of the block, or the depth below the surface of the midpoint of the vertical face of the block. The curve is a straight line, except at the lower end, corresponding to very shallow depths when the curve becomes concave to the "depth" axis, and indicates that the relation described does not hold exactly under conditions of shallow depths.

Under normal conditions, two positions of maximum curvature are obtained, which are equal numerically but differ in sign; in such a case the horizontal surface distance between the two maximum points is equal to twice the depth of the block below the surface, while the point

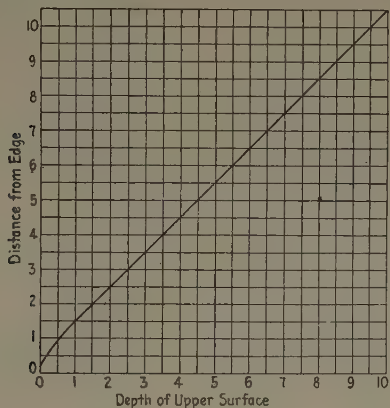


FIG. 9.—INFINITE BLOCK. DISTANCE OF POINT OF MAXIMUM CURVATURE VALUE FROM EDGE, WITH VARYING DEPTH OF BLOCK.

midway between the two positions of maximum curvature lies vertically over the edge of the block.

It is well known that the positions of maximum curvature are given by the points

$$x = \pm \sqrt{d(d + 1)}$$

and it is obvious from this that the positions of maximum and minimum differential curvature values coincide with the positions of one-half maximum gradient, and that the limitations previously deduced for employing the latter in estimating the depth of the block still also hold good when making use of the position of maximum and minimum curvature values.

It can also be shown that at the points of maximum curvature

$$(\beta_1 + \beta_2) = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}$$

while the point of zero curvature value, which coincides with the maximum gradient position is given by $(\beta_1 + \beta_2) = \pi$; relations which are of use in formulating a geometrical solution of the problem of interpretation.

Variation of Differential Curvature with Distance from Face of Block

Fig. 10 has been drawn to indicate how points, at which the differential curvature value attains one-half of its maximum value, move away from the face of the block with varying depth. As the curvature value increases to a maximum and then falls off again on each side of the face, it is obvious that there will be two points on each side of the face, at which the curvature attains one-half of its maximum value, one nearer to the

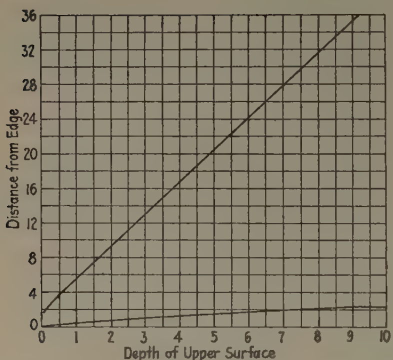


Fig. 10.—Infinite block. Distance from edge at which curvature reaches ½ maximum value.

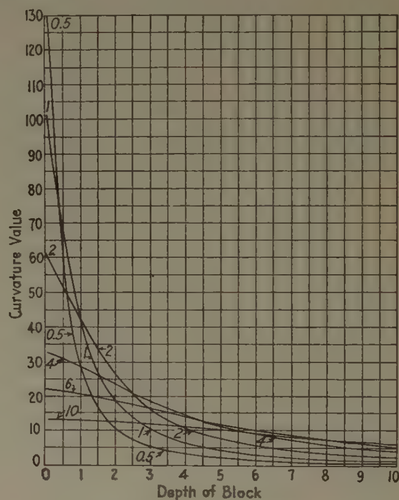


Fig. 11.—Infinite block. Variation of curvature with depth of block, at fixed point.

Horizontal distance of point from edge of block is indicated on each curve.

face than the position of maximum curvature and the other beyond it. The behavior of these points with varying depths of block give rises to the two curves shown in Fig. 10.

Curvature Value at Definite Distance from Edge

Fig. 11 shows the variation of curvature with the depth of the block at a fixed distance from the face of the block. Each curve indicates the curvature value at a definite distance from the face, the distance (in terms of the block thickness) being shown on each line.

In every case, the curvature value decreases as the depth of the block increases. The rate of decrease is very rapid for shallow depths of block and flattens out as the distance from the edge increases, but as the

block gets deeper the rate of decrease rapidly falls off until, for a depth of 10 times the thickness, it is a very gradual slope indeed.

It is at once apparent, from Fig. 11, that, as in the case of the gradient, for blocks at small depth the largest curvature effects are obtained quite near to the edge of the block, but when the blocks are deep-seated, the maximum effects are obtained at a greater distance from the edge, this distance increasing with the depth of the block.

Inclination of Curvature Curve over Edge

Referring again to Fig. 3, it is seen that the dotted line representing the differential curvature passes through the origin in every case, and that the inclination of this curve to the horizontal decreases continuously as the depth increases.

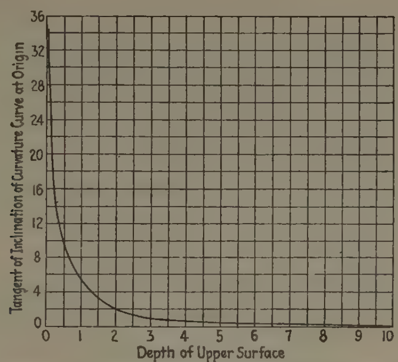


FIG. 12.—INFINITE BLOCK. CURVE CONNECTING TANGENT OF INCLINATION OF CURVATURE CURVE AT ORIGIN, AND DEPTH OF BLOCK BELOW SURFACE.

In Fig. 12, the tangent of the inclination of this curve is plotted against the depth of the block, and from the resulting curve it would appear possible to make use of this relation for purposes of interpretation.

For shallow depths (up to about 2 or 3 units) this method is likely to prove quite satisfactory, but for greater depths the curve has become flattened out to such an extent that any estimate of the depth made from this portion of the curve is likely to prove unreliable.

In employing this curve for the estimation of the depth of any infinite block, care will be necessary to employ suitable scales for the representation of gradient, and for distances, in order that the tangent of the angle of inclination of the curve may be correctly interpreted.

Differential Curvature Interpretation

From a consideration only of the differential curvature quantities resulting from an infinite horizontal layer of this type, it is possible to give a complete interpretation of the block, whether or not any informa-

tion as to the gradient is available. The following features are useful in this connection:

1. The symmetry of the curvature curve indicates the verticality of the step.

2. The differential curvature value is zero at a point vertically over the edge of the block.

3. The midpoint between the two points (numerically equal) of maximum differential curvature lies vertically over the edge of the block.

4. The mean depth is given approximately by the distance from the point of zero curvature value to the position of maximum curvature, or one-half of the distance between the two points of maximum curvature.

5. A comparison of the curvature values with the curves of Fig. 3 enables the ratio of depth to thickness to be ascertained, and this, together with a knowledge of the mean depth, gives an indication of both depth and thickness.

6. The effective density of the block or step may be ascertained from a direct comparison of the numerical values with those of the corresponding example of Fig. 3.

7. An estimate of the depth (for purposes of verification) may be obtained from the slope of the curvature curve in the vicinity of the origin. The reliability of this estimate is good up to a depth only of two or three times the thickness of the block.

It appears to be possible, therefore, to locate the vertical face of an infinite block with reasonable accuracy, and also to determine the depth, thickness and density of such a block, if the curve indicating the curvature quantity along a profile perpendicular to the edge of the block is known with a sufficient degree of precision. The reliability of the interpretation will depend, of course, on the accuracy of the curvature curve, and this will be controlled by the closeness of the station network and the correctness of the individual determinations.

The point of zero curvature and the two points of maximum curvature are of the greatest importance, and it is desirable that the position of these three points should be determined as closely as circumstances permit; the variation of the curvature quantity in the vicinity of these points is very rapid, and great care is necessary if a reliable interpretation is to be obtained.

Comparison of Gradient and Curvature Effects

Ratio of Maximum Gradient to Maximum Curvature

A reference to Fig. 3 shows at once that the value indicated by the dotted curve that represents the maximum curvature effect due to any particular block is always considerably less than the maximum gradient value produced by the same block, while a closer inspection shows that

practically throughout the whole range covered by Fig. 3 the numerical value of the maximum curvature value approximates very closely to one-half of the maximum value of the gradient. This relation may be considered to hold exactly except in the case of particularly small depths of block, when there is a slight deviation from this simple ratio; for bodies lying quite near to the surface this relation does not hold very closely, although in the case of a block at a depth of not less than 0.5 times its thickness, such an estimate would appear to be reasonably accurate.

As the depth decreases, however, the maximum gradient approaches infinity in value, while the limiting maximum value of the differential curvature is equal to

$$2\gamma(\beta_2 - \beta_1) = 2\gamma \cdot \frac{\pi}{2} = \pi\gamma$$
$$= 210E \text{ (approx.)}$$

Intersection of Gradient and Curvature Curves

If the two curves representing the gradient and curvature effects for any block are superimposed, as in Fig. 3, they intersect only once for each arrangement of block. The gradient profile is symmetrical about the vertical line containing the face of the block, while the differential curvature is symmetrical about the point of intersection of this vertical line and the ground surface, so that in the latter case one-half of the curve lies below the horizontal axis.

An examination of these curves reveals the fact that the curvature and gradient curves intersect in all cases at a point beyond the edge of the block, and not above the block.

At all points between the edge of the block and the point of intersection of the curves, the gradient has a larger value than the curvature, while beyond the point of intersection the gradient value falls away more quickly than the curvature, so that for all points beyond this point the curvature value is of greater importance than the gradient.

The effect of this is to render the Eötvös torsion balance more sensitive to curvature effects for distant irregularities than it is to gradients. The differential curvature, therefore, is more suitable than the gradient for the detection of distant features, but at the same time it should be remembered that extraneous anomalies at a distance are more likely to introduce disturbing effects, so that the topographical correction must be extended considerably. It is also evident from these curves that the point of intersection coincides in every case with the position of maximum curvature, and as the maximum curvature is equal to one-half the maximum gradient, it is obvious that at this point of intersection of the curvature and gradient the value of the gradient will be equal approximately to one-half the maximum gradient value, while the curvature will be at its maximum value.

An inspection of the curves of Fig. 3 and Fig. 13 shows that this point of intersection occurs at a distance from the edge, which is approximately equal to the mean depth of the block, so that this provides a ready means of estimating the depth of such a block.

We have, therefore, a point of unique importance which possesses the following distinct and characteristic features.

1. It is the point at which the gradient reaches one-half of its maximum value.

2. It is the point of maximum curvature.

3. It is the point of intersection of gradient and curvature curves, or the point at which the gradient and curvature are numerically equal.

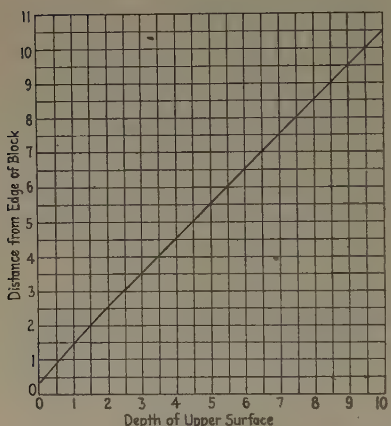


FIG. 13.—INFINITE BLOCK. DISTANCE FROM EDGE OF POINT OF INTERSECTION OF GRADIENT AND CURVATURE CURVES, WITH VARYING DEPTH OF BLOCK.

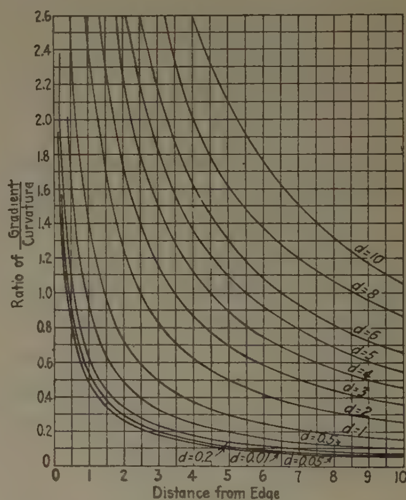


FIG. 14.—INFINITE BLOCK. RELATION BETWEEN RATIO GRADIENT: CURVATURE AT ANY POINT, AND DISTANCE FROM EDGE, FOR DIFFERENT DEPTHS OF BLOCK.

4. The distance of this point from the edge is equal approximately to the mean depth of the block.

It is apparent also that at all positions between this point and the edge the gradient will be greater than the curvature but that at distances more remote the curvature predominates, this relative predominance increasing with the distance from the edge.

Relation between Gradient and Curvature Values

The ratio between the gradient and curvature values at any point is also of considerable importance for purposes of interpretation, and if the values of this ratio for a series of stations along a traverse are plotted, valuable indications may be obtained of the position and depth of any block. If the ratio is plotted of gradient to curvature at each point on a

profile across the edge of an infinite block, a curve is obtained of which one portion approximates to one or another of the curves of Fig. 14. When this comparison has been made, and the approximate curve determined, the depth of the block is at once known and it is a matter of little difficulty to locate the edge of the block.

Further assistance in the work of interpretation may be obtained by reference to Fig. 15, in which the variation of the gradient to curvature ratio with the depth is shown for a fixed point, at a definite distance from the edge of the block. Each curve corresponds to the variations at a definite point and the distance of this point from the edge of the block is indicated on the corresponding curve.

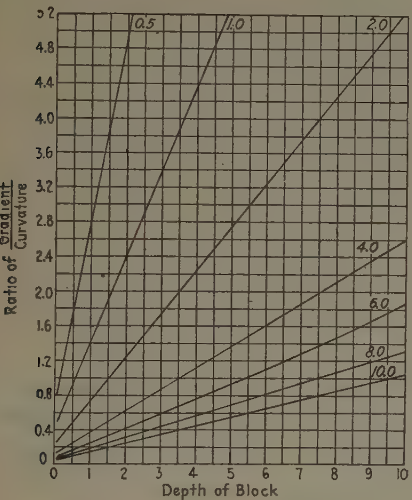


FIG. 15.—INFINITE BLOCK: VARIATION WITH DEPTH OF RATIO GRADIENT : CURVATURE AT FIXED POINT.

Horizontal distance of point from edge of block is indicated on each curve.

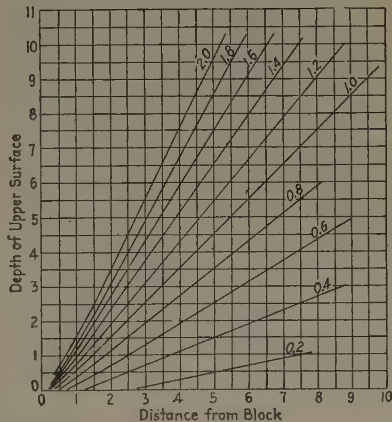


FIG. 16.—INFINITE BLOCK. RELATION BETWEEN DEPTH OF BLOCK, AND DISTANCE FROM EDGE, FOR POINTS HAVING SAME VALUE OF GRADIENT : CURVATURE.

The equations of the various curves are given in Table 1.

TABLE 1.—*Equations of Curves of Gradient to Curvature Ratio* (*Fig.* 15)

Distance from Edge	Equation of Curve, Gradient: Curvature	Column 1 × Column 2
0.5	$2.1d + 0.7$	$1.05d + 0.35$
1.0	$d + 0.4$	$d + 0.4$
2.0	$0.5d + 0.2$	$d + 0.4$
4.0	$0.25d + 0.1$	$d + 0.4$
6.0	$0.17d + 0.1$	$1.02d + 0.6$
8.0	$0.125d + 0.05$	$d + 0.4$
10.0	$0.1d + 0.05$	$d + 0.5$

We see from Column 3 that the product of Columns 1 and 2 is practically constant for a constant value of d, so that the gradient: curvature ratio and the distance from the edge are constant in value approximately and equal to the mean depth of the block below the surface.

From these figures it is apparent that the gradient:curvature ratio increases as the edge of the block is approached, over which point theoretically it becomes infinite. When a point has been obtained at which the gradient value is numerically greater than the curvature, it is known that the distance from the edge is not greater than the mean depth of the block. An effort should be made to obtain the maximum possible value of the gradient:curvature ratio, as this is an indication of the closeness with which the edge has been located.

In the process of interpretation it is frequently desirable to ascertain the gravitational effects of a known anomaly under specified conditions, in order to reconstruct a subterranean disturbing body which shall produce a gravitational anomaly agreeing as closely as possible with the observed values.

In Fig. 16, information concerning the ratio of gradient to curvature is given in a convenient form, enabling this ratio to be obtained at once under any conditions; or conversely, if the ratio is known, to indicate the distance from the edge of the block.

With the aid of these curves it is possible to read off at once the gradient: curvature ratio at any point, and to reconstruct rapidly a gradient: curvature curve for any supposed subterranean irregularity of this nature.

Geometrical Construction

The following geometrical construction may be employed for solving the problem of interpretation, when the positions of maximum gradient and of maximum curvatures are known.

In Fig. 17, let $BT'TC$ represent the subterranean block and let D be the image of B in the ground plane XX'. Join CD and let it meet XX' at A.

On CD as diameter describe a circle to cut XX' in P and R.

Then A is the point of maximum gradient and P and R are the points of maximum curvature. For the point A

$$\angle XAB + \angle XAC = \angle XAD + \angle XAC = \pi$$

which are the conditions of the point of maximum gradient.

Also $\quad \angle XPB + \angle XPC = \angle DPX + \angle XPC = \frac{\pi}{2}$ for the point P

And $\angle XRB + \angle XRC = \angle DRX + \angle XRC = 2\pi - \angle DRC = \frac{3\pi}{2}$

for the point R

which are the conditions *for the two points of maximum curvature.*

In practice, therefore, if P and R are the points of first and second maximum curvature obtained during a traverse; and A is the point of maximum gradient,

and if $$PA = a = AR$$

we know that $$a^2 = d(d + 1)$$

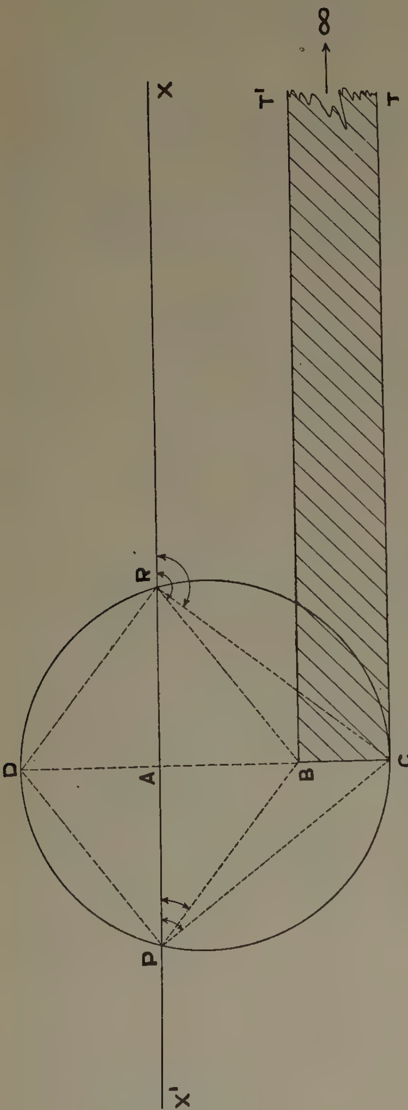


FIG. 17.—GEOMETRICAL CONSTRUCTION FOR USE IN INTERPRETATION.

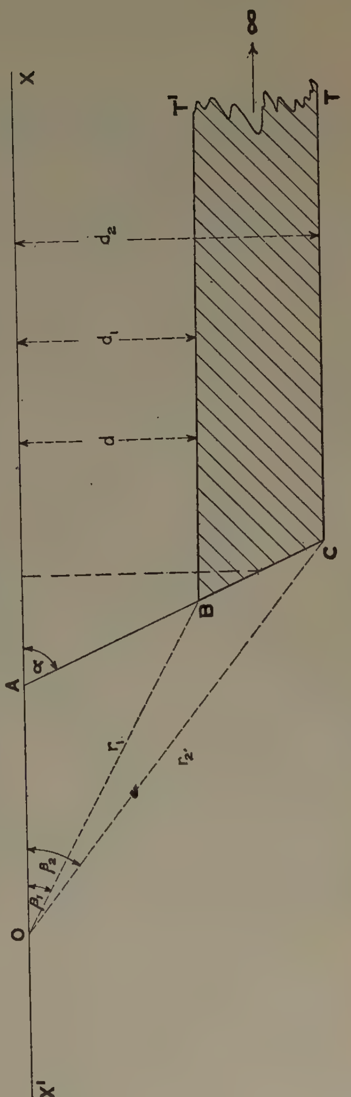


FIG. 18.—INFINITE HORIZONTAL BLOCK WITH INCLINED FACE.

The value of d can be determined from Fig. 14, so that the depths AB and AC are obtained.

A direct comparison of the numerical value of the maximum gradient with the upper curve of Fig. 4 enables the effective density of the block to be ascertained, so that the gravitational anomaly is completely determined.

Combined Gradient and Curvature Interpretation

It is evident that if (as has been stated) it is possible to furnish a complete interpretation of the gravitational anomaly from a study of the gravitational gradient alone, and if it is also possible to furnish a complete interpretation from a consideration of the curvature effects only, it becomes a comparatively easy matter to interpret the anomaly when both the gradient and curvature effects are known.

It should be pointed out, however, that theoretically, under favorable circumstances, neither the gradient nor the differential curvature is capable of furnishing any information concerning the infinite rectangular block that is not readily determined from the other, so that, so long as one of the curves is known completely, a full and detailed interpretation can be made. In such a case, therefore, the only advantage to be derived from a determination of both gradient and curvature values is to provide a check on the interpretation.

Theoretically, therefore, the gradient and curvature are of equal value in facilitating a full and complete interpretation of the problem presented by the infinite rectangular horizontal step or block. In practice, however, conditions are nearly always more complex than those hitherto considered, and the disturbing effects of irregularities are usually more troublesome to some application of the curvature than to the gradient. In consequence, the curvature is of less practical value than the gradient, and, except in the most suitable areas, serves only as an auxiliary check on the results which have been deduced from gradient observations.

In view of these considerations, greater reliance is to be placed upon the gradient value, and any deductions resulting from a comparison of both the gradient and curvature magnitudes should be considered with due respect to this fact.

PART II.—INFINITE HORIZONTAL BLOCK WITH INCLINED FACE

If the edge BC of the block is inclined to the vertical, as in Fig. 18, we get the less simple case of an inclined step or fault, which is more likely to occur in practice than the simple case already considered. A detailed examination of the characteristics of the resulting gravitational effects is justified, therefore, in order to ascertain the main features by

which such occurrences may be readily distinguished from the previous case.

The calculation of these effects for a number of inclinations of the edge or face of the block, and at various depths, is a laborious matter, so that the examination has been restricted to infinite blocks of this type occurring at six different depths only, but in each case a complete set of calculations has been made at intervals of 10° of inclination from 0° to 90°, those between 90° and 180° being readily obtainable from these, as follows:

$$\begin{pmatrix}\text{Gradient at a point } +x \text{ for} \\ \text{an inclination of } \alpha \text{ of the face} \\ \text{of the block}\end{pmatrix} = \begin{pmatrix}\text{Gradient at the point } -x \text{ for} \\ \text{an inclination of } \pi - \alpha \text{ of the} \\ \text{face of the block}\end{pmatrix}$$

$$\begin{pmatrix}\text{Curvature at a point } +x \text{ for} \\ \text{an inclination } \alpha \text{ of the face of} \\ \text{block}\end{pmatrix} = -\begin{pmatrix}\text{Curvature at the point } -x \\ \text{for an inclination of } \pi - \alpha \\ \text{of the face of the block}\end{pmatrix}$$

The results of these calculations are shown in Figs. 19 to 24.

The six depths for which calculations have been made are: $d = 0$; 0.05; 0.1; 0.2; 0.5; 1.0. The first one or two of these may be regarded as typical representations of very shallow deposits, but even the deepest example, in which $d = 1.0$, can hardly be considered to represent a deposit at anything approaching the limiting depths. It is not suggested that the range of depth covered here represents in any way the limit of penetration of this method of investigation, but it is considered that at depths much greater than this, the differences between the effect of an infinite block of the type now under consideration and one having a vertical edge will be so small as to be hardly appreciable in practice.

In this case, as in the former, where both gradients and differential curvature values are shown in the same diagram, the gradients are represented by continuous curves while the curvatures are shown dotted.

An inspection of the curves of gradient and differential curvature at once reveals a number of prominent characteristics which will now be examined in detail, as in the previous case. In the first place, we will confine our attention strictly to the gradient features, after which we will consider the differential curvature values, and subsequently the combined effect of both gradient and curvature.

Gradient Characteristics

The gradient at any point due to an infinite block of this type is given by the expression

$$\frac{\partial^2 U}{\partial x \partial z} \doteq \left\{ (1 - \cos \alpha) \log \frac{r_2}{r_1} - \sin 2\alpha(\beta_2 - \beta_1) \right\} \times \gamma.$$

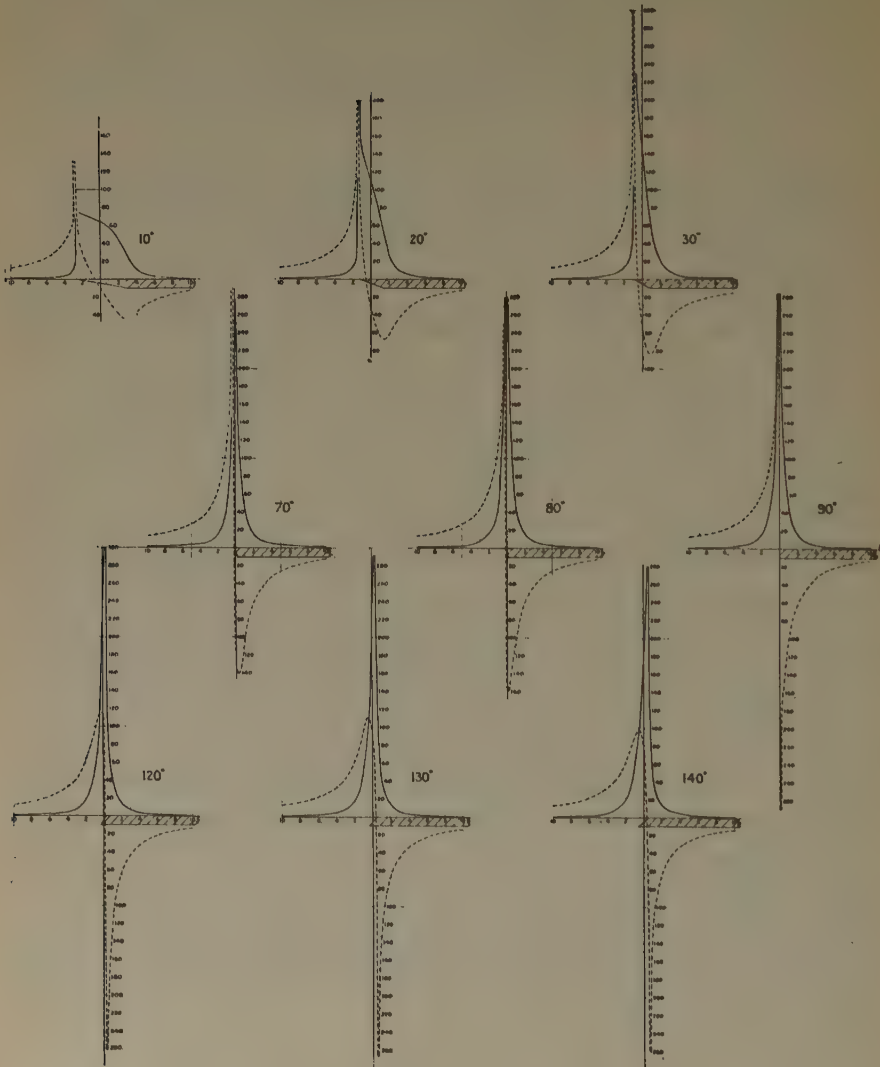
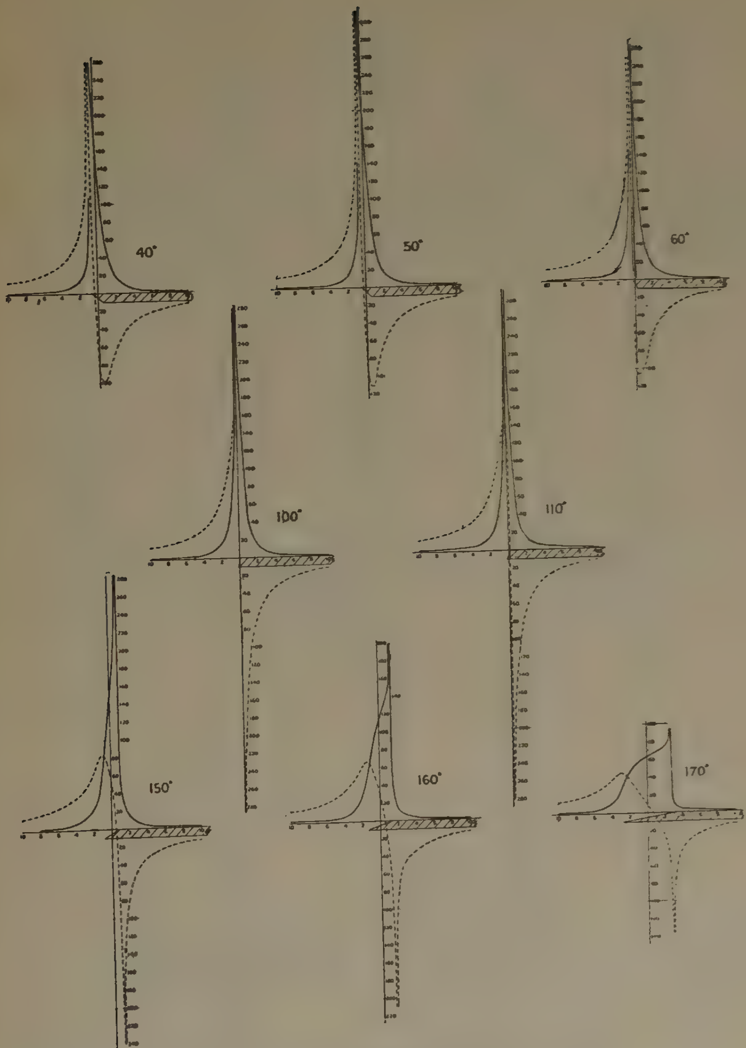


Fig. 19.—Infinite horizontal block with inclined face.



EFFECT OF VARIATION OF α AT CONSTANT DEPTH. $d = 0$.

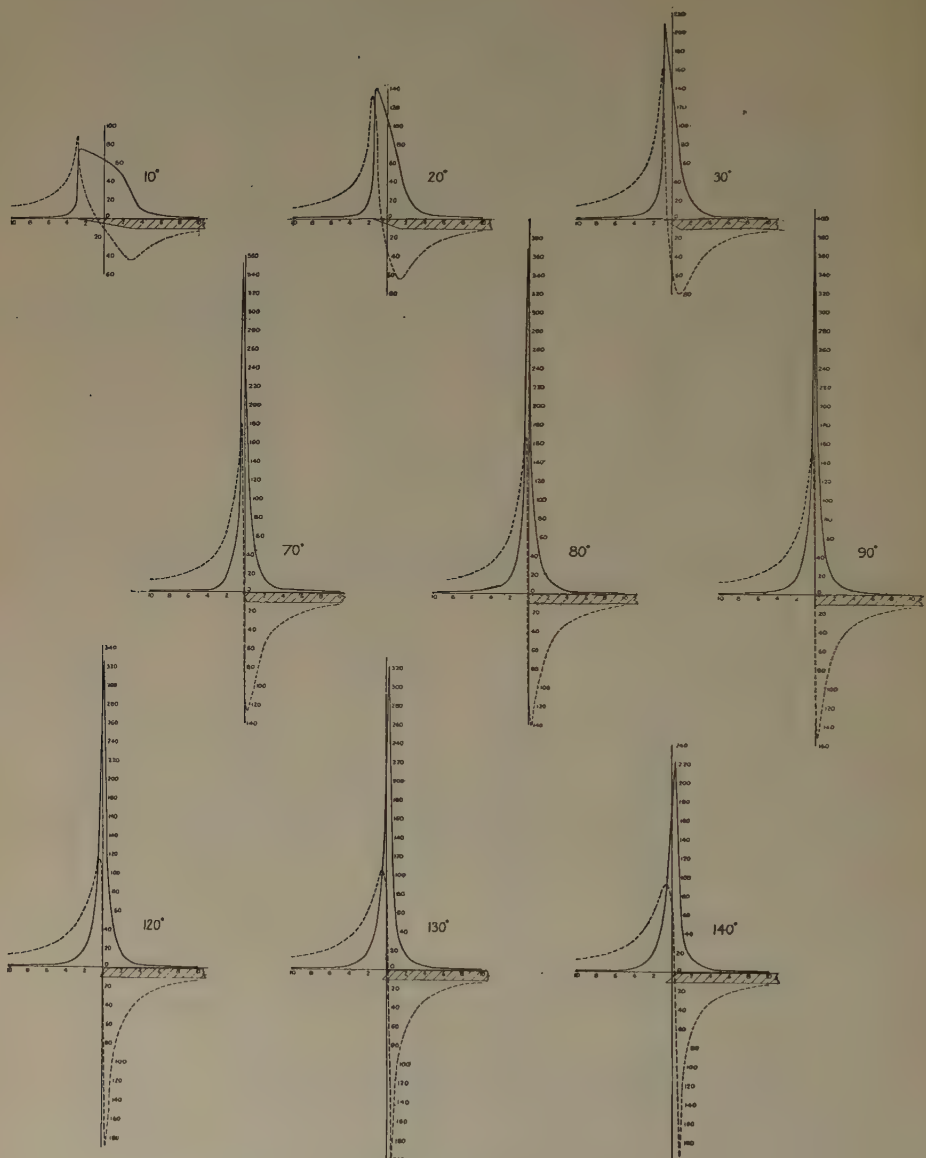
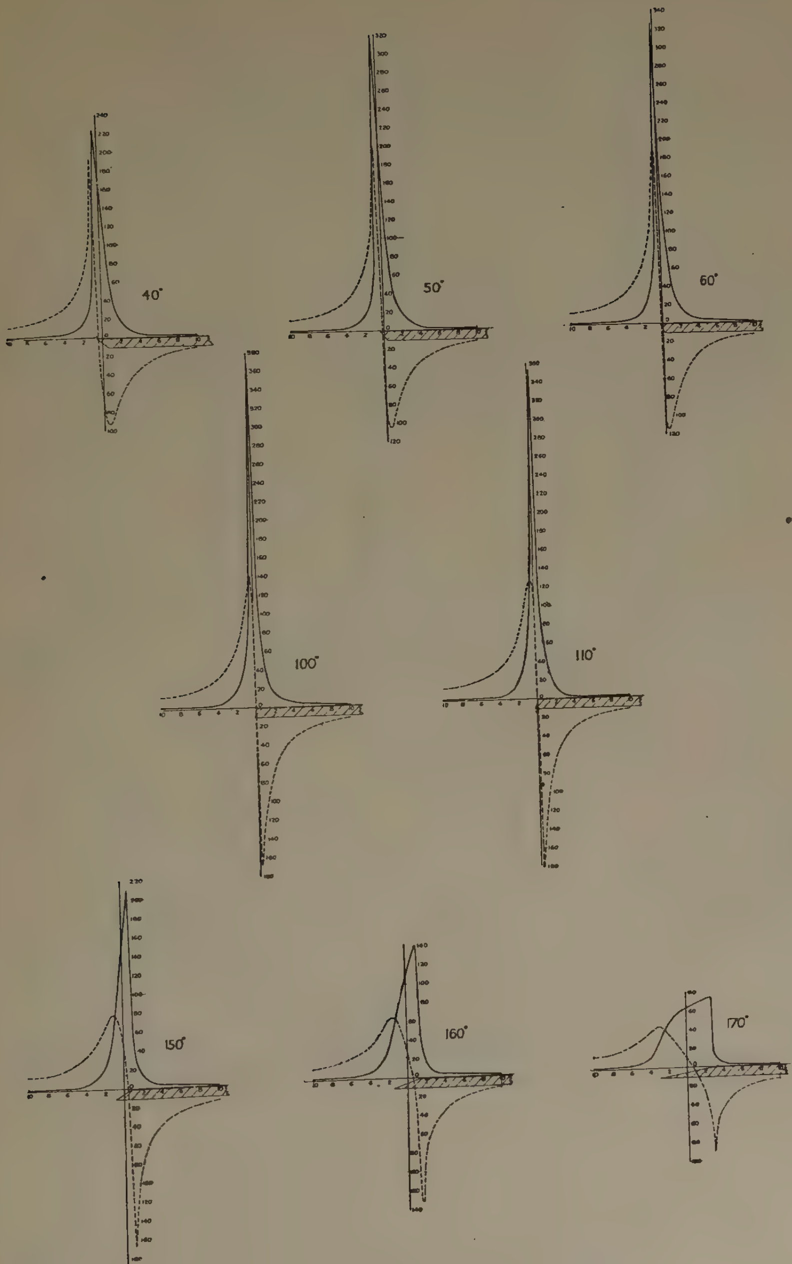


FIG. 20.—INFINITE HORIZONTAL BLOCK WITH INCLINED FACE.



EFFECT OF VARIATION OF α AT CONSTANT DEPTH. $d = 0.05$.

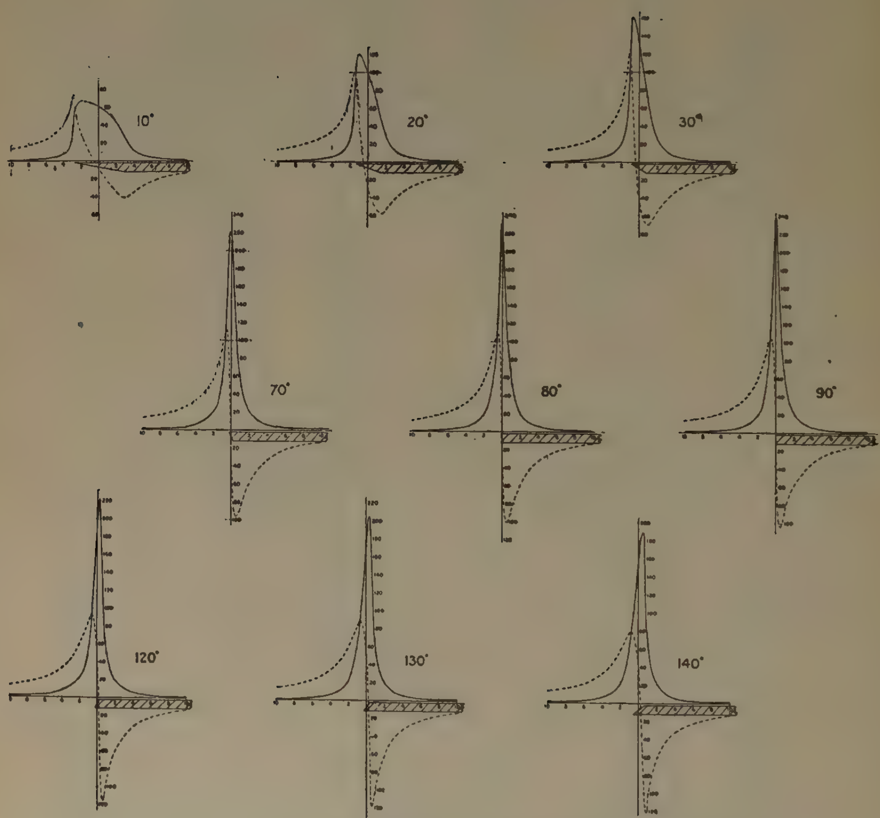
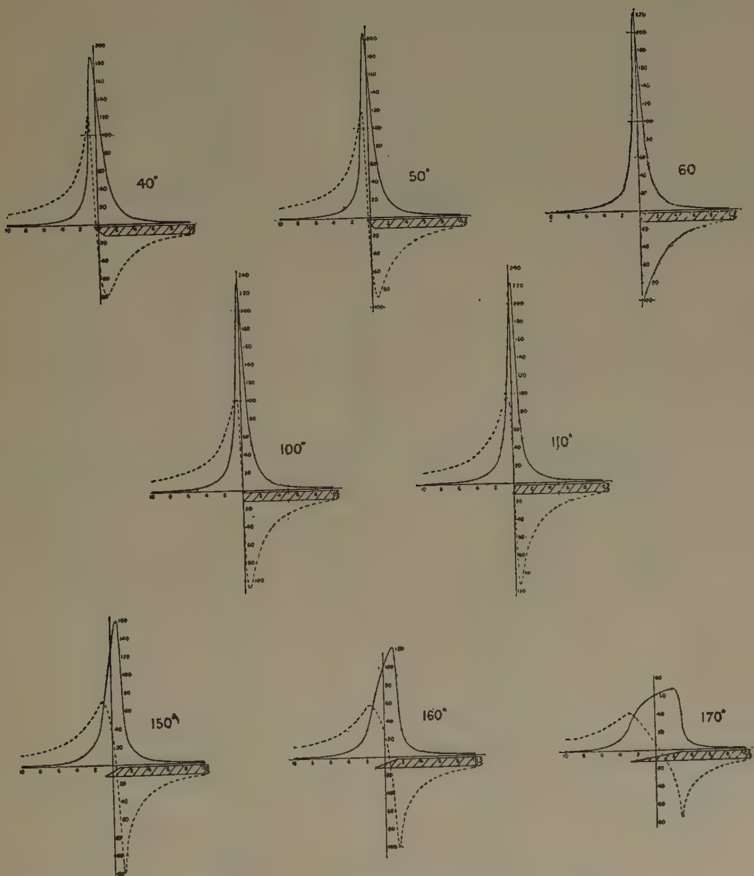


FIG. 21.—INFINITE HORIZONTAL BLOCK WITH INCLINED FACE.



EFFECT OF VARIATION OF α AT CONSTANT DEPTH. $d = 0.1$.

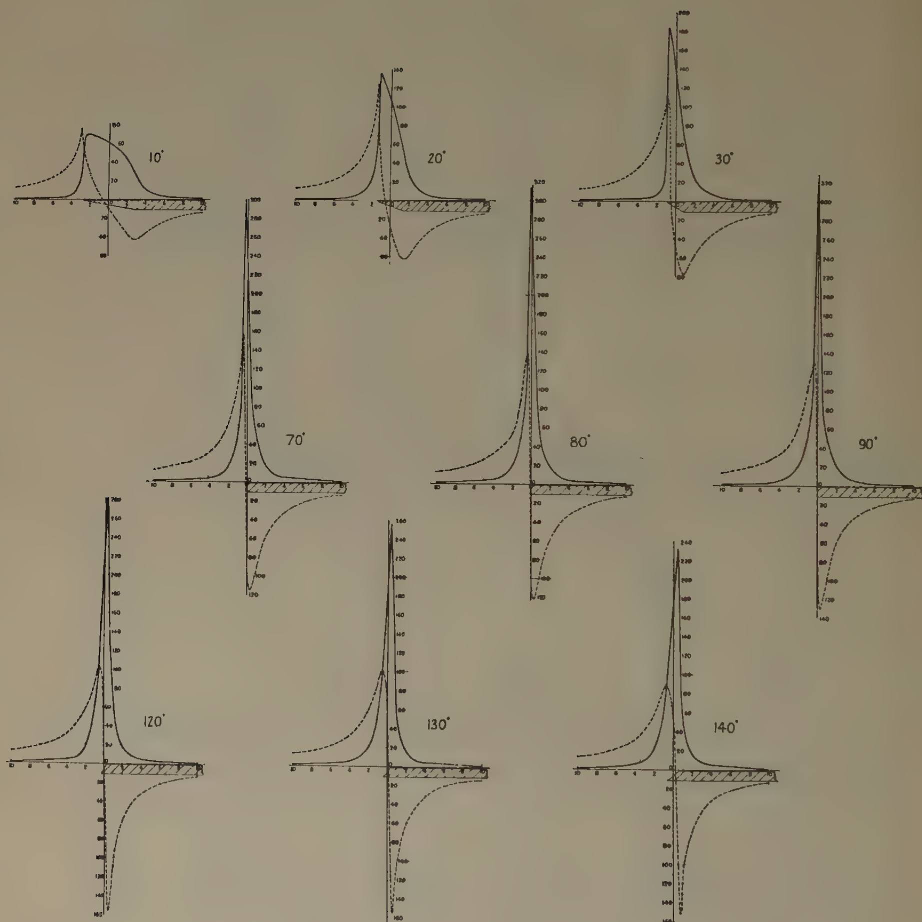
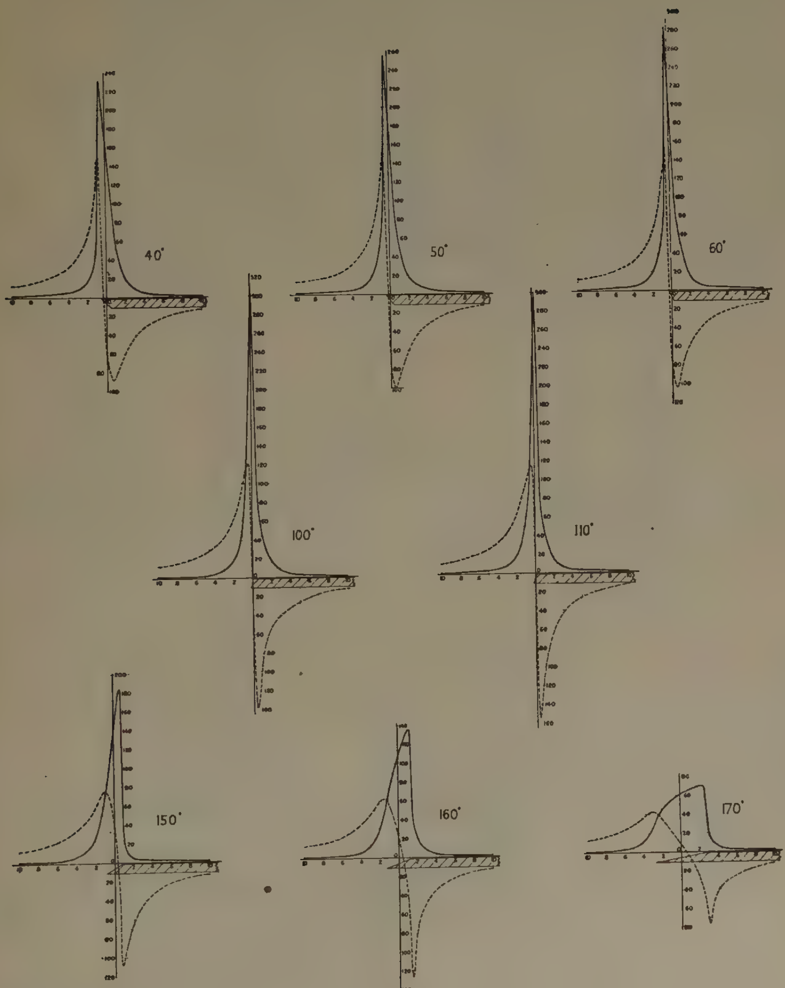


FIG. 22.—INFINITE HORIZONTAL BLOCK WITH INCLINED FACE



EFFECT OF VARIATION OF α AT CONSTANT DEPTH. $d = 0.2$.

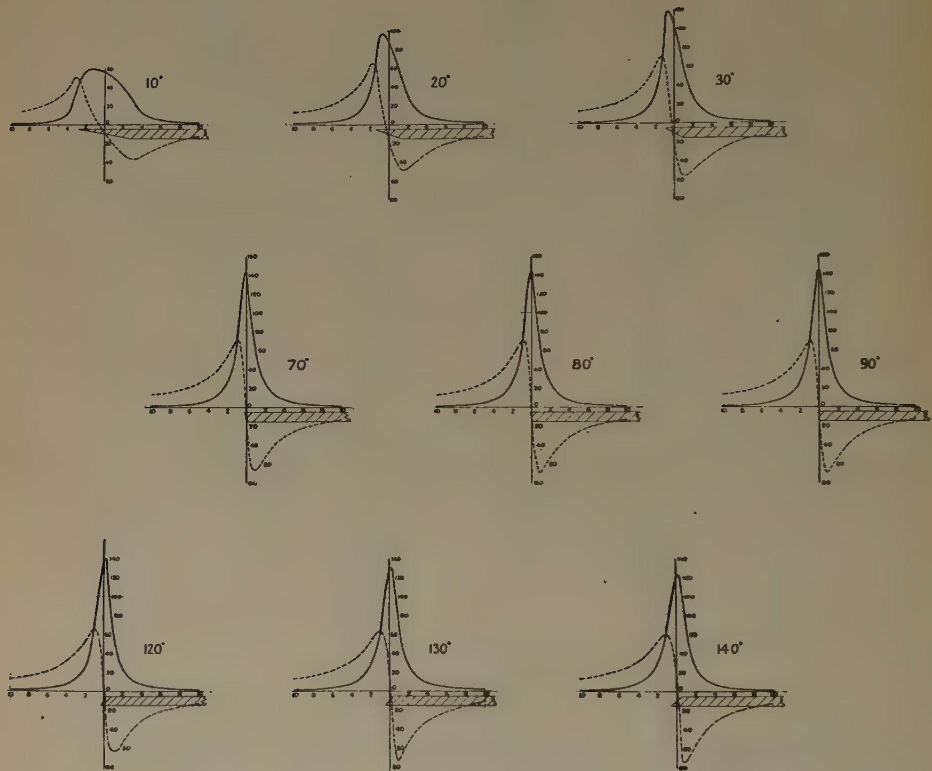
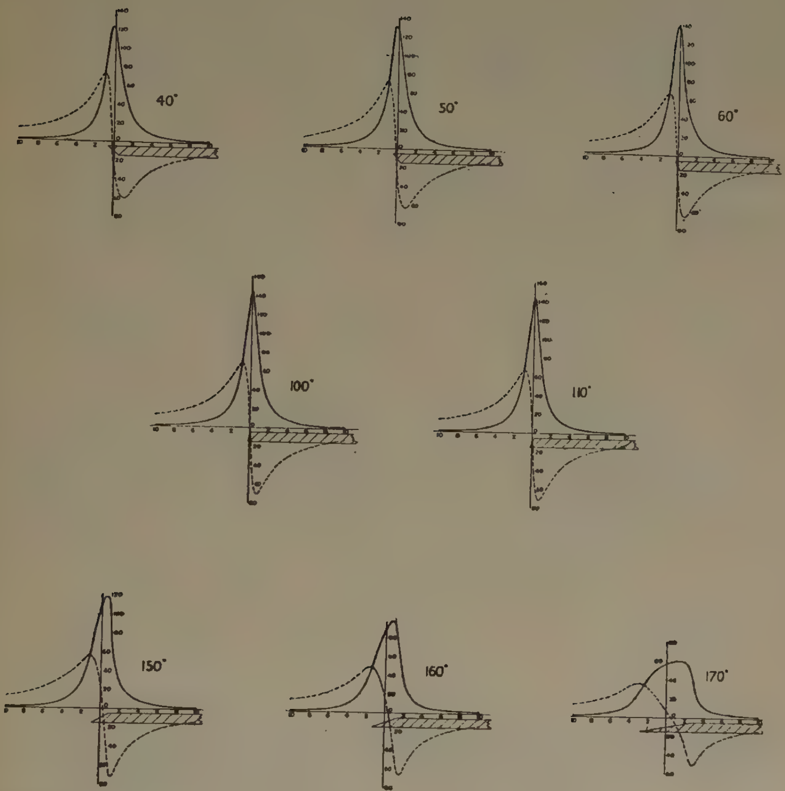


FIG. 23.—INFINITE HORIZONTAL BLOCK WITH INCLINED FACE.



EFFECT OF VARIATION OF α AT CONSTANT DEPTH. $d = 0.5$.

When the vertical edge or face of the infinite block is replaced by an inclined face, the curves of gradient and differential curvature value lose their symmetry and change considerably in general shape. The gradient curve is folded over towards the upper portion of the inclined face, the maximum gradient occurring in the vicinity of the upper extremity of this inclined face, while beyond the maximum value, the gradient curve falls steeply to a low value.

The curve of differential curvature values is also distorted considerably; it no longer passes through the point regarded as the origin but has its zero value shifted towards the upper portion of the inclined plane,

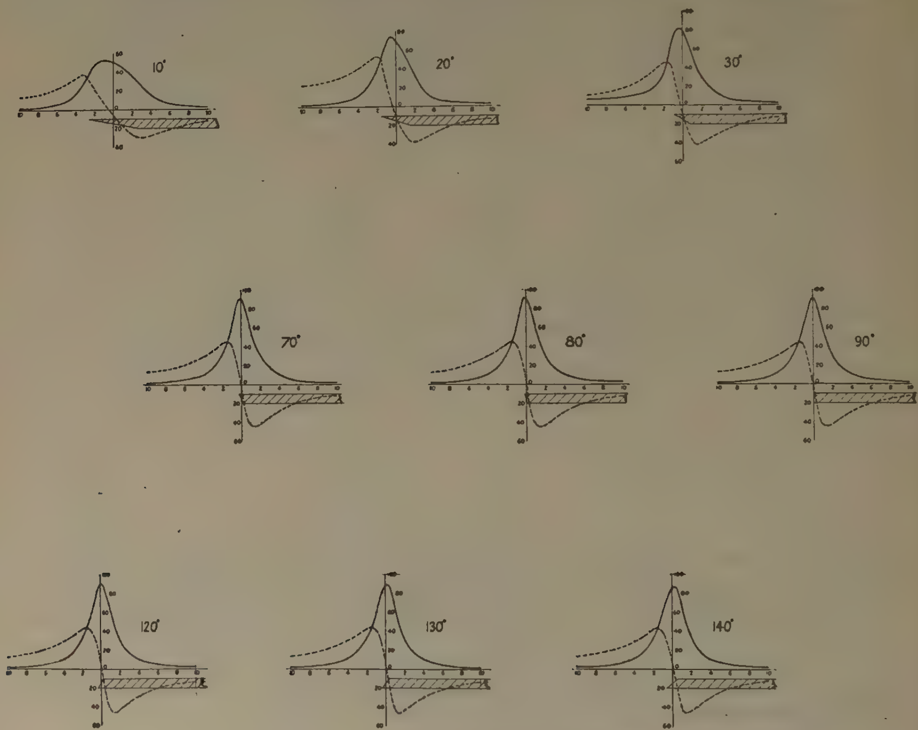


FIG. 24.—INFINITE HORIZONTAL BLOCK WITH INCLINED FACE.

while the two portions of the curve have distinctive shapes, the one extending over the upper edge of the inclined face having a more sharply defined maximum value than the other portion.

As in the previous case, the gradient curve remains entirely above the base line throughout, indicating that in this case the gradient is directed from left to right continuously and varies only in magnitude.

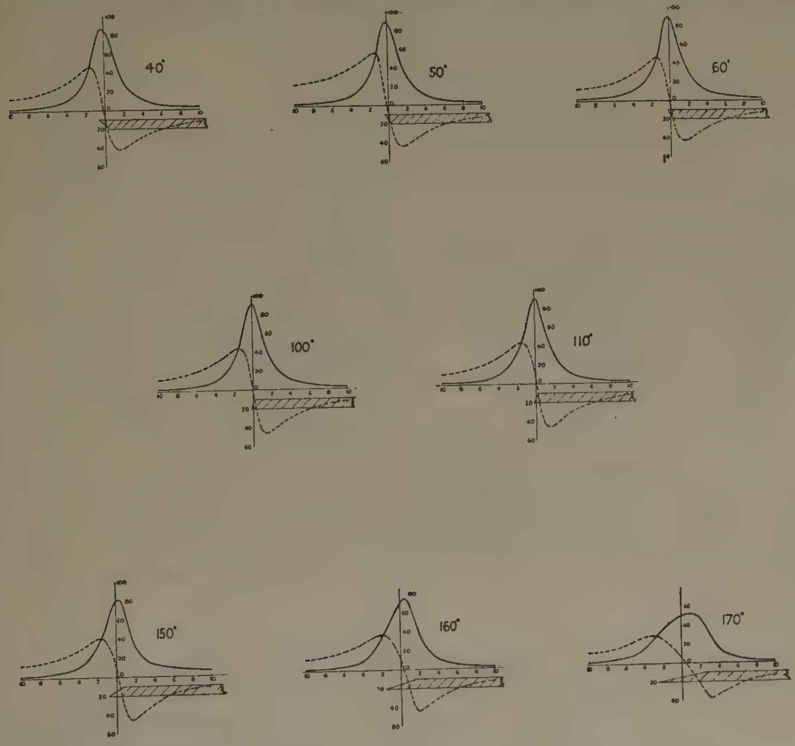
Value of Maximum Gradient

For a deposit at any given depth the numerical value of the maximum gradient increases noticeably with the increase of the angle α from 0° to

90°, this angle α being the inclination of the sloping face to the horizontal. This is rendered evident by Fig. 25, in which the maximum gradient is plotted against this inclination of the edge.

A separate curve is drawn for each depth of block (for each value of d), and as might be expected, the variation of maximum gradient with α is most pronounced at shallow depths, and decreases rapidly as the depth increases.

In all cases, the steepest portion of the curve corresponds to small values of α, the curves becoming flatter in the region of the 90° value of



EFFECT OF VARIATION OF α AT CONSTANT DEPTH. $d = 1.0$.

α. With increasing depth this flatness extends further laterally in both directions, showing that a deep block must have its inclined surface at a much greater inclination to the vertical than is necessary in the case of a shallow block, before it is possible from a consideration of the gradient alone to distinguish it from the rectangular type.

This point is brought out clearly in Fig. 26, which indicates the maximum gradient produced by any depth of block, (within the limits considered), and at any inclination, this maximum gradient being in every case expressed as a percentage of the maximum gradient given by a rectangular block at the same depth.

If, for example, we assume that it is just possible in practice to distinguish with certainty a variation of 10 per cent. from the maximum value given by the rectangular block, these limiting conditions are represented by the 90° curve in Fig. 26. On this assumption, the form of a block at a depth $d = 0.05$ would just be revealed if $\alpha = 72°$, while if $d = 0.1$ the value of α should be 65°, and at a depth of $d = 1.0$, the angle α would have to be reduced to 33° before the block could be distinguished, on these considerations, from the rectangular one.

At greater depths, the angle α would have to be reduced still more, and it would seem from the form of the curves that at a depth of $d = 4$ (approx.) it would be practically impossible to ascertain from the gradient

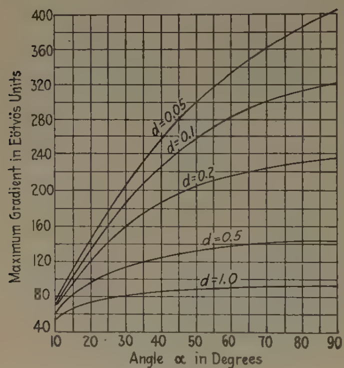


FIG. 25.

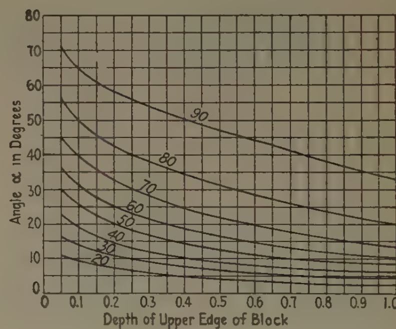


FIG. 26.

FIG. 25.—INFINITE BLOCK WITH INCLINED FACE. EFFECT OF INCLINATION ON MAXIMUM GRADIENT FOR VARIOUS DEPTHS.

FIG. 26.—INFINITE BLOCK WITH INCLINED FACE. VARIATION OF MAXIMUM GRADIENT WITH DEPTH OF BLOCK AND WITH INCLINATION α OF FACE.

Figures on curves indicate maximum gradient, expressed as percentage of maximum gradient given by same block with vertical edge.

alone the form or slope of the edge of an infinite block such as we are now considering, even with a small value of α, so that for all practical considerations and for the purpose of interpretation (from the standpoint of the maximum gradient) all infinite blocks having a depth of d equal to not less than four times the thickness may be treated as though they were of the rectangular type with $\alpha = 90°$.

Position of Point of Maximum Gradient

It is obvious from an inspection of the gradient curves of Figs. 19–23, that the position of the point of maximum gradient moves progressively in a lateral direction as the inclination α increases, and in order that this movement may be examined, it is necessary to take some fixed point to which we may refer the motion of the moving point. For this purpose the midpoint of the inclined edge is taken as the origin and, unless

otherwise stated, all distances are measured horizontally from a point on the surface vertically over this fixed point.

Fig. 27 shows the effect of the inclination α on the position of the point of maximum gradient. A separate curve is drawn for each depth and the extreme edge of the block at any time is shown by a dotted curve.

This dotted curve coincides with the two curves $d = 0.05$ and $d = 0.1$ in its lower portion but elsewhere it lies above them, from which it appears that the maximum gradient always occurs at a point between the midpoint and the upper edge of the inclined surface; therefore always within the confines of the block and not beyond the extreme edge.

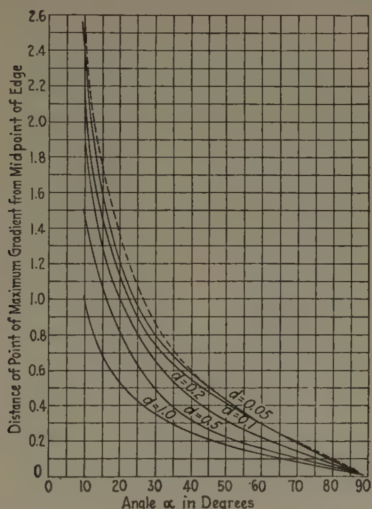


FIG. 27.—INFINITE BLOCK WITH INCLINED FACE. EFFECT OF INCLINATION ON POSITION OF MAXIMUM GRADIENT FOR VARIOUS DEPTHS.
Dotted curve shows position of extreme edge of block.

The rate of travel of the point of maximum gradient is most rapid for small inclinations of the edge and rapidly decreases as α approaches 90°. For blocks near the surface the effect is more noticeable, so much so that the position of maximum gradient for values of α up to 90° may be regarded as coinciding with the extreme upper edge of the block for depths up to $d = 0.05$ and value of α as low as 10°, while for depths of $d = 0.1$ the maximum gradient for this value of α has only moved inwards from the edge an amount equal to 10 per cent. of the distance between the midpoint and the extreme edge of the inclined surface.

For a depth of $d = 0.5$ this value has increased to between 40 and 50 per cent. and for a depth $d = 1.0$ to approximately 60 per cent. This movement of the position of maximum gradient causes the gradient curve to be deformed from its symmetrical shape, for all values of α other than 90°. At small depths and for small inclinations of the edge, the gradient

curve is folded over in a unique manner towards the edge of the upper surface of the block, at which point it falls sharply in value. At greater depths, and also at greater inclinations, there is a tendency for this feature to become less pronounced and for the sharpness of the curve to be replaced by a rounder and less noticeable effect which is not so readily recognized.

It can be shown that the conditions to be satisfied in order to obtain the maximum gradient value are:

$$\beta_1 + \beta_2 = \pi$$

or

$$\beta_1 + \beta_2 = 2\pi$$

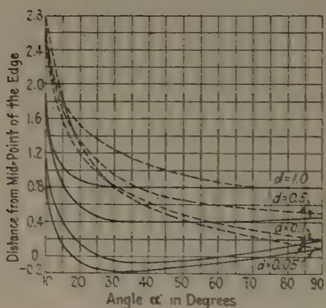


FIG. 28.

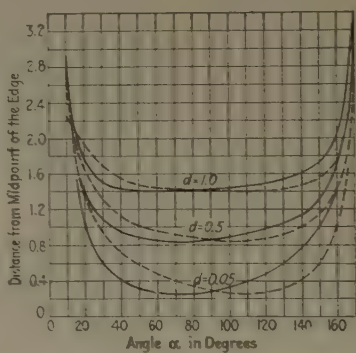


FIG. 29.

FIG. 28.—INFINITE BLOCK WITH INCLINED FACE. POINTS AT WHICH GRADIENT IS REDUCED TO ¾ MAXIMUM VALUE, FOR VARIOUS DEPTHS.

Two curves correspond to each depth; the dotted curves indicate side nearer to upper edge of slope.

FIG. 29.—INFINITE BLOCK WITH INCLINED FACE. POINTS AT WHICH GRADIENT IS REDUCED TO ½ MAXIMUM VALUE, FOR VARIOUS DEPTHS.

Two curves correspond to each depth; the dotted curves indicate side to left of midpoint.

from which it is obvious that this maximum value occurs at a point between the verticals passing through the extremities of the inclined face, as has already been pointed out.

The horizontal distance x of this point from the midpoint of the inclined face is given by the equation

$$x = \frac{(d_2 - d_1)^2 \cot \alpha}{2(d_2 + d_1)} = \frac{\cot \alpha}{2(2d + 1)}.$$

where d_1 and d_2 are the depths of the upper and lower surfaces respectively of the block, and d is the depth of the upper surface expressed in terms of the thickness of the block as unit.

Variation of Gradient with Distance from Edge

In Figs. 28 to 30, curves are drawn showing the distances from the midpoint of the sloping edge, at which the gradient values are reduced

to definite proportions, namely three-fourths, one-half and one-fourth respectively of their maximum values. Owing to the asymmetry of the gradient curve, there are two curves for each depth of block, one of which is shown by a full line, the other by a dotted line. These curves flatten out and approach each other as the inclination approaches 90°, and in general the mean of the two curves for any depth may be regarded as running practically at a constant distance from the midpoint of the inclined edge, at any rate for values of α between the angles of 60° and 120°. In considering the curves for points of three-fourths maximum gradient, the range of the constant mean position between the two points of equal fractional value of the maximum gradient may be used to locate

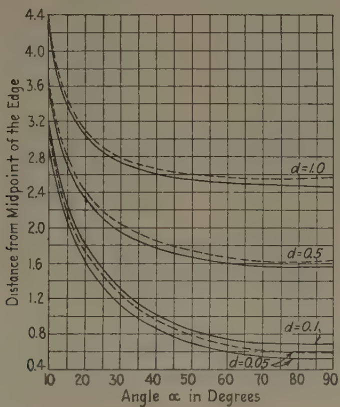


FIG. 30.—INFINITE BLOCK WITH INCLINED FACE. POINTS AT WHICH GRADIENT IS REDUCED TO ¼ MAXIMUM VALUE, FOR VARIOUS DEPTHS.

Two curves correspond to each depth; the dotted curves indicate side nearer to upper edge of slope.

the midpoint of the inclined edge of the block, exactly as in the case of the block with vertical edge.

For inclination of the edge beyond the limits given, the mean position between the two points of equal gradient moves towards the upper edge of the slope, the motion being fairly quick at first, and later, when further from the above limits, more rapid.

Positions at Which Gradient Attains Definite Values

Taking the midpoint of the inclined edge of the block as the fixed point of reference from which all measurements are made, curves have been drawn for a number of depths, in Figs. 31 to 34, showing the distance from this fixed point of reference at which the gradient assumes definite values of 10, 5, and 2 Eötvös units ($1E = 1 \times 10^{-9}$ c.g.s. units) respectively.

Owing to the asymmetric nature of the gradient resulting from this type of block, two curves can be drawn in each case, one for each side of

the fixed point. These two curves have been found in general to lie very close to each other, so that they have been replaced in each case by a mean curve.

An examination of these curves reveals a marked tendency to horizontality, the small variations therefrom being limited to cases where the angle of slope α is below 30°. Above this angle (and therefore between the angles 30° and 150° in practice), the lines may be regarded as horizontal, thereby indicating that within these limits no effect on the position of the points under consideration is produced by a variation of the angle of slope.

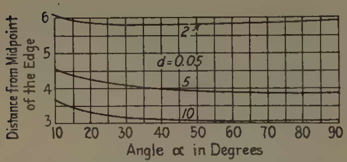


FIG. 31.—INFINITE BLOCK WITH INCLINED FACE. POINTS AT WHICH GRADIENT ATTAINS DEFINITE NUMERICAL VALUES, WHEN DEPTH = 0.05.

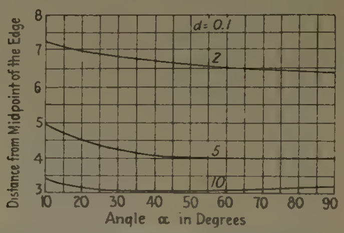


FIG. 32.—INFINITE BLOCK WITH INCLINED FACE. POINTS AT WHICH GRADIENT ATTAINS DEFINITE NUMERICAL VALUES, WHEN DEPTH = 0.1.

The conditions existing for a rectangular block may be applied therefore to a block with inclined edge, and except for inclinations smaller than 30° and greater than 150°, the distance from the midpoint of the sloping edge, of points at which the gradient value is reduced to 10E, 5E and 2E respectively, may be regarded as constant and independent of the inclination, so that the value in each case is equal to that obtained for the rectangular block.

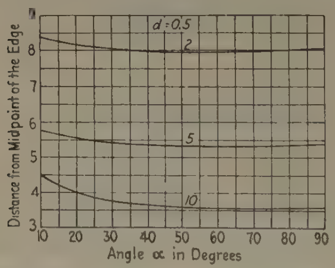


FIG. 33.—INFINITE BLOCK WITH INCLINED FACE. POINTS AT WHICH GRADIENT ATTAINS DEFINITE NUMERICAL VALUES, WHEN DEPTH = 0.5.

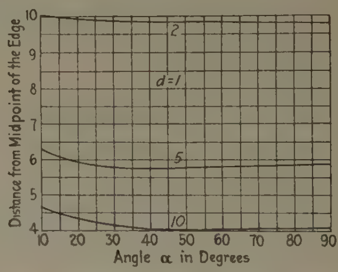


FIG. 34.—INFINITE BLOCK WITH INCLINED FACE. POINTS AT WHICH GRADIENT ATTAINS DEFINITE NUMERICAL VALUES, WHEN DEPTH = 1.

The conclusions reached in the case of the infinite rectangular block in this connection are in general equally applicable to an infinite block with an inclined face.

The actual distance from the midpoint of the inclined face varies considerably with the depth of the block, and in general increases with the depth within the limits of this investigation.

Gradient Value at Definite Distance from Edge

In Figs. 35 and 36, the gradient values are plotted at definite fixed points for all inclinations at depths of $d = 0.05$ and $d = 1.0$ respectively. The distance from the midpoint of the edge is indicated on each curve, and as the gradient curve is asymmetrical about this point, there are two curves in each case, one being measured to the left and indicated

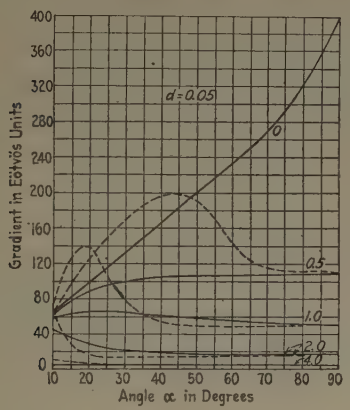


FIG. 35.

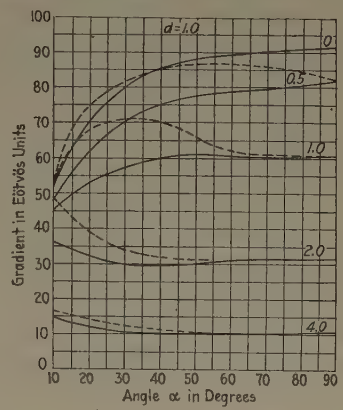


FIG. 36.

FIG. 35.—INFINITE BLOCK WITH INCLINED FACE. VARIATION OF GRADIENT WITH α, AT DEFINITE DISTANCES FROM MIDPOINT OF FACE, WHEN DEPTH = 0.05.

This distance is indicated on each curve. Dotted curves represent points to left of center line; solid lines, those to right.

FIG. 36.—INFINITE BLOCK WITH INCLINED FACE. VARIATION OF GRADIENT WITH α AT DEFINITE DISTANCES FROM MIDPOINT OF FACE, WHEN DEPTH = 1.0.

Distance is indicated on each curve. Dotted curves represent points to left of center line and solid lines those to right.

by a dotted curve while the other is measured to the right and is shown by a continuous curve.

From these curves it is evident that the effect of changing the inclination of the edge is very marked at points in the vicinity of the midpoint of the edge but diminishes rapidly as one moves farther from this point, until, for both depths considered, it has practically disappeared at a distance of 4 units from the point of reference; and even at this distance, differences are noticeable only for values of α which are either small or approaching 180°.

It would appear therefore that for purposes of interpretation, in endeavoring to recognize blocks of this type and to distinguish them from the rectangular type, it is of little use to consider portions of the gradient curves well removed from the midpoint of the inclined face, and it is

fairly safe to say that from the standpoint of gradients, no information concerning the inclination of the edge of an infinite block will be obtained at a distance from the midpoint of the edge greater than four times the thickness of the block.

Gradient Interpretation

The interpretation of an infinite horizontal block of this type terminating in a face inclined to the vertical presents greater difficulties than the infinite rectangular block considered earlier in the paper.

Nevertheless, from a close examination of the gradient curve, it is possible to make deductions as to the form of the block which will approximate closely to the actual conditions.

It should be pointed out first, however, that it is not possible from a consideration of the gradient curve alone to distinguish between an infinite block of the rectangular type and one having an inclined face, if the depth of the block below the surface exceeds four times the vertical thickness of the block.

The gradient curve is deformed from its original symmetrical shape as the face becomes inclined, and appears to be folded over towards the upper limit of the inclined edge, near which point the gradient falls off sharply in value. This effect, of course, is most pronounced in the case of shallow depths and tends to become more gradual as the depth increases.

The maximum gradient always occurs within the limits of the inclined face of the block and not beyond the extreme edge. The distance of this point from the limit of the inclined face (towards the midpoint of this face) increases with the depth and also as the inclination of the edge to the vertical increases.

The effect of inclining the edge of an infinite block is greatest in the vicinity of the edge, and for purposes of interpretation of such a block it is of little use to consider gradients at a distance from the edge greater than four times the thickness of the block.

The shape of the gradient curve is the first indication of an asymmetric block of this type, and a comparison of the shape of the curve in any given case with the curves of Figs. 19 to 24 enables the ratio of depth to thickness to be ascertained and the inclination α of the face of the block to be estimated.

The position of maximum gradient is known to be situated above and near to the upper edge of the inclined face.

Referring now to Fig. 27, we obtain the distance of the point of maximum gradient from the midpoint of the inclined face, which thus enables us to fix the latter point, so that we are able to fix the extremities and the midpoint of the inclined face.

In any particular case, if one considers the two points at which the gradient is a definite fraction (say ¾ or ½) of the maximum gradient, then to a first approximation, the mean distance of these points from the midpoint of the inclined face may be regarded as the corresponding point (at which a gradient of ¾ or ½ the maximum value occurs) for the vertical edge block, and within certain limits this point may be used to locate the midpoint of the inclined face of the block exactly as for the rectangular block.

The effective density of the block may be ascertained from a direct comparison of the numerical values with those of the corresponding example, Fig. 19.

As the traces along the surface of the extremities of the inclined face are known while α has also been estimated, the thickness of the block may be ascertained and its actual depth below the surface calculated from the value of d already obtained.

Characteristics of Differential Curvature

Determination of Curvature Value

The curvature value for this type of infinite block is given by the equation:

$$\frac{\partial^2 U}{\partial x^2} = \left\{ \sin 2\alpha \log_e \frac{r_2}{r_1} + (1 - \cos 2\alpha)(\beta_2 - \beta_1) \right\} \times \gamma.$$

and curves showing the variation of this value with certain conditions are given in Figs. 19 to 24, where they are shown dotted, the full lines indicating the gradient.

In the case of the infinite rectangular block, the curvature curve was seen to be symmetrical about the point on the surface vertically above the edge of the block, at which point the curvature value crosses the axis and changes sign. However, in the case of a block with inclined face, this curve is no longer symmetrical, nor does it change sign in crossing over the midpoint of the inclined face. On referring to the curves of gradient and curvature for varying depths, it will be seen that for any depth of block (within the limits considered), and for any angle of slope, the maximum curvature value always occurs near the upper limit of the inclined face. With angles of inclination, therefore, between 0° and 90° (*i. e.*, an overhanging block) the maximum curvature occurs near, and usually just outside, the extreme edge of the block. Similarly, with angles of inclination between 90° and 180°, the maximum curvature occurs near, and generally just a little farther over the block than the upper limit of the inclined face.

The proximity of this point of maximum curvature to the upper edge of the inclined face appears to vary with the depth of the deposit and the

inclination of the edge. Speaking generally, as the inclination of the face is varied from 90°, either upwards or downwards, the point of maximum curvature approaches more closely to the upper edge of the inclined face while it recedes farther from this point as the depth of the deposit is increased.

The shape of the curvature curve also varies considerably with the inclination of the face, to the vertical. As this angle increases, the two portions of the curve gradually acquire different features, the one in the direction of the upper edge of the inclined face which assumes a higher maximum value being marked by a more sharply peaked curve, the tendency in the other portion being for the maximum value to occur on a well-rounded sweep of the curve.

Maximum Curvature Values

As in the case of the infinite rectangular block, the curvature has two maximum values, one on each side of the midpoint of the edge, but whereas in the previously considered symmetrical case these two maximum values are numerically equal, in the case of the block with inclined face they are unequal.

The two points of maximum curvature, sometimes referred to as the points of "maximum" and "minimum" curvature, will be referred to here as the First and Second Maximum Curvatures, the First Maximum Curvature in any particular case being the one of higher numerical value. The effect of varying the inclination upon the magnitude of the First and Second Maximum Curvatures is shown in Fig. 37, from which it is at once evident that the maximum curvature values are numerically greater in the case of blocks at small depth, and diminish rapidly as the depth increases. It is also noticeable that in the case of shallow depths the First Maximum Curvature varies in magnitude to a far greater extent than the Second Maximum Curvature.

For all depths considered here the First Maximum Curvature curve reaches its maximum value when the inclination of the sloping face is approximately 45°, this maximum value being more pronounced as the depth below the surface is reduced. For greater depths of block this curve is much flatter, and at a depth of 1 unit (the thickness of the block), the First and Second Maximum Curvature curves are practically coincident and horizontal for inclinations between about 25° and 155°.

For greater depths than this, the First and Second Maximum Curvature values may be expected to be practically equal for all angles of slope, and it is improbable that a comparison of the relative magnitudes of these values in any particular case would furnish any information which could be of use in the interpretation of any practical field results, if the depth of the body below the surface were greater than the vertical thickness of the block or layer.

These points are illustrated graphically in Fig. 38 in which the ratio of First Maximum Curvature to the Second Maximum Curvature is plotted against the angle of slope, for various depths of block. From this series of curves, it is possible, if the depth of the block is known, to estimate the inclination of the face, and conversely, if the slope should be known, to estimate the depth of the block below the surface.

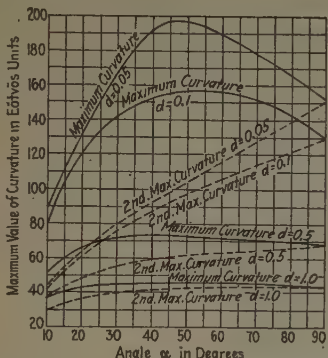


FIG. 37.—INFINITE BLOCK WITH INCLINED FACE. EFFECT OF INCLINATION ON MAXIMUM CURVATURE FOR VARIOUS DEPTHS.

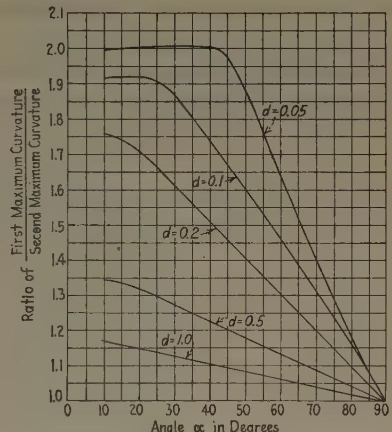


FIG. 38.—INFINITE BLOCK WITH INCLINED FACE. EFFECT OF INCLINATION ON RATIO OF FIRST MAXIMUM CURVATURE TO SECOND MAXIMUM CURVATURE.

Position of Maximum Curvature

It may be shown that the conditions for maximum and minimum curvature values in the case of an infinite horizontal block with inclined face are given by

$$(\beta_1 + \beta_2) = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}$$

from which we see that there are two positions, as may be expected from previous considerations. In Fig. 39 the effect of the inclination on the position of the two points of maximum curvature is shown, and as before, these points are referred to the midpoint of the inclined face. In each of the cases considered, the points of First and Second Maximum Curvature are equidistant from the point of reference, so that the "mean" curve is drawn in each case. The position of the extreme edge of the block is also indicated by a dotted line.

For small angles of inclination (10°) these curves all lie close together, showing that the points of maximum curvature for such small angles occur almost vertically over the extreme point of the inclined edge.

As the angle increases, however, these curves diverge, and also move away from the curve showing the extreme edge of the block, and this

divergence increases with the angle of slope (up to 90°) while it also increases rapidly with the depth of the block.

A comparison of these curves shows that as the angle of slope increases the distance of the maximum curvature from the point of reference tends to reach a constant value in each case, but this effect becomes far more prominent as the depth increases, until when $d = 1$ the distance of the maximum curvature from the point of reference would be practically constant for all inclinations between $d = 40°$ and $d = 140°$. For greater depths of block this range will tend to increase, whereas for blocks nearer to the surface the range rapidly diminishes.

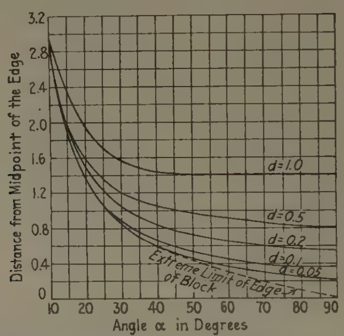


FIG. 39.—INFINITE BLOCK WITH INCLINED FACE. EFFECT OF INCLINATION ON POSITION OF TWO POINTS OF MAXIMUM CURVATURE, FOR VARIOUS DEPTHS.

Positions of both first and second maximum curvatures have been plotted in each case, and mean curves drawn.

From the results obtained, it appears that the midpoint of the inclined face may be regarded as lying midway between First and Second Maximum Curvature. This being so, the distance from this point to the point of maximum curvature will be known, so that the only remaining variables are the depth d of the block, and the inclination α of the face. If one of these quantities is known, the other may be estimated.

The distance of the points of maximum or minimum curvature from the point on the surface vertically over the midpoint of the inclined face may be expressed as

$$\begin{aligned} x &= \tfrac{1}{2}\sqrt{4d_1d_2 + (d_2 - d_1)^2 \cot^2 \alpha} \\ &= \tfrac{1}{2}\sqrt{4d(d + 1) + \cot^2 \alpha} \\ &= \pm\sqrt{d(d + 1) + \frac{\cot^2 \alpha}{4}} \end{aligned}$$

Curvature Interpretation

As in the case of the gradient, the curvature value is asymmetrical. An examination of the various curves shows that the maximum curvature value always occurs at a point very near to the upper limit of the inclined

face. In the case of an overhang (*i. e.*, with α lying between 0° and 90°) the maximum curvature occurs at a point just outside the extreme edge of the block, while for values of α lying between 90° and 180°, the maximum curvature is located farther from the edge than the upper limit of the inclined face, so that in general the maximum curvature may be regarded as occurring at a point farther from the midpoint of the edge than the upper limit of the inclined face.

The midpoint of the inclined edge may be shown to lie midway between the points of First and Second Maximum Curvature. The distance from this point to the point of maximum curvature will thus be known, so that the only remaining variables are the depth of the block and the inclination of the edge.

From Fig. 38, we see that it is possible, by comparing the magnitudes of the first and second curvatures, to estimate the inclination of the face of the block if the depth is known, and vice versa.

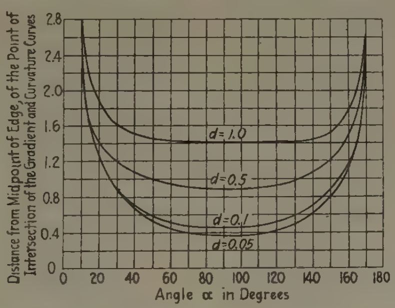


FIG. 40.—INFINITE BLOCK WITH INCLINED FACE. EFFECT OF INCLINATION ON POSITION OF POINT AT WHICH GRADIENT AND CURVATURE CURVES INTERSECT.

Comparison of Gradient and Curvature Effects

Let us now consider the effect of changing the inclination α on the position of the point of intersection of the curvature and gradient curves. In Fig. 40, the distance of this point from the midpoint of the inclined face is plotted against the inclination for four different depths. The resulting curves resemble very closely those of Fig. 29, which indicate the position of the point at which the gradient is reduced to one-half its maximum value. The approximation of these two sets of curves to each other increases with the depth. For small depths, the curve of Fig. 40 corresponds approximately with the mean of the two curves of Fig. 29 for the same depth of block.

It is evident therefore that to a first approximation the gradient and curvature curves intersect at a point at which the value of the gradient is one-half its maximum value.

At small inclinations (10° or 170°) this point of intersection coincides almost exactly with the extreme edge of the block but as the inclination

approaches 90° this point moves away from the extreme edge of the block, so that the point of intersection is more generally outside the boundary of the block in the case of an overhanging block and within the confines of the block in other cases.

Ratio of Gradient to Curvature

Figs. 41 and 42 show the effect of the inclination α on the ratio of gradient to curvature, for definite depths of block, and at definite fixed

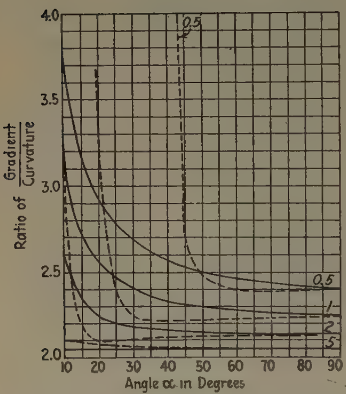


FIG. 41.

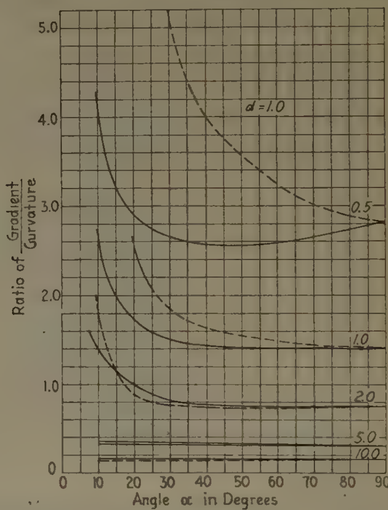


FIG. 42.

FIG. 41.—INFINITE BLOCK WITH INCLINED FACE. EFFECT OF α ON RATIO OF GRADIENT TO CURVATURE, AT DEFINITE FIXED DISTANCES FROM CENTER LINE OF FACE. $d = 0.05$.

Dotted curves refer to points to left of midpoint of face and continuous curves to points to right.

FIG. 42.—INFINITE BLOCK WITH INCLINED FACE. EFFECT OF α ON RATIO OF GRADIENT TO CURVATURE, AT DEFINITE FIXED DISTANCES FROM CENTER LINE OF FACE. $d = 1.0$.

Dotted curves refer to points to left of midpoint of face, and continuous curves to points to right.

distances from the midpoint of the face. From these two sets of curves it appears that as the distance from the midpoint increases the ratio of gradient to curvature reaches a more nearly constant value, until at a certain distance from the midpoint the ratio may be regarded as remaining constant for all angles of slope. As the depth of the block increases, the ratio of gradient to curvature at the same constant distance from the midpoint of the face also increases, and the numerical value of this ratio is an indication of the depth of the block, as in the case of the infinite rectangular block considered previously.

Geometrical Construction

In Fig. 43, let XX' represent the horizontal ground surface and $BCTT'$ the section of an infinite horizontal block with inclined face BC.

Let D be the image of B in XX'.

Join CD to meet XX' in Q.

On CD as diameter describe a circle to cut XX' in P and R. Then P and R are the points of maximum curvature and Q is the point of maximum gradient.

For $$\angle X'QB + \angle X'QC = \angle X'QD + \angle X'QC = \pi.$$

Hence Q is the point of maximum gradient.

Also, $$\angle X'PB + \angle X'PC = \angle DPX' + \angle X'PC = \frac{3\pi}{2}$$

Hence P is a point of maximum curvature.

Similarly, $$\angle X'RB + \angle X'RC = \angle DRX' + \angle X'RC = \frac{\pi}{2}$$

Hence R is also a point of maximum curvature.

Suppose, therefore, in any given example, that P and R are the two points of maximum curvature and Q the point of maximum gradient.

Then if O is the point midway between P and R, O lies vertically over the midpoint of the inclined face.

Let $PQ = a_2$ and $QR = a_1$.

Now through Q an inclined line $CNQD$ is drawn to satisfy the following conditions:

N lies on the vertical line through O.

$$CN = ND = NP = NR.$$

Then $$CQ \times QD = r_2 \times r_1, = a_2 \times a_1.$$

and $$\frac{r_1}{r_2} = \frac{d}{d+1}$$

$$QN = \rho = \frac{r_2 - r_1}{2}$$

Then C, P, D, and R all lie on a circle with $CNQD$ as diameter and radius $= \dfrac{r_1 + r_2}{2}$

If a vertical DLB is drawn through the point D so that $DL = LB$, the two points C and B form the two extremities of the inclined face of the infinite block, which may be represented by the position $BT'TC$.

Differentiation of Gradient and Curvature Curves

If we differentiate the expression for the gradient and differential curvature given on pages 299 and 319, we obtain

$$\frac{\partial(U_{xz})}{dx} = \frac{\gamma\sigma}{x}(\cos 2\beta_2 - \cos 2\beta_1)$$

$$\frac{\partial(U_{xx})}{dx} = \frac{\gamma\sigma}{x}(\sin 2\beta_1 - \sin 2\beta_2)$$

and the ratio
$$\frac{\dfrac{\partial(U_{xz})}{\partial x}}{\dfrac{\partial(U_{xx})}{\partial x}} = \tan(\beta_1 + \beta_2)$$

where x is the horizontal distance measured from the point of intersection of the inclined face and the ground surface.

By selecting certain convenient values of $(\beta_1 + \beta_2)$ we are able to obtain relations which are of assistance for purposes of interpretation. For instance, if we put

$$(\beta_1 + \beta_2) = 45° \text{ or } 225°, \text{ then } \tan(\beta_1 + \beta_2) = +1$$

and
$$(\beta_1 + \beta_2) = 135° \text{ or } 315°, \text{ then } \tan(\beta_1 + \beta_2) = -1$$

so that, if we differentiate the gradient and curvature curves as in Fig. 45, we are able to locate these points at once from an inspection of the resulting curves, the former corresponding to the points of intersection of the differential curves and the latter to the points at which the curves have equal and opposite values.

If x_{45}, x_{135}, x_{225}, x_{315}, represent the distances of the respective points from O, measured in the positive direction, then $x_{45} + x_{135} + x_{225} + x_{315} = O$ from which the center point O can easily be found, while it can also be shown that $-(x_{45} + x_{225}) = (x_{135} + x_{315}) = (d_1 + d_2)$, where d_1 and d_2 are the depths below the ground level of the upper and lower surfaces respectively of the block.

Referring to Fig. 44, let V, W, Y and Z be the points at which $(\beta_1 + \beta_2)$ is respectively equal to 45°, 135°, 225° and 315°, Q the point of maxium gradient, and P and R the points of maximum curvature.

$$QD = QB = r_1 \qquad OF = k = \frac{d_1 + d_2}{2}$$

$$QC = r_2$$

$$ON = h = \frac{d_2 - d_1}{2}$$

$$OP = OR = a$$

$$OQ = x$$

$$QN = \rho$$

It can be shown that

$$h^4 + h^2(a^2 - k^2) - k^2x^2 = 0$$

from which h may be obtained and the center N of the circle located. $\frac{d_2 + d_1}{2}$ and $\frac{d_2 - d_1}{2}$ are now both known, giving d_1 and d_2 separately.

The diameter $CNQD$ is thus fixed, giving the points C, D and consequently B, so that the block is completely determined.

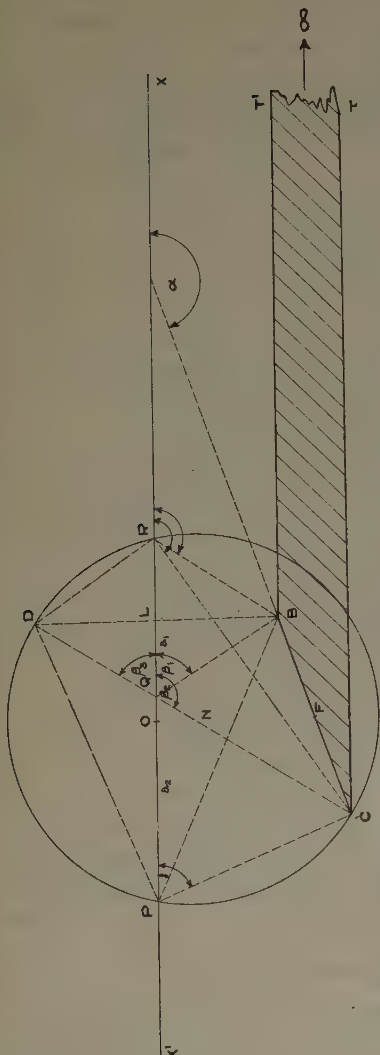


FIG. 43.—HORIZONTAL GROUND SURFACE AND SECTION OF INFINITE BLOCK WITH INCLINED FACE.

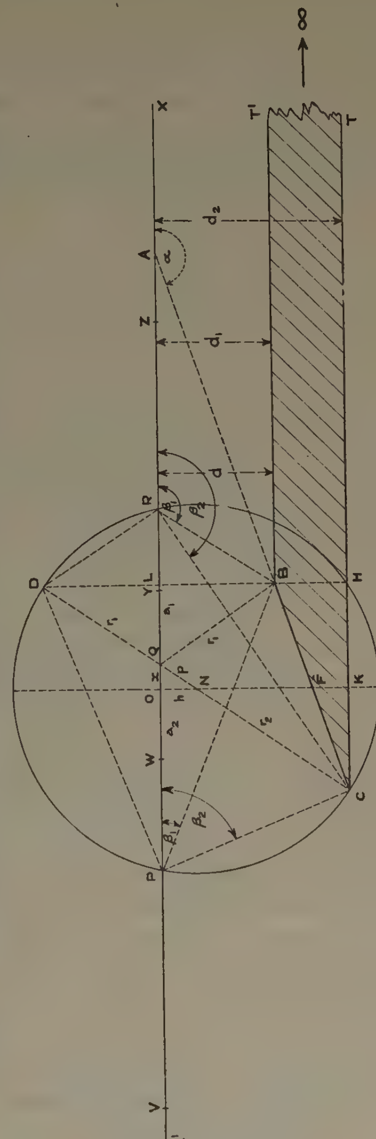


FIG. 44.—GEOMETRICAL CONSTRUCTION.

In Fig. 45, the curves obtained by differentiating the gradient and curvature curves are given, while in addition there are drawn the curve showing the variation of $(\beta_1 + \beta_2)$ with distances along the surface, and the curve showing the ratio of the ordinates of the two differential curves at any point. The points where this ratio attains a value of ± 1 are

seen to coincide with angles 45°, 135°, 225°, and 315°, while the point of maximum gradient, where the differential of the gradient curve crosses the axis, coincided with the 180° value of $(\beta_1 + \beta_2)$, and the maximum curvature values occur at the 90° and 270° positions.

It is obvious, therefore, that in practical field work, if a traverse is taken over an infinite block of this type, from which the gradient and differential curvature profiles can be obtained, we are provided with a series of seven interrelated points from a selection of which it is possible to deduce a complete solution of the subterranean body, giving the

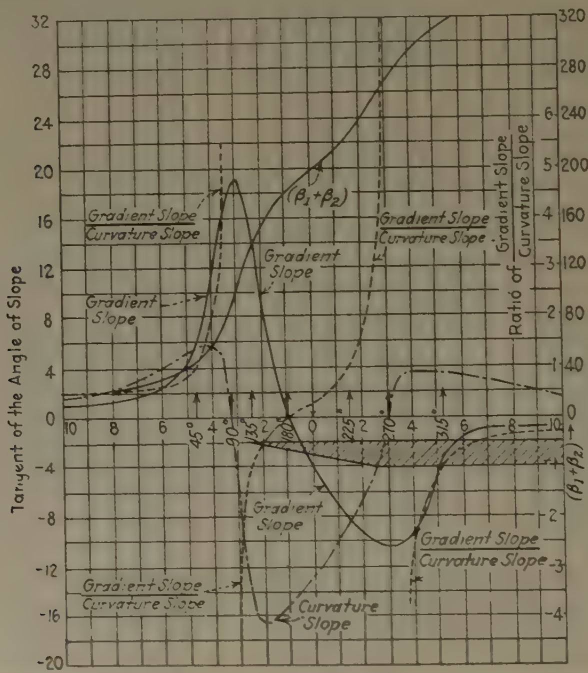


FIG. 45.—DIFFERENTIATION OF GRADIENT AND CURVATURE CURVES. DEPTH OF BLOCK $d = 1.0$; INCLINATION $\alpha = 10°$.

depth, thickness and effective density of the block, and the inclination of its boundary face.

Practical Example

In Fig. 44, let V, W, Y, and Z be the points corresponding to an actual field survey in which the ratio of the slopes of the gradient and curvature curves is ± 1, so that $(\beta_1 + \beta_2)$ attains the values of 45°, 135°, 225°, and 315°, respectively at these points, while at the same time Q represents the point of maximum gradient and P and R those of first and second maximum curvature respectively.

We therefore know from measurement the values of x and a. Let the maximum gradient value at the point Q be 36.3 Eötvös units.

The problem is to obtain a complete interpretation of the subterranean anomaly, giving the position, depth, thickness, boundary slope and effective density of the body.

The point O may be obtained from the points V, W, Y, and Z from the relation

$$-(x_{45} + x_{225}) = (x_{135} + x_{315})$$

while the point O also lies midway between the points of maximum curvature P and R, thus providing a verification of the previous determination.

From the relation

$$-(x_{45} + x_{225}) = (x_{135} + x_{315}) = d_1 + d_2 = 2k$$

we obtain the sum of the depths of the upper and lower surfaces of the block.

Now let us draw a circle passing through P and R, and having its center at the point N, a distance h vertically below the point O, given by the equation $h^4 + h^2(a^2 - k^2) - k^2x^2 = 0$.

Continue the line NQ so as to form a diameter $CNQD$, and draw the vertical DLB through D making BL equal to LD, so that B is the reflection in the ground surface of the point D and BT' and CT are drawn horizontally through B and C.

Then $CBT'T$ is the subterranean anomaly required, the angle XAC being the inclination α of the inclined face (say $\alpha = 170°$).

The thickness of the block is $(d_2 - d_1)$; the ratio of depth to thickness is $d = \dfrac{d_1}{d_2 - d_1}$ = say 0.5 (in this instance), so that the depth of the upper surface is 0.5 times the thickness of the block. Referring now to Fig. 23, we find that for this type of disturbing feature at a depth of $d =$ 0.5 and $\alpha = 170°$, the maximum value of the gradient corresponding to a unit density difference is 60.5 Eötvös units, while in the example cited the maximum gradient value of the point Q is 36.3 Eötvös units, from which we conclude that the effective density difference in this instance is 0.6.

Combined Gradient and Curvature Interpretation

The interpretation of gravitational anomalies due to infinite blocks of this type with inclined face presents greater difficulties than that of infinite rectangular blocks.

To a first approximation, and except at small inclinations, the curvature and gradient curves intersect at a point at which the gradient value is one-half of the maximum gradient, as in the case of the rectangular block.

At small inclinations of the face ($\alpha = 10°$ and $170°$) this point coincides almost exactly with the extreme edge of the block, but as the

inclination approaches 90° it moves away outwards from the midpoint of the edge, so that the point of intersection has a tendency to be outside the boundary of the block, rather than within it.

From Fig. 41, it is evident that within fairly wide limits of the inclination, the ratio of gradient to curvature at a definite known distance from the midpoint of the inclined face has a numerical value that is a direct indication of the depth of the block, exactly as in the case of the rectangular block. The numerical values already obtained in the case of the block with vertical face are also applicable in this case.

The depth of the block is a controlling factor and shallow blocks with inclined faces are more readily distinguished than those occurring at depth. The deeper a block of this type is buried, the more nearly does its effect approximate to that of a block with vertical edge, and as the depth increases a greater inclination of the face to the vertical is necessary for it to be distinguished, until at a depth of about four times the thickness of the block it is practically impossible to distinguish an infinite block with an inclined face from the vertical-edge type.

It is not possible, except by comparing the shape of curves such as those given in Figs. 19–24, to make a complete interpretation of an infinite horizontal block with an inclined face by the use of only one of the gravitational elements—gradient and differential curvature. By the combined effect of gradient and curvature, however, it is possible, given reasonably good gradient and curvature profiles, to give a fairly accurate interpretation of the subterranean block and to indicate its position, depth, thickness, effective density, and the inclination of its boundary plane.

PART III.—VARIATION OF TOTAL GRAVITY

On examining the variations in the curve of total gravity for a mass distribution such as has been here considered, it is found that the value of gravity increases as one proceeds from the left through the edge of the denser material of the step or block and reaches its greatest value at an infinite distance to the right.

The curve of the total gravity does not accurately reproduce the form of the disturbing body but furnishes a rough approximation to the position and shape of the gravitational feature, and indicates the actual magnitude of the variation in the total gravity value.

This increase of gravity is given by the formula

$$\Delta g = 2\pi\gamma(\sigma' - \sigma)t$$

where t = the thickness of the step or block
and γ = the gravitational constant.

It appears, therefore, that a change of this magnitude occurs during a traverse from an infinite distance on the left to an infinite distance on

the right, and that this variation is entirely independent of the depth d of the step below the surface of the earth. As γ is of constant value, it would appear that the only two controlling factors are: $(\sigma' - \sigma)$, the density difference between the block and the enclosing rock, and t, the thickness of the step, or of the block. As the density difference $(\sigma' - \sigma)$ is for our purposes regarded as constant and of unit value, the equation is resolved into $\Delta g = 2\pi\gamma t$. In this particular portion of the investigation, the height of the step t is maintained constant (although the depth below the surface is varied considerably) so that one would expect the variation of gravity to have the same constant value in all cases due to an infinite traverse along a line perpendicular to the edge of the block, whatever the depth of the block may be.

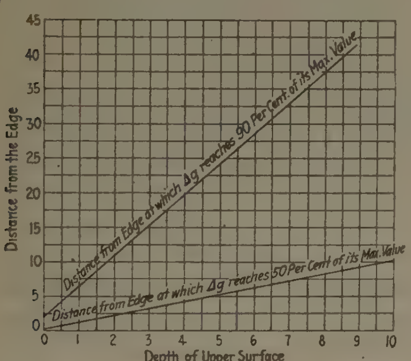


FIG. 46.—TOTAL GRAVITY. EFFECT OF INFINITE RECTANGULAR BLOCK.

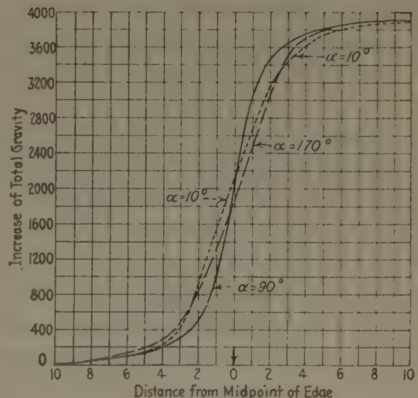


FIG. 47.—TOTAL GRAVITY. EFFECT OF INCLINATION α OF FACE OF AN INFINITE BLOCK ON VALUE OF Δg. DEPTH OF BLOCK $d = 0.05$.

If, for any depth of block, we integrate the gradient curve corresponding to a traverse at right angles to the edge of the infinite step or block, we obtain the total-gravity curve corresponding to this traverse. A comparison of these values obtained for various depths over a wide range serves to establish the constancy of the value irrespective of depth. It is easily seen from the gradient curves that a shallow deposit gives a more rapid change with a steeper slope on the total-gravity curve in the vicinity of the edge, which flattens out rapidly while still near the edge. With a deep deposit the effect is not so sudden and persists over a greater distance.

In actual practice, therefore, it is not necessary to proceed to an infinite distance from the edge of the block in either direction, for as one recedes from the edge the changes in the value of gravity diminish very rapidly, so that when the total-gravity curve is plotted, its steep slope quickly flattens out and becomes practically horizontal.

The maximum theoretical value of Δg due to an infinite block is only obtained at an infinite distance from the edge, but if a definite fraction of this value be considered, a curve can be obtained connecting this value with the depth of the upper edge of the step or block. Curves shown in Fig. 46 indicate the distance from the edge of the block, with various depths at which Δg reaches 90 and 50 per cent. of its maximum theoretical value. Both these curves are in the form of straight lines, which should serve to assist in the interpretation of total-gravity curves for a block or step of this nature.

From the 50 per cent. curve it is obvious that the depth of the upper surface of the step is equal to the horizontal distance from the edge of the point atwhich the total-gravity curve reaches 50 per cent. of its maximum value. This enables the depth of the step below the surface to be estimated, while the 90 per cent. curve would serve to verify this value.

Variation of Total Gravity

As has already been shown, the maximum change in the value of total gravity due to a vertical step of infinite extent is independent of the depth below the surface. Similarly, it may be shown that the maximum variation in the value of total gravity for a block of a definite thickness is independent of the value of α, the slope of the edge. As may be expected, and can be seen from Fig. 47, the slope of the total-gravity curve in the vicinity of the edge varies with the inclination α of the edge, the slope of the curve increasing with the inclination until the latter approaches 90°, where it attains its maximum value. Beyond this value the slope of the curve continually decreases.

If, for any one depth of the step, the total-gravity curves are drawn corresponding to blocks having various inclinations of the edge, a remarkable similarity is noticed. The blocks with faces inclined at small angles give curves which commence to rise earlier than the others, and, having a smaller inclination, are practically straight for a considerable distance before they proceed to flatten out well beyond the midpoint of the edge. The value of total gravity over the midpoint of the edge decreases as the slope of the edge increases from 0° to 180°, but this variation is insufficient to permit of it being usefully employed for the purpose of interpretation.

DISCUSSION

(*Donald H. McLaughlin presiding*)

D. C. Barton, Houston, Tex. (written discussion).—The following observations may supplement in a minor way Captain Shaw's important contribution to the literature on the interpretation of torsion balance surveys:

1. If graphs such as Fig. 3 are plotted on semilogarithmic paper with the gradient or differential curvature plotted logarithmically, the density no longer affects the

amplitude of the curve, but does determine its vertical position on the chart. Until the density is known, and the observed values corrected to those corresponding to sp. gr. 1.0, it is difficult to fit an observed curve to one of the standard curves. If semilogarithmic plotting is used, the difficulty is obviated and the effective specific gravity of the anomalous mass can be determined by the difference of vertical position on the chart of the observed and standard curve. Such semilogarithmic plotting, of course, will not have any advantage in graphs involving the ratio of any gradient to the maximum gradient or the ratio of gradient to differential curvature.

2. In the case of the infinite plate with the vertical face, Shaw mentions that the slope of the tangent to differential curvature profile at its intersection with the axis of OE may be used as a criterion of the depth of the block. For the gradient, the somewhat parallel property is the radius of curvature of the gradient profile at the point of its maximum. Some standard scale of plotting must be used and the observed values must be plotted logarithmically or corrected to those corresponding to a specific gravity of 1.0. Although the radius of curvature is affected by the thickness of the plate in reference to the depth of the plate, the effect of the lower portion of a plate that is thick relatively to its depth is small compared to that of the upper portion of the plate, and for thick blocks a better determination of the depth of the top can be made by the radius of curvature method than by the half maximum gradient method.

3. In connection with the infinite plate with the inclined face: if x_1, x_{max} and x_2 are the abscissas respectively of the upper corner of the block, the point of maximum gradient and the lower corner of the block, if d_1 and d_2 are the depths respectively to the top and bottom of the plate, and if α is the angle of inclination of the face, it can be shown that

$$\frac{x_{\max} - x_1}{x_2 - x_{\max}} = \frac{d_1}{d_2}$$

by using Shaw's formula for the position of the maximum gradient,

$$x_{\max} = \frac{(d_2 - d_1)^2 \cot \alpha}{2(d_2 + d_1)},$$

and remembering that $x_1 = +\frac{1}{2}(T - 2) \cot \alpha$ and $x_2 = -\frac{1}{2}(T - 2) \cot \alpha$.

From formula A, it can be seen that: (*a*) as d_1 approaches zero or d_2 approaches infinity, the point of maximum gradient approaches the position of the upper corner of the face; and (*b*) if d_1 approaches d_2 or, with Shaw's assumption that $d_2 = d_1 + 1$, if d_1 approaches infinity, the point of maximum gradient approaches the position of the midpoint of the face; and (*c*) with the assumption that $d_2 = d_1 + t$ where t is the thickness of the plate, the ratio of formula A may be written $\frac{d_1 + t}{d_1}$ or $1 + \frac{t}{d_1}$, and it can be seen that as t becomes infinitely small compared to d_1, the point of maximum gradient approaches the position of the midpoint of the face, and that as t becomes infinitely large compared to d_1 the point of maximum gradient approaches the position of the upper corner of the face—that is, the point of maximum gradient lies above the upper half of the inclined face of the block and approaches the position of the upper corner of the face as the plate becomes infinitely thick, and approaches the position of the midpoint of the face as the plate becomes infinitely thin.

This point of maximum gradient is also the point of zero differential curvature.

4. The practical applicability of all these pretty properties of a plate infinite in three directions is not as great as their mathematical beauty might lead the unwary to suspect. The unfortunate fact holds that in practice it is very difficult to determine with sufficient accuracy the gradient and differential curvature profiles of a single anomalous mass, for: (*a*) the mass relations of the subsoil commonly are heterogeneous enough to produce an error of at least 1 to $3E$ vectorially in any direction; (*b*) few

structural situations are simple enough to warrant the representation of them by a single homogeneous anomalous mass; (*c*) the gravity anomaly produced by a structure in which one is interested is usually obscured by other anomalies, and commonly it is rather difficult to unscramble the anomalies with sufficient accuracy for quantitative calculations; (*d*) the Fates often seem to conspire to keep one from determining the profile at a critical point. A natural obstacle such as a lake, a swamp, too irregular and too rough topography, or irregular outcrop of some extra dense formation may preclude stations at the critical point, or an obdurate landowner may not allow the instruments on his land, or the exigencies of lease or competitive situations, or the demands of superiors for speed may not allow time for sufficient stations. The result in actual practice is that in general the interpreter has to base his conclusions on inadequate data. But even in situations where the accuracy of the determination gradient and differential curvature profile does not warrant quantitative calculations by such formulas as Captain Shaw's, yet those formulas are valuable in giving a qualitative, approximate idea of the anomalous mass producing the observed gradient, and in most commercial work, determination of the depth and thickness of such an infinite plate with an accuracy ± 25 per cent will be of great practical service. Captain Shaw's paper is well worth the serious attention of all students of interpretations.

C. A. Heiland, Golden, Colo. (written discussion).—The computations of this paper are intended to facilitate indirect methods of gravity interpretations; *i. e.*, methods where we first assume a definite shape of the disturbing formations, compute, as Shaw has done, their effect, compare this theoretical effect with the actually observed effect and change the shape and depth of the subterranean formation until theoretical and observed results check. Shaw has carried out such computations for various cases, so that all observers who have to use similar computations may use his curves without going through the entire calculations. Unintentionally, he has thus made the major portion of the treated indirect method a direct method, as it will often be possible to read now directly from the gradient and curvature curves the approximate subterranean conditions. Nobody so far has computed the effect for so many cases.

The author has dealt in his paper only with the effects produced by formations which are infinite in three directions: in the strike (y direction) and in one direction ($-x$) perpendicular to it. In view of the general title of the paper, as well as of the fact that the author himself has stated expressly in the introductory remarks that it is very dangerous to apply such infinite formations where in reality finite formations should be used, I would suggest that a few words be devoted to the changes which are produced in the curves if the formative approach the finite rather than the infinite case. This is, of course, readily done for formations which are finite in the $-x$ direction, if one end is so far away from the other that at the surface the effect of the far end is the same as the negative effect of the close end. If, however, the ends move closer together, so that the effects of the ends are superimposed, the conditions become somewhat more difficult, and their effect on the amplitude of the anomaly of gradient and curvature must be taken into consideration.

In the same manner, a finite extent in the strike of the formation (y direction) must be taken into consideration. A few words may have been devoted to a discussion of the question how much the extent in the y direction may deviate from infinity without affecting materially the amplitude of the anomaly in each depth of the formation.

H. Shaw (written discussion).—Dr. Barton's suggestion of plotting the curves on semilogarithmic paper is most interesting, but would, I am afraid, present greater difficulties in practice than are at first apparent, for the features which show up so

markedly in the existing curves would be greatly masked. The steep portions of the curves would tend to be leveled out so that increased difficulty would be encountered in fitting an observed curve to one of the standards.

In his third observation, Dr. Barton reaches the same conclusion as is set out in the paper but in a simpler manner, in which he has expressed the conditions both clearly and concisely.

It is, I think, agreed that in practice gravitational anomalies do not occur in such simple and infinite forms as have been treated in this paper, but it is hoped, by an analysis of the gravitational effects of various anomalies, to progress in gradual stages, from these simple cases to increasingly complex ones. In this way it is considered possible to obtain some indication of the gravitational effects to be expected from certain typical formations, and the influence on those results of certain modifications in the general geological conditions. The geophysicist should then be able at a glance to get some idea of the type of formation under investigation, and with a closer comparison of the curves, to make a fairly approximate quantitative interpretation.

In replying to Dr. Heiland's remarks, I should perhaps first point out that the paper in its present form is not to be regarded as a complete treatment of the subject, but rather as the first of a series of papers in which it is proposed to cover anomalies of gradually increasing complexity. Indeed, this method of treatment has already been extended to cover finite bodies both in the x direction and the y direction, and these results, together with others which I have already worked out in great detail, are practically ready for publication.

As mentioned early in the paper, the original intention was to commence with the simplest formations and, progressing through more complicated anomalies, gradually approach the more complex structures. It was quickly realized that this was a long and arduous task, and well beyond the limits of one paper, so that in preference to treating the subject cursorily in one paper, it was decided to undertake a detailed and critical examination of the question, in the hope that the results obtained, as exemplified by the diagrams, may prove useful for purposes of reference to all who are engaged upon such interpretation work.

The present paper of course deals only with the simplest formations, but I anticipate very soon to be able to deal with the irregular features mentioned by Dr. Heiland.

Interpretation of Gravitational Anomalies, II*

By H. Shaw,† London, England

(New York Meeting, February, 1930)

In the author's previous paper an attempt was made to analyze in detail the gravitational effects arising from certain subterranean anomalies of simple form, and extending to infinity in three directions. As a result of this critical examination, certain features—which were considered to be characteristic of the anomalies under investigation—were disclosed, a close observance of which enables a complete interpretation to be reached.

The most simple case was first considered; *viz.*, that of a heavy horizontal plate, bounded above and below by horizontal planes, and extending to infinity on one side of a vertical plane containing the y axis, in the three directions $+y$, $-y$, and $+x$. After this, the investigation was extended to the case of a similar infinite horizontal block having an inclined face.

In each case the form and behavior of the gradient curve and the curve of differential curvature were analyzed, and entirely distinct interpretations were put forward, based upon (1) the gradient, (2) the differential curvature and (3) a comparison of these two quantities. In continuation of the previous work, and as a preliminary to further analysis of more complex structures, certain factors will now be examined which characterize (1) the simple infinite vertical block and (2) an infinite block which is inclined to the vertical, formations which are of considerable practical interest, being typical of important geological structures—intrusive dikes, lodes, veins, etc.—and because, even though in nature these may be irregular, valuable guides to a more complete interpretation are obtained by assuming, as a first approximation, that they have the simple characteristics to be studied hereafter.

In the previous paper certain preliminary assumptions were made in order to render the resulting deductions universally applicable and entirely independent of the particular scale employed. Consideration was limited to homogeneous deposits of unit positive effective density (and this condition is retained in the present investigation), so that in the case of

* This is a continuation of a subject treated by the author in an earlier paper. See page 271.

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any particular anomaly having a different effective density, the corresponding gravity values may be obtained at once by applying the appropriate density factor. It is proposed to continue the practice previously adopted of representing the variations of the gradient by continuous unbroken curves, those of the curvature being shown by broken lines when occurring on the same diagram as the gradient.

The curves are drawn to scale and on the assumption of a unit density difference, the vertical scale is expressed directly in Eötvös units (E = 1×10^{-9} c.g.s. units).

In the case of the gradient profiles, the portions of these curves above the axis (positive gradients) indicate that the gravity gradient is directed towards the right, the reverse being true when the curve is below the axis. Positive values of the curvature (above the axis) correspond to upward concavities of the level surface or to synclinal formations, while the portions below the axis indicate anticlinal formations.

As in the earlier work, consideration will be limited almost entirely to an analysis of the features and characteristics of the gradient and differential curvature, similar methods being employed.

PART I.—INFINITE VERTICAL BLOCK

We will first consider the case of a vertical block rising from an infinite depth and also extending to infinity in the two directions $+y$ and $-y$. The z axis we will take as the axis of symmetry of the section, so that a profile across the slab perpendicular to its length will be along the x axis. For purposes of convenience, the thickness of the block in the direction of the x axis will be maintained constant throughout the investigation, this thickness being regarded as our unit of length. All distances, therefore, such as the depth of the block below the surface, will be expressed in terms of this thickness as unit, so that scale effect is eliminated, and the results are rendered entirely independent of the actual dimensions of the body and of the system of units employed.

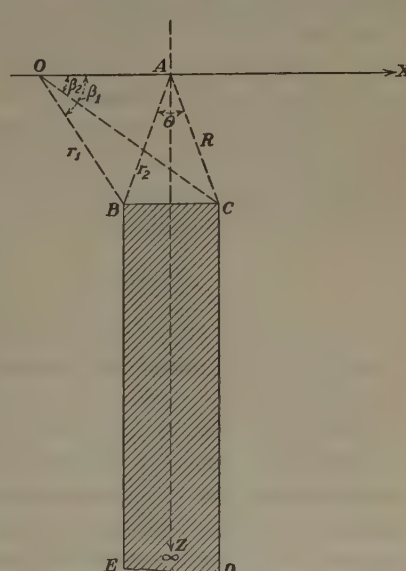

Fig. 1.—Infinite vertical block.

Referring now to Fig. 1, consider a slab bounded by two vertical surfaces BE and CD, terminating above in the horizontal surface BC, and extending to infinity vertically downwards in the direction of the

z axis and also to infinity along the y axis in both directions. It will suffice for our purpose therefore if we examine the gravitational effects over a profile along the x axis; *i.e.*, perpendicular to the infinite length of the block.

In this way the gradient and differential curvature magnitudes are greatly simplified, as all the differentials containing y become zero.

Under these conditions the second differentials acquire the following values.

$$\frac{\partial^2 U}{\partial x^2} = 2\gamma\sigma(\beta_2 - \beta_1) \qquad \frac{\partial^2 \overline{U}}{\partial x \partial y} = 0$$

$$\frac{\partial^2 \overline{U}}{\partial y^2} = 0 \qquad \frac{\partial^2 \overline{U}}{\partial x \partial z} = 2\gamma\sigma \log_e \frac{r_2}{r_1}$$

$$\frac{\partial^2 \overline{U}}{\partial y \partial z} = 0$$

$$\therefore \text{Gradient} = \sqrt{\left(\frac{\partial^2 \overline{U}}{\partial x \partial z}\right)^2 + \left(\frac{\partial^2 \overline{U}}{\partial y \partial z}\right)^2} = \frac{\partial^2 \overline{U}}{\partial x \partial z} = 2\gamma\sigma \log \frac{r_2}{r_1}$$

$$\text{Curvature} = \left(\frac{\partial^2 \overline{U}}{\partial y^2} - \frac{\partial^2 \overline{U}}{\partial x^2}\right) = 2\gamma\sigma(\beta_1 - \beta_2)$$

The angles β_1 and β_2 are expressed in radians.

The values are calculated and plotted along a profile across blocks of different depths between the extreme values of $d = 0.05$ and $d = 20$ times the width of the block. As in the previous case, however, it should be remembered that we are considering infinite layers, and that the conclusions and deductions resulting from this examination are strictly applicable only to infinite structures.

As before, we will consider the three methods of interpretation available by employing (1) the gradient alone, (2) the differential curvature alone, and (3) the combination of these two values.

Gradient Characteristics

From the above formulas it is at once evident that the gradient at a point $+x = -$(gradient at a point $-x$).

Curvature at a point $+x =$ curvature at a point $-x$, as may be seen also from Fig. 2.

The first important difference, therefore, between the infinite horizontal slab and the infinite vertical slab is that in the former the curvature shows a reversal of sign on crossing the edge of the block but the gradient does not, while in the case of the vertical slab, the gradient reverses on crossing the center line without reversal of the curvature.

A comparison of the curves of Fig. 2 with those of Fig. 3 in the previous paper suffices to show that the curves of gradient and curvature have apparently been interchanged. Speaking generally, the gradient curve

has taken up the position previously occupied by the curvature, and vice versa. It is therefore evident from a glance at the gradient and curvature curves in any particular case, whether the gravity anomaly giving rise to these curves approximates more nearly to an infinite horizontal slab or to an infinite vertical one.

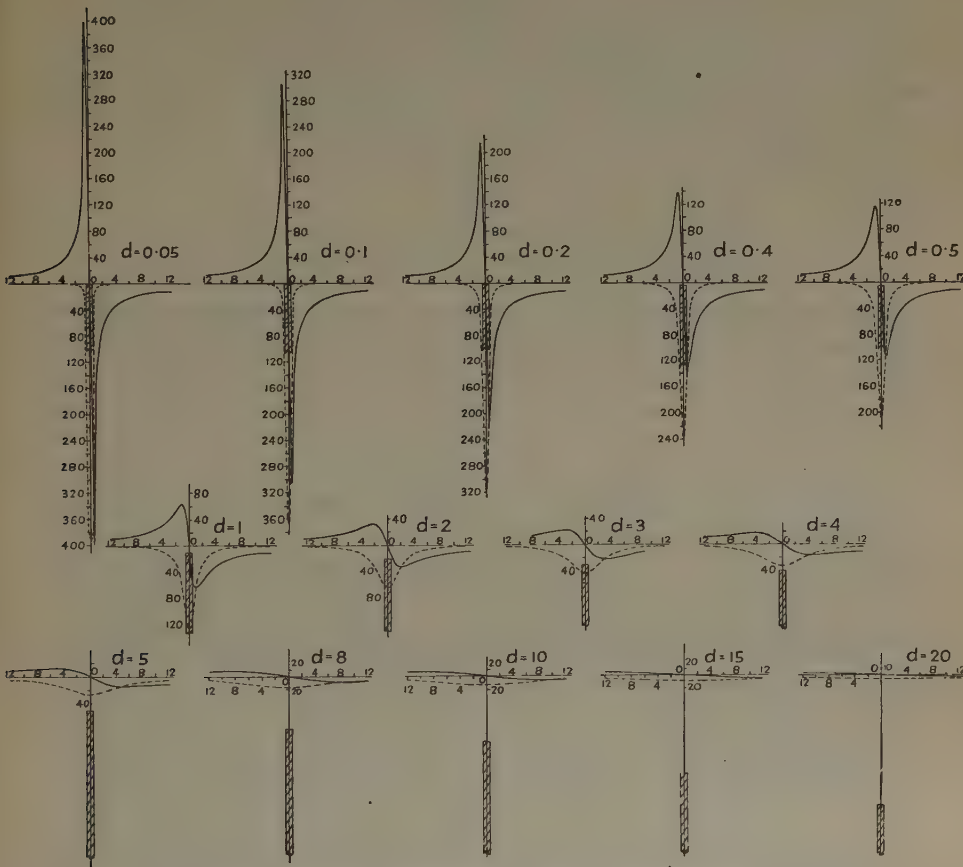

FIG. 2.—EFFECT OF INFINITE VERTICAL BLOCK AT DEPTHS FROM 0.05 TO 20.

For the moment however, we propose to rely entirely upon the gradient, so that the accuracy with which the profile curve of the gradient can be obtained controls the reliability of the resulting interpretation. It is important therefore that the gradient curve should be determined with great care, while in the vicinity of the critical points it is desirable to close up the station interval in order to increase the precision of the resulting deductions.

Probably the most important feature about the gradient curves of Fig. 2 is the symmetry about the point O, vertically over the center line

of the block. This feature is due to the symmetry of the slab about the *YOZ* plane, and any deviation from this symmetry will be immediately rendered evident by distortion of the resulting curves from their existing symmetrical form.

Further, in every case the gradient value changes sign (that is, it reverses in direction) over the midpoint of the block, at which point it has a zero value. As we proceed from this point in either direction, the gradient rapidly attains a maximum value, after which it diminishes again, but less rapidly. The slope also of the central portion of the curve between the maxima is seen to decrease as the depth of the block increases.

Variation of Maximum Gradient with Depth

As may be expected, the value of the maximum gradient varies markedly with the depth of the block below the surface, and this variation is indicated by the lower curve of Fig. 3. The maximum value diminishes rapidly with increase of depth up to about five or six times the width of the block (*i. e.*, five or six units) beyond which depth the maximum gradient value diminishes less rapidly, and eventually becomes asymptotic to the "depth." If, as before, we assume 10E and 5E to be definite limiting values of the gradient that can be determined and recognized with certainty under field conditions, we obtain limiting depths of 6.5 and 13.0 times the width of the block respectively, to which it will be just possible to locate an infinite slab of this form and of unit effective density. In the case of a different effective density, these figures require suitable modification, and as may be seen from Fig. 3, when the maximum gradient becomes small, the depth of the block to which it is due increases very rapidly. As however the density cannot be determined accurately, any estimate of the depth, which is derived from the lower portion of this curve, is likely to be misleading. This point is important when the effective density of the block is greater than unity. Fig. 4 shows the effect of the density upon the limiting depth to which a block may be located, and demonstrates that in all cases the limiting depth varies directly as the effective density.

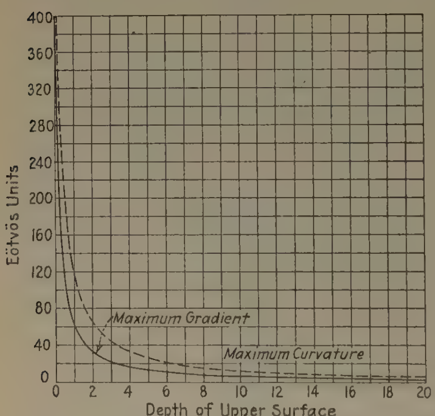

FIG. 3.—RELATION BETWEEN MAXIMUM GRADIENT AND DEPTH OF UPPER SURFACE AND MAXIMUM CURVATURE AND DEPTH OF UPPER SURFACE, OF INFINITE VERTICAL BLOCK.

From the figures given above it is evident that a vertical block extending downwards to a depth of 6.5 or 13 times the width of the block will only just be distinguishable from the infinite block; so that a block

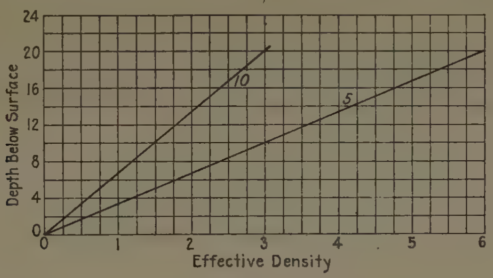

FIG. 4.—LIMITING DEPTH OF LOCATION OF INFINITE VERTICAL BLOCK AT DIFFERENT DENSITIES WITH GRADIENT AT 10E AND AT 5E.

which extends down, say, to 7 or 14 units may for all practical purposes be considered to be infinite, according to whether 10E or 5E is regarded as the limiting detectable gradient.

Position of the Maximum Gradient

The relation between the position of the maximum gradient and the depth of the block is shown in Fig. 5, from which it appears that as the depth is increased the point of maximum gradient recedes from the center line of the block, so that its distance from this central line is approximately equal to the depth of the block below the surface. The curve (Fig. 5) is a straight line for all depths greater than the width of the block and for shallower depths than this, it becomes convex to the depth axis, indicating that the above condition does not hold in the case of shallow blocks in which the depth is less than the thickness.

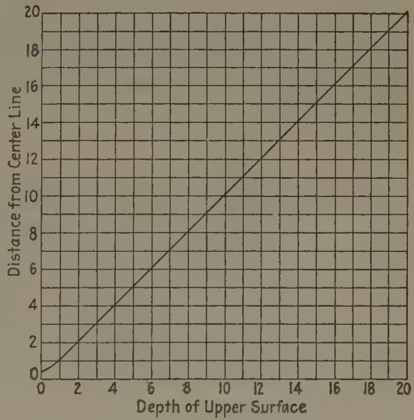

FIG. 5.—DISTANCE OF POINT OF MAXIMUM GRADIENT FROM CENTER LINE OF INFINITE VERTICAL BLOCK WITH VARYING DEPTH.

Normally, two positions of maximum gradient are obtained, one on each side of the block, at which the gradient values are numerically equal but of opposite sign. In general therefore, the horizontal distance between the two points of maximum gradient is equal to twice the depth of the block.

It can be shown that for points at which the gradient has a maximum value

$$x = \sqrt{d^2 + (\tfrac{1}{2}\text{ width of block})^2} = \sqrt{d^2 + \tfrac{1}{4}} \text{ (when the block is of unit width)}$$

or again that $(\beta_1 + \beta_2) = \frac{\pi}{2}$ or $\frac{3\pi}{2}$

while at the point of zero gradient value, on the center line of the block

$$x = 0$$

and
$$(\beta_1 + \beta_2) = \pi$$

Variation of Gradient with Distance from Midpoint

It is possible that in certain circumstances valuable information may be obtained by investigating the positions at which the gradient assumes definite proportions of its maximum value and Fig. 6 shows how points at which the gradients are ½ and ¾ respectively of the maximum value move away from the center line of the block with varying depth. As the gradient increases to a maximum value and then falls away again on each side, it is evident that for each value there will be two points on each side of the center line at which the gradient attains this value, so that for each definite fractional value considered, two curves will be obtained.

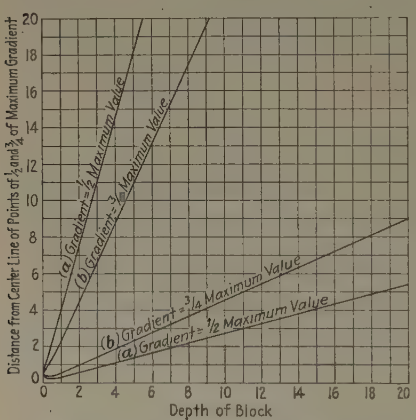

Fig. 6.—Distance from center line of points of infinite vertical block having gradient values a definite fraction of maximum value, when gradient equals one-half maximum value and three-quarters maximum value.

For the examples shown in Fig. 6, the two branches for each particular fraction are complementary. Near the center line of the block and for shallow blocks this relation does not appear to hold, but in cases where either of these values is greater than unity, the approximation is very good, and the inclination of the two branches of the curve may be regarded as complementary.

Gradient Value at Definite Distance from Center Line

If, for points at different distances from the center line, curves are drawn connecting the gradient value and the depth of the block, we obtain Fig. 7. Each curve shows the gradient value at a definite distance from the center line, this distance being indicated (in terms of the thick-

ness of the slab) on each line. In each case an increase of depth is accompanied by a decrease of gradient, rapidly in the case of shallow blocks but more gradually as the depth increases. It is obvious, therefore, from Fig. 7 as well as from Fig. 2, that for a shallow block the greatest gradient effect is obtained near the center line of the block, but when the block is at a greater depth, the maximum effect is obtained at a greater distance from the center line, this distance increasing with the depth of the block.

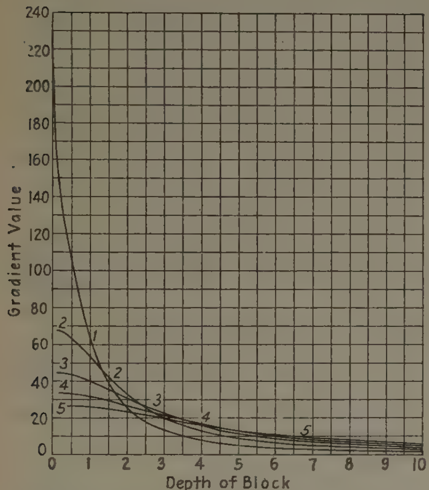

FIG. 7.—VARIATION OF GRADIENT WITH DEPTH OF INFINITE VERTICAL BLOCK WITH DEPTH OF BLOCK AT FIXED POINT. Horizontal distance of point from edge of block is indicated on each curve.

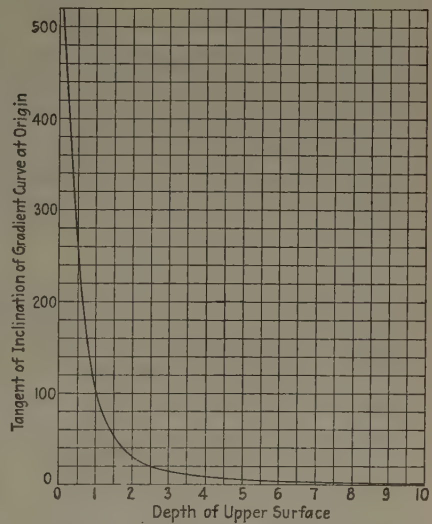

FIG. 8.—CURVE CONNECTING TANGENT OF INCLINATION OF GRADIENT CURVE OF INFINITE VERTICAL BLOCK AT ORIGIN, AND DEPTH OF BLOCK BELOW SURFACE.

Inclination of the Gradient Curve at the Origin

As may be seen from Fig. 2, the gradient attains a zero value in every case at a point vertically over the center line of the block, and the inclination of the gradient curve in the vicinity of this point varies considerably with the depth of the block. In Fig. 8 the tangent of this inclination has been plotted against the depth of the block, and the resulting curve shows a very rapid variation of this value at depths up to about 3 or 4 times the width of the block, but at greater depths the curve becomes flattened too much to permit of a reliable estimate of the depth being made in this way.

Gradient Interpretation

For an infinite vertical slab of the type under consideration, it is possible to give a complete interpretation from an examination of the

gradient curve alone, the following characteristics being of importance in this connection:

1. The symmetry of the gradient curve indicates the symmetry of the disturbing feature.

2. The gradient value is zero at a point vertically over the center line of the body.

3. The midpoint between the two positions (numerically equal) of maximum gradient lies vertically over the center line of the body.

4. The depth below the surface is given approximately by the distance from the point of zero gradient to the point of maximum gradient, or one-half of the distance between the two points of maximum gradient, except in the case of shallow blocks, in which the depth is less than the thickness.

5. A comparison of the gradient curve with the curves of Fig. 2 enables the ratio of the depth to width of the block to be ascertained, and this, together with a knowledge of the depth, gives an indication of the width of the slab.

6. The effective density of the slab or block may be ascertained from a direct comparison of the numerical value with those of the corresponding example of Fig. 2.

7. An approximate estimate of the depth (for purposes of verification) may be obtained from the slope of the gradient curve in the vicinity of the center line. An estimate of the depth by this method is reliable only up to a depth of from 3 to 4 times the width of the slab.

It appears to be possible therefore from the gradient profile alone to locate a point vertically over the center line of the block, and also to determine the depth, width and density of such a slab, if the gradient curve is known with sufficient accuracy. The reliability of the interpretation will naturally be dependent upon this accuracy and will be controlled by the closeness of the station network, especially in the vicinity of the points of zero gradient and maximum gradient. It is essential therefore that the position of these three points should be determined as carefully as circumstances permit, in order that the resulting interpretation may be reliable.

Characteristics of Differential Curvature

Fig. 2 shows that the differential curvature function is in all cases symmetrical about the center line of the slab, and always of the same sign. From this it appears that the curvature vector is always parallel to the face of the slab, and perpendicular to the line of traverse.

Variation of Maximum Curvature with Depth

It is obvious that for a vertical block of the type under consideration, the gravitational effects at points on the earth's surface will depend largely on the depth at which the block lies below the surface.

The numerical value of the maximum curvature is given by: Maximum Curvature = $2\gamma\sigma\theta$ (Fig. 1). If now this maximum value of the curvature is plotted against the depth, we obtain the upper curve of Fig. 3, which is asymptotic to both the horizontal and vertical axes. This maximum value falls away rapidly as the depth increases up to about 8 or 9 times the width of the block, beyond which the decrease becomes more gradual. If, as in the case of the gradient, 10E and 5E are assumed to be the minimum curvature values that in practice can be detected with certainty, it follows from Fig. 3 that, with an effective density difference of unity, such an infinite vertical slab may under favorable conditions be located by means of the curvature values to a depth of 13 and 25 times the width of the block. In the event of the effective density differing from unity, a corresponding modification of these figures would be necessary, the limiting depth in all cases being directly proportional to the effective density, as is evident from Fig. 9.

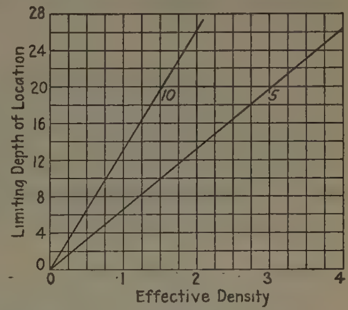

FIG. 9.—LIMITING DEPTH OF LOCATION OF INFINITE VERTICAL BLOCK AT DIFFERENT DENSITIES WITH CURVATURE VALUE OF 10E AND 5E.

A comparison of these figures with those obtained for the gradient would appear to indicate that under similar conditions an infinite vertical block can be located by means of the curvatures down to a depth equal to twice that which is possible in the case of the gradient. It should be emphasized however that curvatures can be employed efficiently only in relatively flat and unbroken ground, and any departures from these ideal conditions, even at a distance, have a far greater disturbing effect on the curvature than on the gradient.

Position of Maximum Curvature

It is obvious that the curvature value reaches its maximum numerical value over the center line, or at the point where $x = 0$, and the curves of Fig. 2 illustrate that for an infinite vertical block the curvature function is symmetrical about the center line of the block. The magnitude of the maximum curvature value decreases rapidly as the depth of the block is increased, and this depth is obviously the controlling factor. In the case of a shallow block the curvature falls away rapidly in the immediate vicinity of the center line, but in the case of a similar block buried at greater depths, not only are the variations numerically smaller, but they also change more gradually and extend laterally to much greater distances. In the case of shallow blocks, therefore, a large curvature value is obtained which falls off so rapidly that it is evident over only a

short distance, while on the other hand, the effect of a deep-seated block is to give rise to smaller curvature values, which however change only gradually, but are distinguishable over considerable distances.

The general shape of the curve obtained in any particular case may be compared with the curves of Fig. 2, when the ratio of depth to width of block may be estimated fairly closely, after which the effective density of the block can be obtained by a direct comparison of the numerical values.

Variation of Curvature with Distance from Center Line

If now we take certain definite proportions (*e. g.*, ½ or ¾) of the maximum curvature value, and in each case draw a curve connecting the distance of this point from the center line with the depth of the block, we obtain the series of curves shown in Fig. 10, from which it is at once evident that the distance from the center line bears a definite relation to the depth of the block. As with the infinite horizontal block, the curve indicating the points of one-half maximum curvature is of particular interest, for it furnishes a simple means of determining the depth of the block. An inspection of this curve shows that for all depths greater than about one, the ordinates and abscissas are for all practical purposes equal, so that the distance from the center line may be regarded as equal to the depth of the block. If therefore two positions have been located at which the curvature is equal to one-half its maximum value, one on either side of the point of maximum curvature, not only will the center line of the block be known, but also its depth below the surface. In the case of a shallow block however, in which the depth is less than the width of the block, this approximation is less accurate, the error increasing with the shallowness of the block. At a depth of 1.5 units, the error in estimating the depth of the block in this way is approximately 2 per cent., while the error is increased to approximately 10 per cent. when the depth is reduced to unity, beyond which point it would be inadvisable to employ this means of estimating the depth.

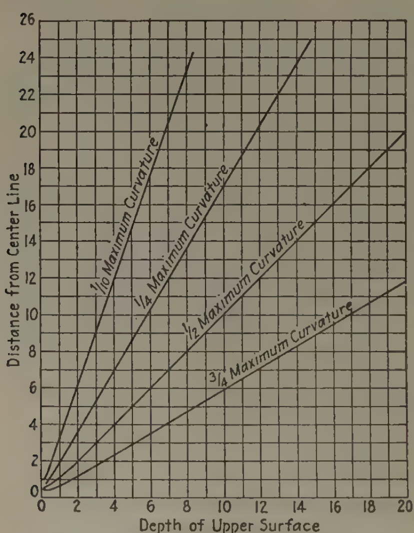

Fig. 10.—Distance from center line of infinite vertical block at which curvature assumes definite fraction of its maximum values.

It can be shown (Fig. 1) that the condition to be satisfied at the point of one-half maximum curvature is

$$x = R = \sqrt{d^2 + \tfrac{1}{4}}$$

or

$$(\beta_1 + \beta_2) = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}$$

and as this is precisely the condition to be satisfied for the point of maximum gradient, we shall return later to a consideration of the characteristics of this point.

Positions at Which Curvature Attains Definite Values

Fig. 11 shows the distance from the center line at which the curvature falls to certain definite numerical values, and the variation of this distance with the depth of the block. This enables us to see immediately the

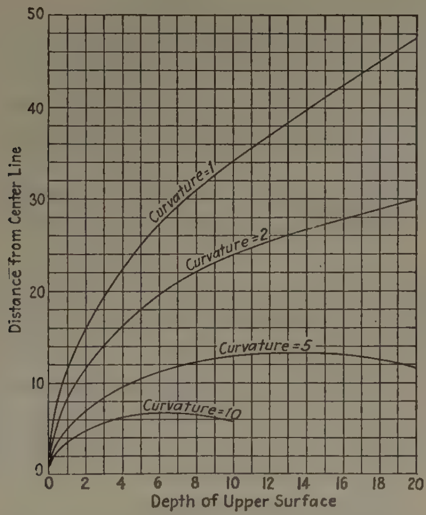

FIG. 11.—DISTANCE FROM CENTER LINE OF INFINITE VERTICAL BLOCK AT WHICH CURVATURE BECOMES EQUAL TO DEFINITE NUMERICAL VALUES.

distance from the center line at which any given vertical slab will exert a definite known effect, while for a block of known density the value most closely approximating to the one obtained in practice can be chosen or interpolated from these curves. In every case, as the depth of the block increases from zero the curve rises, indicating that the distance from the center line at which the curvature attains a definite value increases with the depth of the curve. After reaching a definite maximum value, the curves tend to fall again, but it is only with higher curvature values that this becomes evident.

Curvature Interpretation

The foregoing analysis of the curvature effects resulting from an infinite vertical block would appear to indicate the possibility of furnishing a complete interpretation of the anomaly from an examination of the curvature values only, as follows:

1. The symmetry of the curvature value indicates the symmetry of the anomaly.
2. The maximum curvature is found vertically over the center line of the block.
3. The depth below surface of the anomaly is equal approximately to the horizontal distance between the point of maximum curvature and the point of one-half maximum curvature, except in the case of very shallow blocks.
4. The horizontal distance between the two points at which the curvature attains one-half its maximum value is equal to twice the depth of the anomaly except in the case of shallow bodies, a point midway between these positions being vertically over the center line of the block.
5. A comparison of the shape of the curvature curve with the curves of Fig. 2 enables the ratio of depth to width to be ascertained, and this, together with a previous knowledge of the depth (from either No. 3 or No. 4), gives an indication of the width also.
6. The effective density of the block may be evaluated from a direct comparison of the numerical values with those of the corresponding curve of Fig. 2.

Therefore an anomaly of the type under consideration may be determined completely from a consideration of the curvature results alone, without any reference to the gradient values. As in the case of the gradient however, the reliability of the interpretation is dependent upon the accuracy of the curve of differential curvature values and the closeness of the station network in the vicinity of certain critical points.

Comparison of Gradient and Curvature Effects

It is evident from Fig. 2 that for any depth of the block the maximum curvature value is greater than the maximum value of the gradient, while a closer comparison of these two curves shows that in nearly every case the maximum curvature value is exactly twice that of the maximum gradient.

As the depth of the block becomes shallow however, this relation ceases to hold, and while the maximum gradient value approaches infinity, the limiting value of the differential curvature, when the block reaches the surface, is

$$
\begin{aligned}
2\gamma\sigma(\beta_2 - \beta_1) &= -2\gamma\sigma\pi \\
&= -420 \text{ E (approx.)}
\end{aligned}
$$

so that near these limiting conditions the maximum gradient is greater than the maximum curvature.

Intersection of Gradient and Curvature Curves

If, as in Fig. 2, the curves representing the gradient and the curvature effects for any particular block are superimposed, they intersect at one point only. At all points between the center line and the point of intersection the curvature has a larger numerical value than the gradient, while beyond this point the gradient is of the greater magnitude.

In an infinite vertical block of the type under consideration, therefore, the gradient will be more readily detected at a distance than the curvature, while the latter will be more pronounced in the immediate vicinity of the block.

In Fig. 2 the curves of gradient and curvature intersect in each case at the point of maximum gradient, and as the maximum gradient is equal to one-half maximum curvature it is obvious that at the point of intersection of the two curves the value of the curvature will be equal approximately to one-half the maximum curvature value while the gradient will be at its maximum value. As may be seen from Fig. 12, the distance of this point of intersection from the center line is equal to the depth of the block, except in a very shallow anomaly, so that we have here a point of unusual importance which possesses the following characteristics:

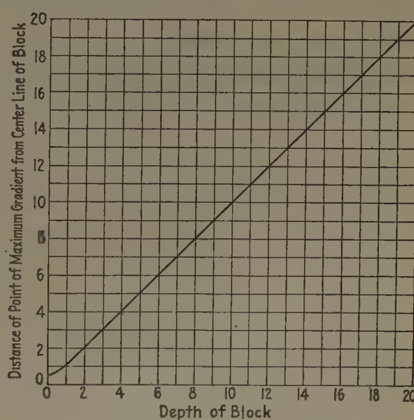

FIG. 12.—DISTANCE FROM CENTER LINE OF INFINITE VERTICAL BLOCK OF POINT OF INTERSECTION OF GRADIENT AND CURVATURE CURVES WITH VARYING DEPTH OF BLOCK.

1. It is the point at which the curvature reaches one-half its maximum value.
2. It is the point of maximum gradient.
3. It is the point of intersection of the gradient and curvature curves or the point at which the gradient and curvature values are numerically equal.
4. The distance of this point from the center line is equal to the mean depth of the block, except where the depth is less than the width of the block when the point of intersection is at a greater distance from the center line than the depth.

Relation between Gradient and Curvature Values

Valuable indications of the positions and depth of any block may be obtained from a consideration of the ratio between the gradient and

curvature values along a traverse across the anomaly. If along such a profile the ratio of gradient to curvature is plotted, the resulting curve approximates to one or other of the curves of Fig. 13, from which a close estimate of the depth of the block may be obtained.

If we take as the general equation of the line

$$r = ml + c,$$

where r = ratio of gradient to curvature,
m = tangent of angle of slope,
l = distance from center line,
c = value of gradient curvature ratio at origin,

then from the curves of Fig. 13 we get the values given in Table 1, so that the product is approximately equal to unity in all except the very shallowest cases.

TABLE 1

Depth, d	m	c	$d \times m$
20	0.05		1.0
10	0.1		1.0
8	0.125		1.0
6	0.167		1.0
4	0.25		1.0
3	0.33		1.0
2	0.5		1.0
1	1.0		1.0
0.5	2.0	− 0.2	1.0
0.2	5.3	− 1.3	1.06
0.1	11.5	− 4	1.15
0.05	40.6	−20	2.03

Interpretation may also be assisted by reference to Fig. 14, in which the variation of the ratio of gradient to curvature with the depth is shown for a fixed point, at a definite distance from the center line. Each curve corresponds to the conditions at a definite point and the distance of this point from the center line of the block is indicated on the corresponding curve. Each curve of this figure forms one branch of a hyperbola, so that throughout each curve the product of

$$\text{Depth} \times \frac{\text{Gradient}}{\text{Curvature}} \text{ ratio} = \text{Constant},$$

the value of the constant being different for each curve. This constant is in all cases equal to the number shown on the curve (which is the

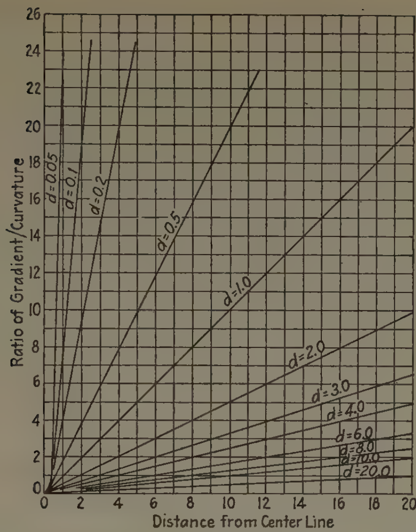

FIG. 13.—RELATION BETWEEN GRADIENT/CURVATURE RATIO OF INFINITE VERTICAL BLOCK AT ANY POINT AND THE DISTANCE FROM CENTER LINE FOR DIFFERENT DEPTHS OF BLOCK.

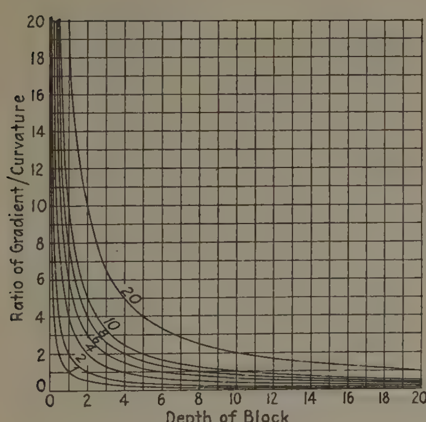

FIG. 14.

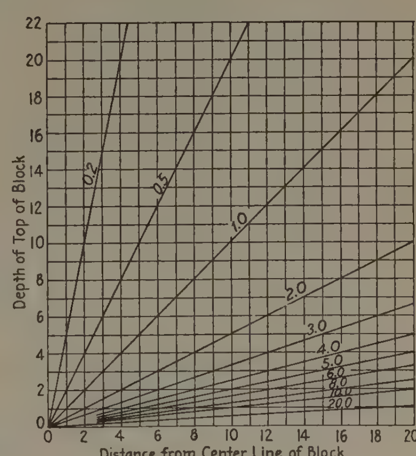

FIG. 15.

FIG. 14.—VARIATION WITH DEPTH OF GRADIENT/CURVATURE RATIO OF INFINITE VERTICAL BLOCK AT FIXED POINT.

Horizontal distance of point from center line of block is indicated on each curve. All curves are hyperbolas; that is, depth $\times$ gradient/curvature = constant.

FIG. 15.—RELATION BETWEEN DEPTH OF INFINITE VERTICAL BLOCK AND DISTANCE FROM CENTER LINE FOR POINTS HAVING SAME VALUE OF GRADIENT/CURVATURE RATIO.

Gradient/curvature value is indicated on each line.

horizontal distance from the center line of the block), so that we have in general

$$\text{Depth} \times \frac{\text{Gradient}}{\text{Curvature}} \text{ ratio} = \text{horizontal distance from the center line}$$

or $\frac{\text{Gradient}}{\text{Curvature}}$ ratio at any point = horizontal distance of that point from the *center line*, divided by the depth of the block.

For purposes of interpretation it is frequently desirable to determine the gravitational effects of a known anomaly under given conditions, in order to reconstruct a subterranean body which shall produce gravitational effects agreeing as closely as possible with the observed values. Further information is given in a suitable form for this purpose in Fig. 15, from which the ratio of gradient to curvature may be obtained at once under any conditions, or conversely, if this ratio is known the distance from the center line may be determined.

As may be seen from Fig. 15, these curves are all linear, the inclination of the individual members being given in Table 2.

TABLE 2

(1) $\frac{\text{Gradient}}{\text{Curvature}}$ Ratio	(2) Slope of Curve (Tangent of Inclination of Curve)	(3) Column 1 × Column 2
20	0.05	1.0
10	0.1	1.0
8	0.125	1.0
6	0.167	1.0
5	0.2	1.0
4	0.25	1.0
3	0.33	1.0
2	0.50	1.0
1	1.0	1.0
0.5	2.0	1.0
0.2	5.0	1.0

It is evident therefore that for an infinite vertical block the product at any point on the surface of

$$\frac{\text{Gradient}}{\text{Curvature}} \text{ ratio} \times \frac{\text{depth of block}}{\text{distance from center line}} = 1$$

as we have already seen above. Another interesting feature worthy of notice here is the difference between the infinite horizontal block and the infinite vertical block. In the former, the gradient always predominates in the immediate vicinity of the edge of the block but in the vertical block the differential curvature function is the greater, while at a distance from the center line exceeding the depth of the block this condition is reversed.

Combined Gradient and Curvature Interpretation

We have already examined the possibility of obtaining a complete interpretation of a gravity anomaly of this type from a consideration of the gravity gradient alone, and also a similar possibility of reaching the same complete interpretation from an analysis of the curvature effects only. It will be evident therefore that there will be little difficulty in interpreting the anomaly when both the gradient and curvature values are known.

We have seen that under favorable conditions, although both the gradient and curvature values are capable of giving a complete and independent solution, neither is capable of furnishing any information concerning an infinite vertical block which is not readily obtainable from the other. So long, therefore, as one of these values is known in its entirety, a full and detailed interpretation can be made, and the only advantage to be derived from a knowledge of both the gradient and curvature values is to provide a check on the interpretation.

Theoretically the gradient and curvature are of equal assistance in presenting a complete interpretation of this type of infinite block, but in practice extraneous effects frequently prove very troublesome, the difficulties being greater with the differential curvature than with the gradient. Consequently, there is a tendency to place greater reliance upon the gradient than the curvature in practice, and any deductions resulting from a comparison of these two magnitudes should be considered with due respect to this fact.

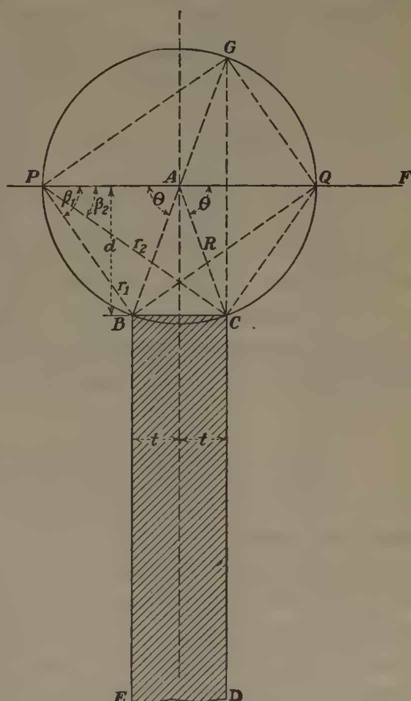

Fig. 16.—Geometrical construction for use in interpretation.

Geometrical Construction

The following geometrical construction may be employed for solving the problem of interpretation of an infinite vertical block, when the positions of maximum gradient and of maximum curvature are known.

In Fig. 16 let A be the point of maximum curvature and zero gradient and let P and Q be the points of maximum gradient and one-half maximum

curvature. With center A and radius PA draw a circle passing through P and Q. Comparing the shape of the gradient and curvature curves available with those of Fig. 2 enables an estimate of the depth d to be made in terms of the thickness of the block. From the center A, two radial lines AC and AB are drawn to cut the circle at the points C and B, so as to make

$$\angle QAC = \angle PAB = \theta = \tan^{-1} 2d$$

Join BC, and from these points draw vertically downwards to E and D. Then $BCDE$ is the infinite vertical block.

Proof.—Produce DC upward to cut the circle again at G. Join BP, CP, BQ, CQ. Then

$$\angle QPC + \angle QPB = \angle QPG + \angle QPB = \frac{\pi}{2} \text{ for the point } P$$

Similarly

$$\angle PQC + \angle PQB = \angle PQG + \angle PQB = 2\pi - \frac{\pi}{2} = \frac{3\pi}{2} \text{ for the point } Q$$

which are the conditions for the two points of maximum gradient.

Also $$\angle QAB + \angle QAC = \angle QAB + \angle QAG = \pi$$

which is the condition for the point of maximum curvature.

A direct comparison of the maximum curvature and gradient values, with the corresponding curves of Fig. 3, gives an indication of the effective density of the block, so that the anomaly is then completely known.

Comparison of Horizontal and Vertical Infinite Blocks

If we consider two infinite blocks at the same depth and of unit thickness, one placed vertically and the other horizontally, it will appear that for relatively small depths the vertical block is more readily located than the horizontal block either by the gradient or the curvature, but at a depth of approximately five times the thickness, this superiority of the vertical block disappears. When employing the gradient only it is possible to locate the horizontal block down to twice the limiting depth of the vertical block, but when curvature is used the penetration in the case of the vertical block is twice that for the horizontal block to give the same maximum curvature value.

PART II.—INFINITE INCLINED BLOCK

If the block previously considered is inclined to the vertical, we get the case of an inclined dike or lode, which occurs more frequently in practice, and it may be of advantage to compare the characteristics of the gravitational effects resulting from such an anomaly with those of the vertical block.

As in previous cases, calculations of these effects have been made for blocks at a number of definite depths, and in each case a complete computation has been made at inclination intervals of 10° from 90° to 0°, those between 90° and 180° being readily obtained from these. The

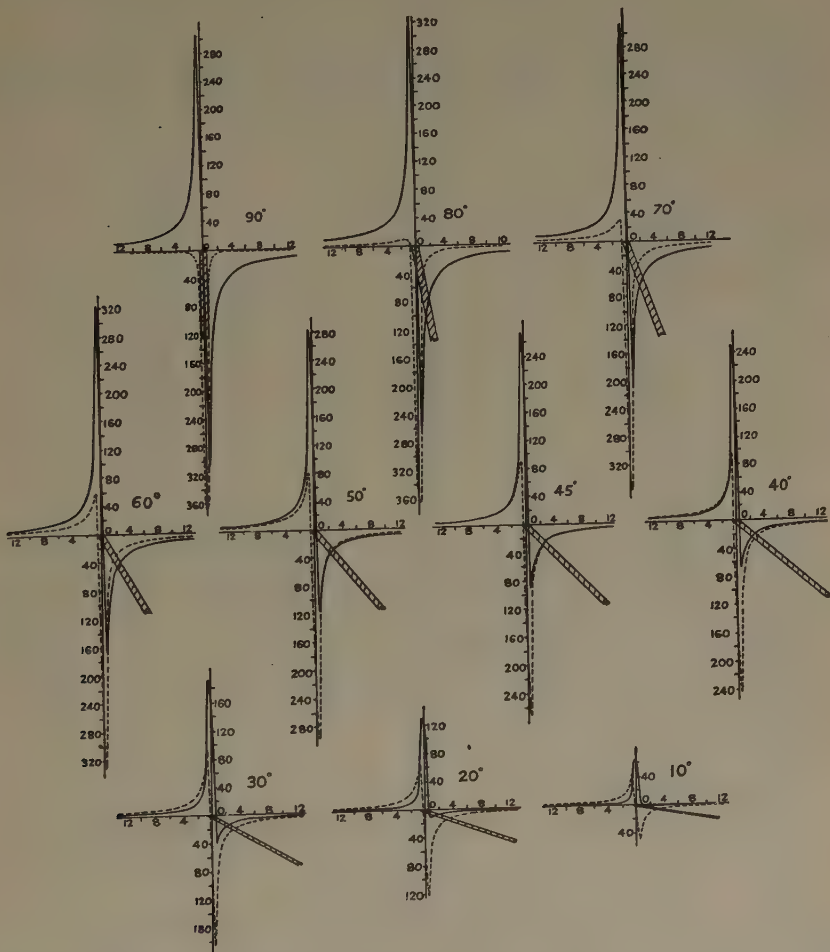

FIG. 17.—EFFECT OF VARIATION OF α AT CONSTANT DEPTH, $d = 0.1$, INFINITE INCLINED BLOCK.

curves giving the results of these computations are shown in Figs. 17 to 21.

The five depths chosen in this case are $d = 0.1, 0.2, 0.5, 1.0$ and 2.0, which cover the range considered to be most useful in actual interpretation work, while the practice of representing gradients by continuous lines and curvatures by dotted lines has been continued.

Gradient Characteristics

The gradient at any point due to an infinite block of this type (Fig. 22) is given by the expression:

$$\frac{\partial^2 \overline{U}}{\partial x \partial z} = \left\{ (1 - \cos 2\alpha) \log \frac{r_2}{r_1} - \sin \alpha \, (\beta_2 - \beta_1) \right\} \times \gamma$$

and as the angle α departs from 90° the curves both of gradient and differential curvature lose their symmetry and assume definite characteristic forms.

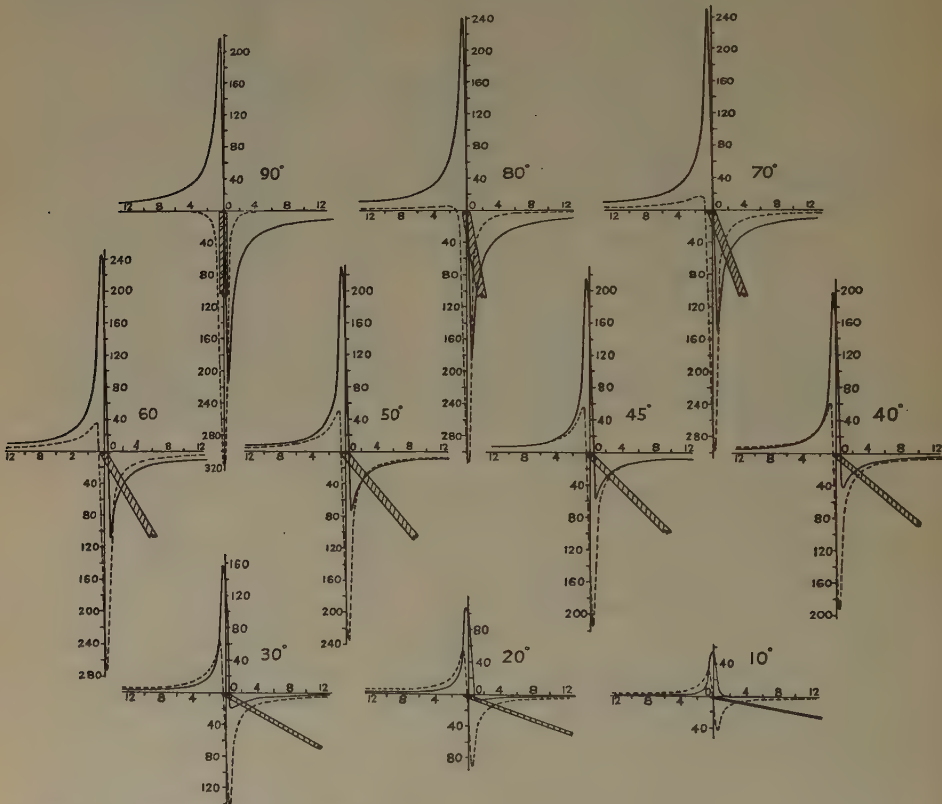

Fig. 18.—Effect of variation of α at constant depth, $d = 0.2$, infinite inclined block.

The gradient curve on the side towards which the dike dips decreases rapidly in comparison with the other portion of the curve, until in all but the shallowest cases the maximum value in this portion of the curve has fallen to not more than 10 per cent. of the other maximum value by the time that α has decreased to 30°.

Figs. 17 to 21 show that as the dip of the block decreases the gradient curve tends to lean over away from the direction of dip while the right-

hand portion of the curve gradually diminishes in magnitude. With low values of α this portion of the gradient curve disappears altogether, so that under these conditions the gradient curve lies entirely above the base line, indicating that no reversal of gradient occurs on traversing an anomaly presenting such features as are here indicated.

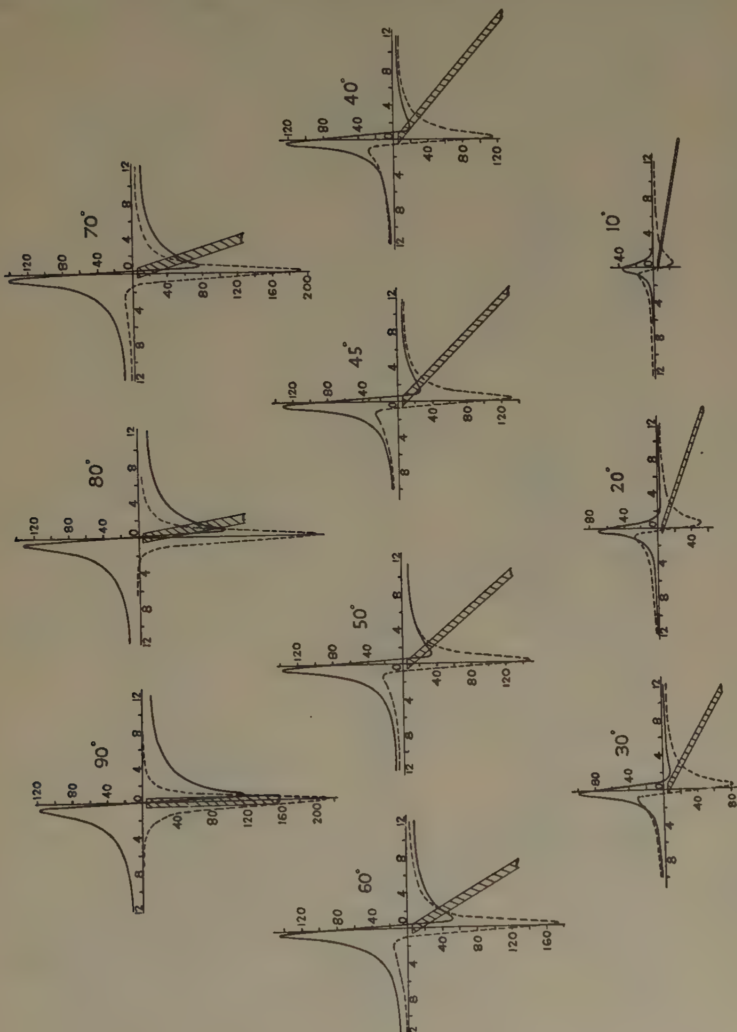

FIG. 19.—EFFECT OF VARIATION OF α AT CONSTANT DEPTH, $d = 0.5$, INFINITE INCLINED BLOCK.

If a vertical line is drawn through the midpoint of the upper surface of the block, which we will refer to as the origin, the position of the first maximum gradient gradually approaches this center line with decrease of α, while the position of the second maximum gradient at the same time recedes from the origin, but at a greater rate so that the intermaximum

distance is always greater than twice the depth of the block below the surface.

Value of Maximum Gradient

In an inclined dike at any given depth from the surface the maximum value of the gradient varies considerably as the inclination α of the dike

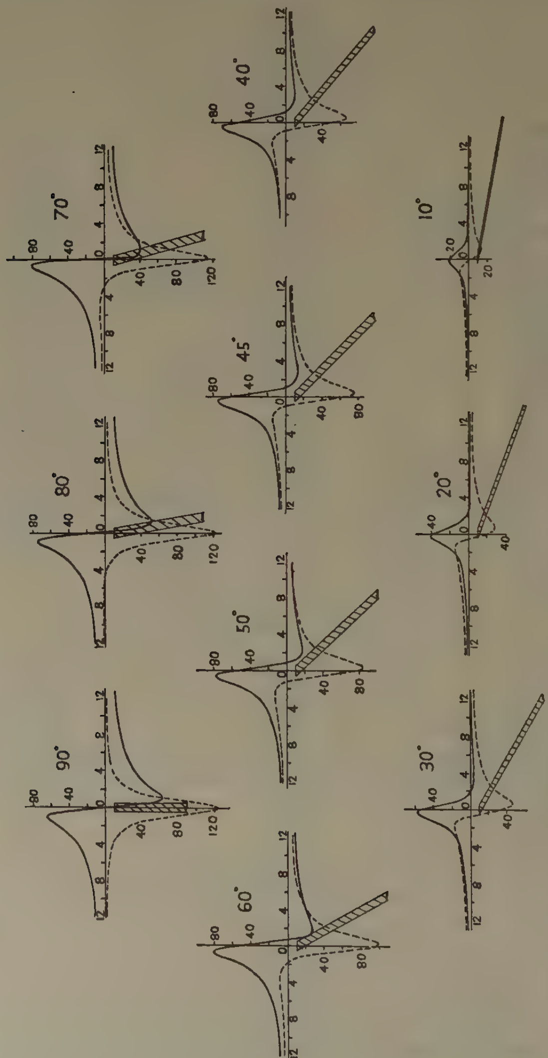

Fig. 20.—Effect of variation of α at constant depth, $d = 1.0$, infinite inclined block.

to the horizontal changes. In Fig. 23 this value of the maximum gradient is plotted against the inclination α, the full lines show the variation of the maximum gradient, on the side away from the dip. This will be

referred to as the first maximum gradient, or simply the maximum gradient. The maximum gradient reached on the other side of the dike, (referred to as a second maximum gradient) is indicated by the dotted curves.

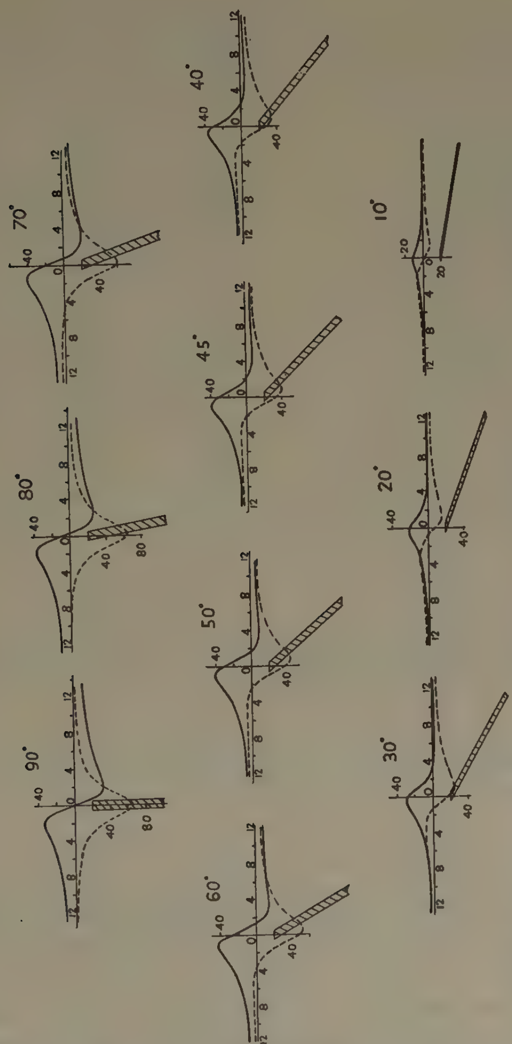

FIG. 21.—EFFECT OF VARIATION OF α AT CONSTANT DEPTH, $d = 2.0$, INFINITE INCLINED BLOCK.

As the dike departs from the vertical the maximum gradient increases in value, and the angle at which its maximum value is reached varies with the depth of the anomaly, but within the range of depths investigated appears to lie between the angles $\alpha = 75°$ and $\alpha = 50°$. As the inclination decreases beyond this point, the maximum gradient value falls rapidly in every case.

The second maximum gradient, on the other hand, diminishes rapidly and continuously as the inclination departs from the vertical, at which inclination it has a value equal to that of the first maximum gradient.

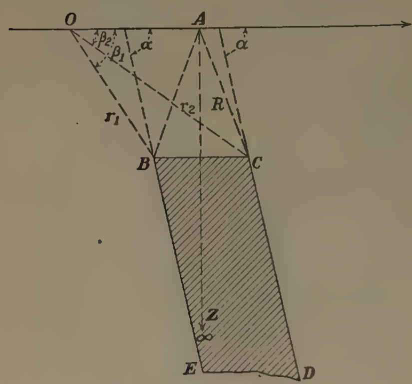

FIG. 22.

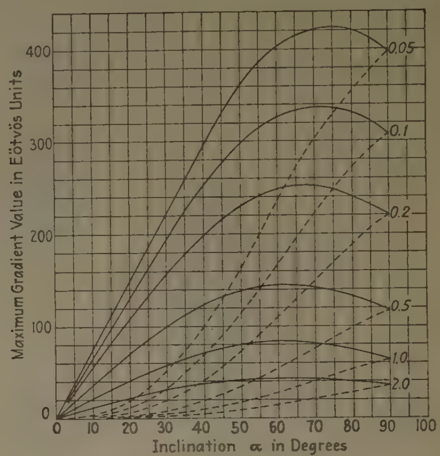

FIG. 23.

FIG. 22.—INFINITE INCLINED BLOCK.

FIG. 23.—VARIATION OF MAXIMUM GRADIENT WITH INCLINATION OF BLOCK. Depth of block is indicated on each curve. Dotted curves indicate variation of second maximum gradient.

Position of Maximum Gradient

It may be shown that the conditions for maximum gradient values are given for an infinite block inclined at an angle α to the horizontal.

$$(\beta_1 + \beta_2) = n\pi + \alpha$$

from which we see that there are two positions of maximum gradient, as would be expected from previous considerations.

Gradient over Midpoint of Block

Nikiforov[1] has developed a method of determining from the gradient curve the midpoint of the upper surface of an inclined block rising from an infinite depth, and has shown that the gradient at this point is the algebraic sum of the first and second maximum gradient values.

The method adopted is to add algebraically the two maximum values of the gradient and then to find the point lying between the positions of these maxima at which the gradient is equal to this algebraic sum.

If the gradient at a point on the surface vertically over the midpoint of the block is plotted against the inclination α of the block, an interesting

[1] Nikiforov: Physical Principles of the Gravitational Method of Prospecting. *Bull.* Inst. Prac. Geophys. Leningrad (1925) **1,** 153–257.

series of curves is obtained (Fig. 24). It is at once evident that the gradient at this point always has its maximum value when $\alpha = 45°$; furthermore, that corresponding to any smaller value of the gradient, there are always two values of α which are in every case complementary: *e. g.*, in an inclined infinite block of depth $d = 0.5$, the gradient at a point on the surface over the midpoint of the block when $\alpha = 70°$ is 67E, but the same gradient value would also be obtained at this point if $\alpha = 20°$.

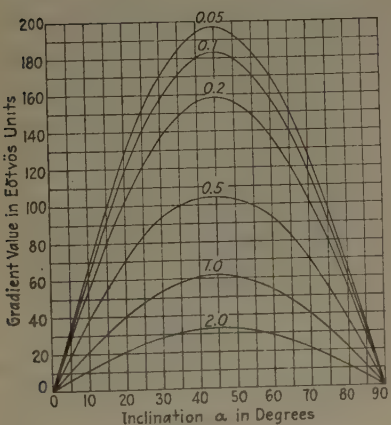

FIG. 24.—GRADIENT OF INFINITE INCLINED BLOCK AT POINT VERTICALLY OVER MIDPOINT OF BLOCK.

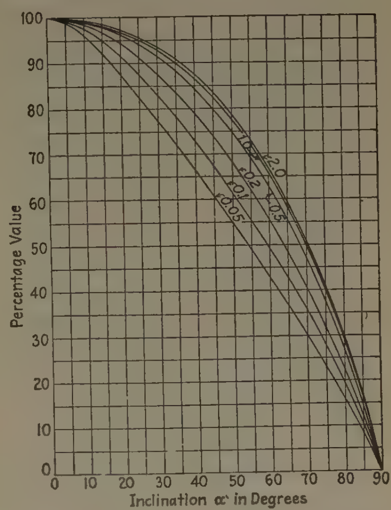

FIG. 25.—GRADIENT OF INFINITE INCLINED BLOCK AT ORIGIN, EXPRESSED AS PERCENTAGE OF MAXIMUM GRADIENT.

Depth is indicated on each curve.

The gradient value at a point over the midpoint of the block, when expressed as a percentage of the first maximum gradient, gives the curves of Fig. 25, from which it may be possible under favorable conditions to estimate the inclination of a block when the depth is known, and vice versa. For the shallowest block, $d = 0.05$, the curve approximates fairly to a straight line, but as the depth increases the deviation of the corresponding gradient curve from the straight line increases also.

Position of Zero Gradient

From Fig. 26 it is apparent that as the inclination of the dike decreases, the position of the point of zero gradient moves in the direction of dip. This motion is less pronounced with shallow bodies than with deeper ones, but even for a dike at a depth of only 0.05, the point of zero gradient moves beyond the upper edge of the block when the value of α has been reduced to about 28°. With a block of depth $d = 2$ a value of $\alpha = 76.5°$ would produce a similar displacement of the point of zero

gradient, while if the inclination of such a block were reduced to 45° the displacement would be equal to twice the width of the block. Such a displacement is of the greatest importance in practice, when in order to test the results of a gravity survey, locations are being made for suitable test bore holes. A bore hole sited at the point of zero gradient in the above case would not reach the dike even at a depth of 8, although the latter approaches to within 2 units of the surface.

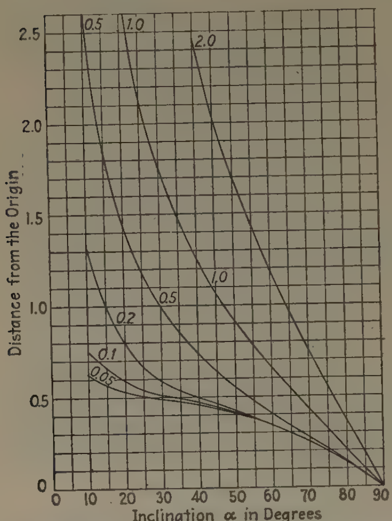

FIG. 26.—DISTANCE OF POINT OF ZERO GRADIENT OF INFINITE INCLINED BLOCK FROM ORIGIN.

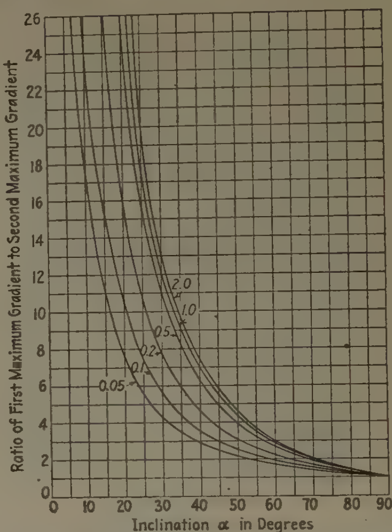

FIG. 27.—RATIO OF MAXIMUM GRADIENT 1/MAXIMUM GRADIENT 2 OF INFINITE INCLINED BLOCK.

Depth indicated on curves.

Comparison of the First and Second Maximum Gradients

If the ratio of the first maximum gradient to the second maximum gradient is plotted against the inclination α of the block, we see at once from Fig. 27 that this ratio increases slowly at first as α decreases from 90°. Before long this ratio begins to increase more rapidly and becomes very large. The increase occurs earlier in the case of deeper deposits and at any particular inclination the value of the "ratio" is always higher for a deep deposit than for a shallow one. It should be possible, therefore, by the application of Fig. 27, to give a fairly close estimate of the dip in any particular case for which the depth is known.

Gradient Interpretation

The interpretation of a gradient curve resulting from an infinite inclined block presents greater difficulties than have been encountered hitherto. From a general inspection of the shape of the gradient curve,

however, it should be possible to recognize the chief characteristics and to ascertain the direction of dip. Taking the algebraic mean of the first and second maximum gradient values, and locating a point at which the gradient reaches this value, we obtain the position of the mid-point of its upper surface.

A comparison of the first and second maximum gradient values enables the dip to be estimated fairly closely (within 5°, or at the most 10°), and it is now possible by employing the ratio of the gradient at the origin to the maximum gradient there, and making use of the "dip" obtained above, to make a determination of the depth of the block. A further application of these results to Figs. 27 and 25 will furnish a still closer and more satisfactory interpretation, so that it becomes possible to obtain a complete solution of the problem from a study of the gradient alone.

Characteristics of Differential Curvature

Determination of Curvature Value

The differential curvature value for this type of infinite block is given by the equation

$$\frac{\partial^2 \bar{U}}{\partial x^2} = \left[\sin 2\alpha \,.\, \log \frac{r_2}{r_1} + (1 - \cos 2\alpha)(\beta_2 - \beta_1) \right] \times \gamma$$

and curves showing the variation of this value under certain conditions are given in Figs. 17 to 21, where they are shown dotted, the full lines indicating the gradient. These diagrams show that the differential curvature value loses its symmetry when the infinite block departs from the vertical position. The left-hand portion of the curve at once crosses the axis, forming an upper portion which continually increases in relation to the remaining part of the curve. At the same time the point at which this curve of differential curvature crosses the axis (corresponding to a zero curvature value) gradually moves inwards towards the center line. The lower portion of the curve also changes with the inclination, the maximum curvature value gradually moving away from the center line and giving the curve the appearance of folding under in the direction of dip.

Variation of Maximum Curvature with Depth

Except for very shallow blocks, the effect of decreasing the inclination to the horizontal is to diminish the value of the differential curvature. For blocks very near the surface $d = 0.05$, a slight increase is at first apparent, as may be seen from Fig. 28, but as the deviation from the vertical increases, this temporary increase is followed by a far more

rapid decrease. In the region of $\alpha = 90°$ the curves are all flat, and as the inclination departs from this value the steepness of the slope increases in all cases with the shallowness of the block. It is evident from a glance at this diagram that the possibility of locating a dike or lode by means of the differential curvature is greatest when the dike is vertical, and that as the dip of the dike decreases below about 70° the possibility of locating it by this means also diminishes rapidly.

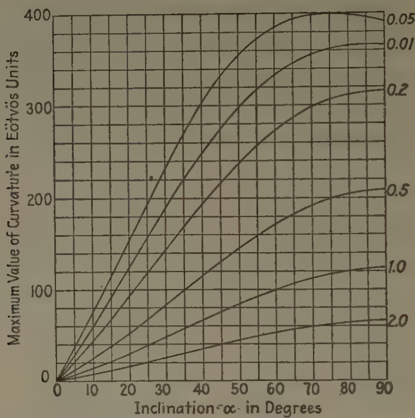

FIG. 28.—VARIATION OF MAXIMUM CURVATURE WITH INCLINATION OF BLOCK.

It can be shown that the conditions to be satisfied in order to obtain the maximum curvature values are

$$(\alpha + \beta_1 + \beta_2) = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}$$

giving the two positions of first maximum and second maximum curvature.

Curvature over Midpoint of Block

Jung[2] has applied Nikiforov's method to the consideration of differential curvature and has shown that the curvature value over the midpoint of the upper surface of an infinite inclined block is the algebraic sum of the first and second maximum curvature values. This method of determining the origin (or the midpoint of the upper surface) is completely analogous to that previously adopted for the gradient, and the results obtained by the two methods should correspond fairly closely in practice.

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[2] K. Jung: Die Bestimmung von Lage und Ausdehnung einfacher Massenformen unter Verwendung von Gradient und Krümmungsgrösse. *Ztsch. f. Geophys.* (1927) **3**, 257–280; (1929) **5**, 238–252.

Differential Curvature Interpretation

The curve showing the differential curvature results along a traverse over an infinite inclined block should by its general form indicate the main characteristics, and enable the direction of dip to be determined. The tendency of this curve to fold under in the direction of dip, and the small part of the curve above the axis in the direction away from the dip, are the main features.

The midpoint of the block may be located by ascertaining the point at which the differential curvature value is equal to the algebraical mean of the first and second maximum curvature values, exactly as in the case of the gradient.

Comparison of Gradient and Curvature Effects

If now we consider the effect of changing the inclination α on the ratio between the maximum gradient and the maximum curvature, we get the series of curves shown in Fig. 29. From this figure it is at

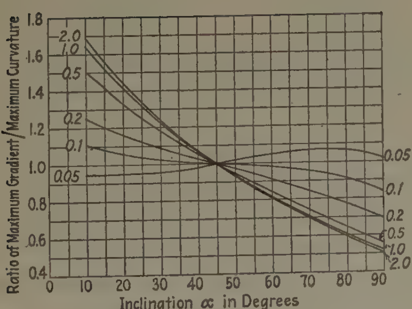

FIG. 29.—VARIATION OF MAXIMUM GRADIENT/MAXIMUM CURVATURE RATIO WITH INCLINATION OF BLOCK.

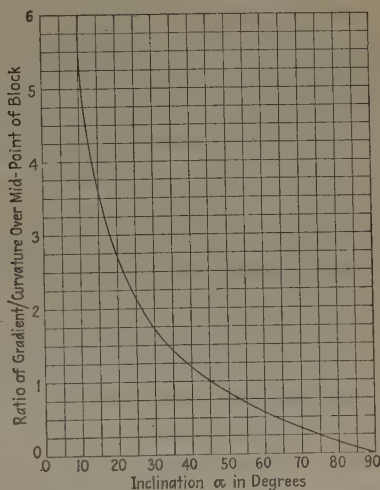

FIG. 30.—VARIATION WITH INCLINATION OF GRADIENT/CURVATURE RATIO OVER MIDPOINT OF BLOCK.

once obvious that the maximum gradient and maximum curvature are in all cases equal when the block is inclined at an angle of 45°.

For all but the shallowest blocks ($d = 0.05$) the ratio of maximum gradient to maximum curvature is greater than unity for small values of α (less than 45°), while for inclinations greater than 45° the maximum curvature is greater than the maximum gradient.

It would seem fairly simple, therefore, by a comparison of these two maximum values to ascertain whether the inclination α is greater

or less than 45°, and the deviation of the maximum gradient-maximum curvature ratio from unity will furnish a rough indication of the variation of α from 45°.

A further useful relation between the gradient and curvature values may be obtained by comparing these values at the origin, and plotting the ratio of the gradient to the curvature at this point against the inclination. The resulting curve, Fig. 30, corresponds almost exactly with the curve representing cot α, so that

$$\cot \alpha = \frac{\text{gradient at origin}}{\text{curvature at origin}}$$

as has been demonstrated by Jung.

This would appear, therefore, to furnish a simple means of determining α, when the origin is known, from a consideration only of values determined at this point.

Combined Gradient and Curvature Interpolation

An inspection of the curves of gradient and differential curvature, and a comparison of these with Figs. 17 to 21, should enable one to judge whether an inclined infinite block of the type now under consideration is present. Proceeding, then, the midpoint of the block may be located independently by the gradient and by the curvature, and the two locations should coincide, within acceptable limits. Having determined the origin, the ratio of the gradient value at this point to the curvature value there gives at once the cotangent of the inclination α, so that it remains now only to ascertain the depth. This may be done with the aid of Fig. 24.

It would appear therefore that in the case of an infinite inclined block it is possible to furnish a complete interpretation from a consideration of the gradient only, but that the interpretation is greatly facilitated when both the gradient and curvature values are available. The interpretation of such an anomaly from an examination of the differential curvature values alone is a matter of some difficulty, while the errors due to extraneous causes that occur in the practical measurement of the curvature would render an interpretation obtained from these observations less reliable than the solution derived from a comparison of the gradient values.

Effect of Impregnating Waters on Electrical Conductivity of Soils and Rocks*

By Karl Sundberg,† Stockholm, Sweden

Electrical investigations carried out in regions containing sedimentary rocks showed that sediments generally are good electrical conductors, a fact which at the present time is used for structural investigations by electrical methods. It is of fundamental importance, not only for these investigations but also for electrical prospecting in general, to know the electrical properties of various soils and rocks. It will be useful, therefore, to discuss this question with the hope that the general points of view discussed in this paper will perhaps be completed by further observations and determinations of material from various localities.

This paper is based principally on the study of literature, and the purpose is primarily a discussion of the main factors that determine the electrical conductivity of soils and rocks. It will follow, however, that the factors discussed here also frequently decide how far a certain orebody can be regarded as a good electrical conductor compared with the surrounding rocks.

All rocks are more or less porous, and the pores are filled with liquids and gases. Their electrical conductivity, therefore, depends on the following factors:

1. The electrical conductivity of minerals composing the rock.
2. Electrical conductivity of the liquid filling the pores.
3. Proportion of the volume of liquid filling pores of the rocks to the solid rock material.
4. Shape of the pores.
5. Arrangement of the particles of gas and liquids.
6. Temperature.

Dry minerals, with the exception of certain ore minerals, are complete nonconductors from the point of view of applied geophysics; in the present investigation we deal with rocks which have no conducting minerals, and factor 1, therefore, is not important. The same is true of soils. The liquids contained in soils and rocks in most cases are waters, sometimes oil. As oil is a nonconductor of electricity, the impregnating waters will determine the electrical conductivity of soils and rocks.

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* Scheduled for New York Meeting, February, 1932.

† Chief Engineer, Aktiebolaget Elektrisk Malmletning.

Accordingly, we can divide the discussion presented here into the following headings:

1. Occurrence and composition of waters in soils and rocks.
2. Water content of soils and rocks.
3. Determination of electrical conductivity of soils and rocks.
4. Order of magnitude of conductivity of soils and rocks.

1.—Occurrence and Composition of Water Contained in Soils and Rocks

The electrical conductivity of impregnating waters and thus of soils and rocks is determined by the amount and content of dissolved substance. The relation between conductivity and a given analysis will be discussed in part 4.

Waters may be classified in two large groups: Surface waters actually on the surface and waters below the surface. Meinzer[1] distinguishes two main water zones below the surface: the "zone of aeration" and the "zone of saturation." The rocks in the zone of saturation are saturated with water under hydrostatic pressure and their interstices are filled with water, termed ground water or phreatic water. Above this zone is the "zone of aeration," in which the interstices are partly filled with atmospheric gases and partly with water, held by molecular attraction and not by hydrostatic pressure. This water is called suspended subsurface water or vadose water. The waters in these two main zones are subdivided in the following way, according to Meinzer:

A. Vadose water (zone of aeration).
   1. Soil water, which may be discharged into the atmosphere by evaporation (direct from the soil or by the action of plants).
   2. Intermediate vadose water between the soil-water and fringe-water belts.
   3. Fringe water, drawn up from the zone of saturation by capillary action.

B. Ground water (zone of saturation).
   Gravity ground water, under the control of gravity (can be drained into wells).
   2. Ground water not under the control of gravity (cannot be
   1. drained into wells but is retained by the rock formations).

The zone of fringe water is so narrow that it may be omitted in our discussion. Meinzer's subdivision of the ground waters has been made from the viewpoint of water supply, the gravity water obviously being the only water that can be obtained from wells. It is important for our purpose to distinguish between the water that constitutes the surface

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[1] O. E. Meinzer: The Occurrence of Ground Water in the United States. U. S. Geol. Survey *Water Supply Paper* 489 (1923).

of the saturation zone (the water table) and waters in deeper formations. The configuration of the water table depends both upon topography and geological structure, whereas deeper waters are generally considered to conform with the structure of the geological column in which they occur.[2] Consequently, a contour map on the water table does not give a true picture of the structure of the water-bearing bed, but a contour map on the top of deeper waters does. The water that forms the ground-water table might be termed superficial ground water, and waters below, deep ground waters. The superficial ground water reaches from the ground-water table down to a depth, where a change in the chemical composition of the water takes place. This point corresponds to the top of the shallowest deep ground water, the bottom of which is determined by the point where a new change in chemical composition takes place, and so on.

Summarizing, for our purpose, we have to consider surface waters, soil water, intermediate vadose water, superficial ground water and deep ground water.

In geochemical, agricultural, hydrological and geological publications a great number of analyses of waters are given by Clarke,[3] Mills and Wells,[4] and others. Studying such analyses we find that the electrical resistivity of surface waters varies between wide limits. The purest surface waters contain only about 2 mg. per liter of substance in solution and show a specific resistance of around 300,000 ohm-cm., whereas water in salt lakes may contain as much as 7 per cent salt and show a resistivity of only 10 ohm-centimeters.

The chemical composition of soil waters and intermediate vadose waters is comparatively unknown because of the difficulty in extracting such waters. Meinzer has observed that it is sometimes necessary to heat material to over 100° Celsius to drive off the last of the moisture held by adhesion in fine-grained material. In geoelectrical investigations practical experience has shown that the electrical resistivity of fine-grained material is generally less than for coarse-grained, therefore the impregnated waters are generally more salty in the former case than in the latter. Probably this is explained by the larger aggregate surface for retaining water by capillarity in a practically stagnant condition in the case of fine-grained material, by which there is more opportunity for the water to dissolve material and become a good electrolyte. Determinations of resistivities of certain clays show that soil waters and intermediate vadose waters may have as low specific resistance as 10 ohm-centimeters.

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[2] A. C. Veatch: Water Conditions in Northern Louisiana, Southern Arkansas and Adjacent Regions. Louisiana Geol. Survey Report, 1905.

[3] F. W. Clarke: The Data of Geochemistry. U. S. Geol. Survey *Bull.* 770 (1924).

[4] R. V. Mills and R. C. Wells: The Evaporation and Concentration of Waters Associated with Petroleum and Natural Gas. U. S. Geol. Survey *Bull.* 693 (1919).

The chemical composition of superficial ground waters is rather well known on account of the great practical importance of such waters. The composition and resistivity vary between almost as wide limits as the surface waters.

The composition of deep ground waters is not very well known because of their difficult accessibility. In this connection it might be pointed out that by means of drilling it is possible to prove the presence of water only in rocks which give it up, whereas, of course, other rocks may contain great quantities of water. The composition of deep ground waters has been studied particularly in mines and in drilling for oil. According to analyses published by Clarke[5] and others the resistivity of waters in mines may be as low as 30 ohm-cm. Of waters encountered in oil fields, the "connate" waters which are residual in marine sediments are especially interesting. Mills and Wells[6] published a great number of analyses of such waters from the Appalachian fields. The specific resistance of "connate" waters may go as low as a few ohm-centimeters.

2.—Water Content of Soils and Rocks

When water penetrates soil or rocks a certain quantity of water is retained through adhesion. This water-retaining capacity when expressed in percentages of the total volume of rock or soil is called the "specific retention." According to Meinzer[7] and Fallon[8] the specific retention and thus the water content in the "zone of aeration" varies from a small percentage for coarse sands to around 40 per cent for fine clays.

In the "zone of saturation" the water content apparently is the same as the pore volume. The pore volume of a material consisting only of spherical grains of uniform size is independent of the size of grains and depends only upon the manner in which the grains are arranged. The most compact arrangement will be obtained when the lines joining the centers of the spheres form an equilateral parallelopiped having face angles of 60° and 120°. For such hexagonal arrangement the pore volume amounts to 25.95 per cent. When the spheres are arranged in the lowest possible manner, the lines joining the centers will form the sides of cubes, and for such cubic arrangement a pore volume of 47.64 per cent is obtained.[9] Höfer,[10] Blumer[11] and Meinzer[12] published values

[5] F. W. Clarke: *Op. cit.*

[6] R. V. Mills and R. C. Wells: *Op. cit.*

[7] O. E. Meinzer: *Op. cit.*

[8] Fallon, Bodenkunde.

[9] O. E. Meinzer: *Op. cit.*

[10] H. Höfer von Heimhalt: Grundwasser und Quellen. Braunschweig, 1920.

[11] H. Blumer: Die Erdöllagerstätten, Grundlagen der Petroleumgeologie. Stuttgart, 1922.

[12] O. E. Meinzer: *Op. cit.*

of the pore volume of various rocks. Mills and Wells[13] determined experimentally the pore volume of sands from the Appalachian oil fields.

Blumer gives the following approximate values of pore volumes: Igneous rocks and crystalline schists, 0.5 to 2 per cent; phyllite, 3 to 4; ordinary clay and sand, 8 to 15; loose sand, porous conglomerates and cellular limestones, 25 per cent and more.

3.—Determination of Electrical Conductivity of Soils and Rocks

Direct Experimental Determination

It might seem that the experimental determination of the electrical conductivity of a soil or rock is very simple. If, however, we consider

Table 1.—*Resistance of Samples between Two Electrodes*

Material	Humidity, Percentage by Volume	Specific Resistivity, Ohm-cm.
Yellow river sand	0.86	83,000
Yellow river sand	3.3	17,000
Yellow river sand	9.5	9,500
Garden soil	3.3	167,000
Garden soil	10.0	15,600
Garden soil	17.3	6,000
Clay	4.4	145,000
Clay	9.2	15,000
Clay	16.1	5,000
Clay	28.0	1,600
Clay	45.0	1,450
Clay	58.6	1,410

the subject a little more closely we find that such is not the case, because often it is impossible to obtain samples in the same state as they are found in nature. For this reason it is obvious that the determination of electrical conductivity of soils is the simplest. But even here we encounter difficulties, partly because while a sample is being taken its humidity changes, and partly because in general it is impossible to retain the natural consistency of samples. Besides, there is the difficulty of having to consider the resistance between the electrode and the sample (because the usual method requires a sample of the material investigated to be placed between two electrodes). A direct determination of the resistance of rocks which are not close to the surface in general should be regarded as impossible because samples cannot be obtained with "natural" properties. For the determination of electrical conductivity of soils, therefore, in general or at least frequently, we are obliged to use indirect

[13] R. V. Mills and R. C. Wells: *Op. cit.*

methods. If such determinations are to be made on rocks we always have to resort to indirect methods.

There are only a few determinations of the conductivity of soils and of rocks in the literature. Within the past few years, however, more and more determinations of rock conductivity have been made and partly published. Results of direct determinations of the resistance of samples between two electrodes were probably first published by Eichhoff.[14] Lately Pullen, of the U. S. Bureau of Mines, has studied this method very carefully.[15] Some of the values obtained by Eichhoff are given in Table 1.

Unfortunately, the conductivity of the waters used for these determinations is not given, but the statement is made that it was ordinary tap water, which, as is clearly shown, has a rather high conductivity.

Our organizations have made many direct determinations of the resistivity of drill cores and ore and rock samples. The general technique is described in one of our publications.[16] When working with drill cores it is advantageous to use mercury electrodes. Fig. 1 gives an example of such determinations of resistivity at Egbell oil field in Czechoslovakia. The formations tested were young Tertiary clay, marls and sands of Pontic and Sarmatic age. The Pontic sediments are deposited in fresh, the Sarmatic in brackish waters, consequently it is reasonable to expect the former to have higher average resistivity than the latter. Fig. 1 shows this to be the case.

Both potential and electromagnetic prospecting methods can be used to determine the average resistivity of soils and rocks in place. Kœnigsberger[17] has determined the resistance between two electrodes and calculated the specific resistance from this.

The best known method for the study of the resistivity of the surface is the four-electrode potential method by Wenner,[18] which has been practically developed by Gish and Rooney of the Carnegie Institute,[19] F. W. Lee[20] was the first to suggest the use of a "megger" for measurements of this kind.

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[14] J. Zenneck: *Ann. d. Phys.* (1907) **23**, 858.

[15] M. W. Pullen: Tentative Method for Making Resistivity Measurements of Drill Cores and Hard Specimens of Rocks and Ore. U. S. Bur. Mines *Circ.* 6141 (1929).

[16] K. Sundberg, H. Lundberg, and J. Eklund: Electrical Prospecting in Sweden.

[17] J. G. Kœnigsberger: Field Observations of Electrical Resistivity and Their Practical Application. Geophysical Prospecting, A. I. M. E. (1928).

[18] F. Wenner: A Method of Measuring Earth Resistivity. U. S. Bur. Stds. *Bull.* 12 (1916).

[19] O. H. Gish and W. J. Rooney: *Terrestrial Magnetism* (1925) **30**, and (1927) **32**.

[20] F. W. Lee: Measuring the Variation of Ground Resistivity with a Megger. U.S. Bur. Mines *Tech. Paper* 440 (1928).

F. W. Lee, J. W. Joyce and P. Boyer: Some Earth Resistivity Measurements. U. S. Bur. Mines. *Circ.* 6171 (1929).

In the Lake Superior copper country, Rooney[21] obtained the following figures for the resistivity of different formations: Glacial drift, 825 to

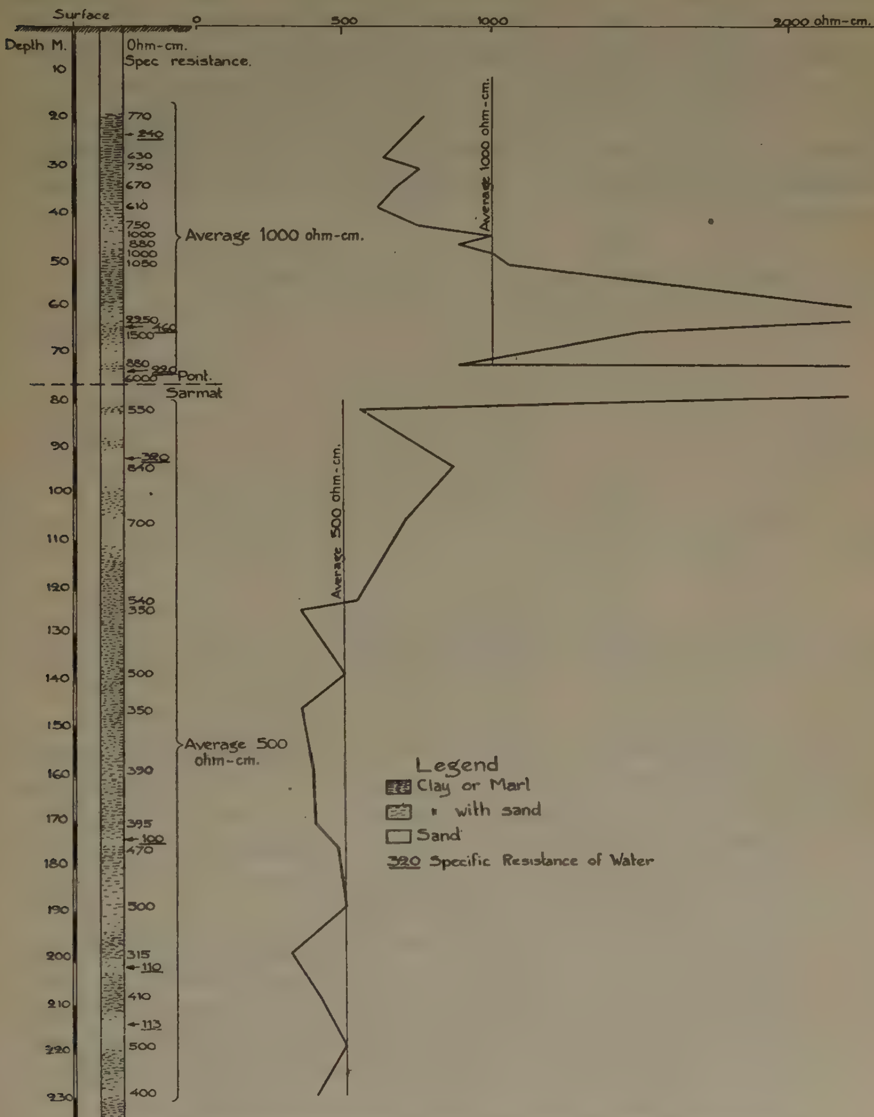

FIG. 1.—RESISTIVITY OF DRILL CORES. (*From Egbell, Czechoslovakia.*)

396,000 ohm-cm.; lavas, 11,900 to 4,355,000; shale, 12,300 to 18,100; conglomerate, 104,000 to 141,500; sandstone, 3,500 to 41,400.

[21] W. H. Hotchkiss, W. J. Rooney and J. Fisher: Earth-resistivity Measurements in the Lake Superior Copper Country. Geophysical Prospecting, A. I. M. E. (1929).

We have carried out extensive potential measurements for investigation of the surface conductivity, according to a principle somewhat differing from Wenner's. Figs. 2 and 3 show results of such investigations in Texas. Fig. 2 gives a schematic resistivity profile for the surface formations in Central Texas, from which it is apparent that the different geologic formations generally have different electrical conductivity. As shown by the detailed profiles in Fig. 3, the resistivity generally varies greatly within the different formations, because of the inhomogeneous stratigraphic character of the individual beds of which one formation is composed.

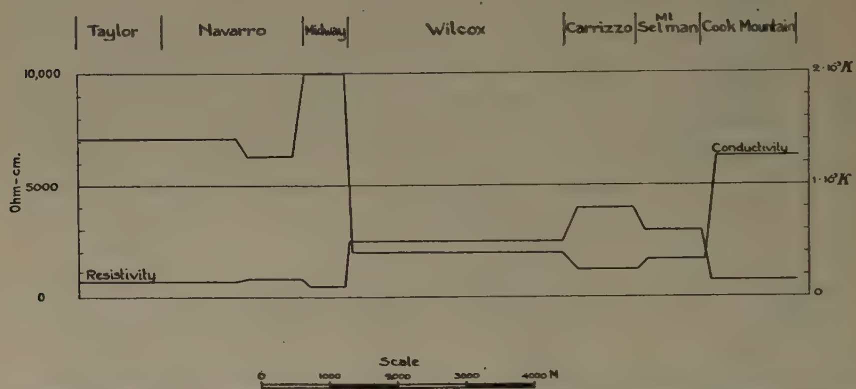

FIG. 2.—AVERAGE RESISTIVITY OF SURFACE FORMATIONS, CENTRAL TEXAS. GENERALIZED PROFILE THROUGH THE SERIES, TAYLOR-COOK MOUNTAIN.

Schlumberger[22] recently made an interesting application of the potential method for determining the resistivity of formations penetrated by a drill hole. Fig. 4, after Schlumberger, shows a resistivity profile at Pechelbronn, Alsace, showing the different electrical character of the tested formations: Upper Pechelbronn formation and Hydrobia marl, the former a litoral formation, the latter marine.

Electromagnetic methods have not yet been much used to determine the surface conductivity. Abraham[23] determined the damping of an antenna, embedded in the soil. By measuring the magnetic field vector around a loop or earthed cable, carrying alternating current, the resistivity can also be determined. The general theory is given by Pollaczek.[24]

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[22] C. and M. Schlumberger: Communication sur le carottage électrique. IIᵉ, Congres International de Forage, Paris, 1929; also Electrical Logs and Correlations in Drill Holes, *Min. & Met.* (1929) **10,** 515.

[23] M. Abraham, H. Rausch von Traubenberg u. J. Pusch: Ueber ein Verfahren zur Bestimmung des spez. Leitfähigkeit des Erdbodens. *Phys. Ztsch.* (1919) **20,** 145.

[24] F. Pollaczek: Ueber das Feld einer unendlich langen wechselstromdurchflossenen Einfachleitung. *Elektrische Nachrichtentechnik* (1926) **3,** 339.

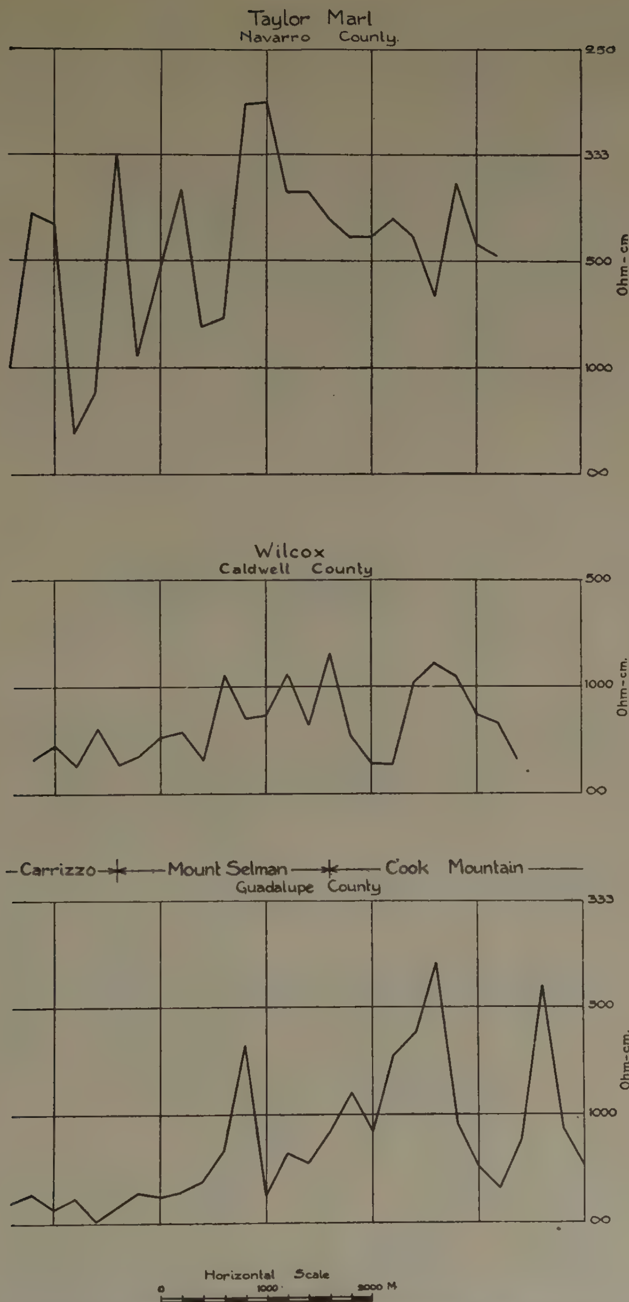

FIG. 3.—ACTUAL RESISTIVITY OF SURFACE FORMATIONS, CENTRAL TEXAS.

Klewe[25] gives results (Table 2) of measurements in northern Germany, using two long parallel grounded wires and measuring the mutual inductance between the wires at different frequencies.

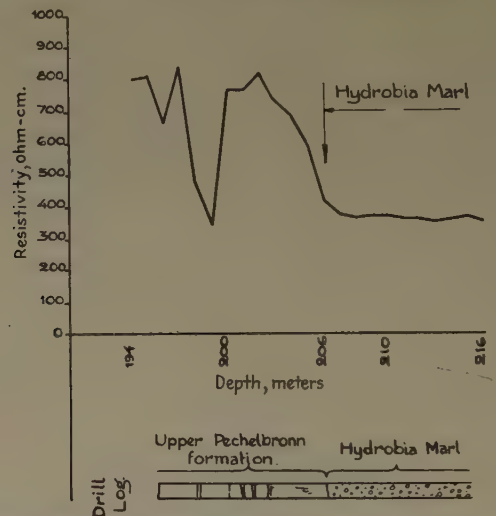

FIG. 4.—RESISTIVITY OF FORMATIONS AT PECHELBRONN, ALSACE. (*After Schlumberger.*)

TABLE 2.—*Results of Measurements in Germany (Klewe)*
The Figures Give the Conductivity in 10^{-15} C.G.S. as Unit

Frequency	Distance between Parallel Wires, Meters						
	1	10	100	300	1000	1500	3000
OLDENBURG (SWAMPY GROUND)							
$16\frac{2}{3}$	455	368	504	675	540	580	520
100	146	139	166	186	169	150	144
300	105	90	103	107	91	86	89
800	82	77	62	83	73	75	77
2000	79	77	87	84	81	90	70
DÖBERITZ (SANDY GROUND)							
$16\frac{2}{3}$	503	700	936	1190	1060		
300	392	246	224	243	201		
800	332	171	148	178	157		
2000	264	112	98	116	169		

Unfortunately, details regarding the geology are unknown. It is interesting to notice that, generally speaking, the average conductivity

[25] H. Klewe: Gegeninduktivitätsmessungen an Leitungen mit Erdrückleitung. *Elektrische Nachrichtentechnik* (1929) 6, 467.

does not vary much with increasing distance between the wires, but varies greatly with frequency. The conductivity increases with decreasing frequency because with lower frequency deeper layers with better conductivity come within reach of the magnetic field.

Indirect Determination of Specific Resistivity

For the reasons given above, it is very desirable to have methods for the indirect determination of the specific resistance. In general, it is possible to determine the percentage of pore volume v of a rock and the specific resistance ρ of the liquid filling the pores. It seems that from these two determinations it should be possible to obtain the resistivity ρ_x of the rock. Of course, ρ_x depends not only on v and ρ but also on the shape of the pores. However, the influence of the shape of the pores probably is not of decisive importance.

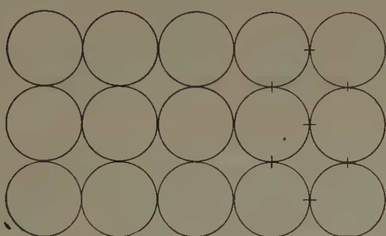

FIG. 5.—CUBIC ARRANGEMENT OF SPHERICAL GRAINS.

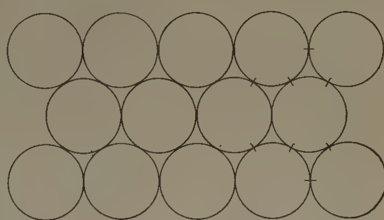

FIG. 6.—RHOMBIC ARRANGEMENT OF SPHERICAL GRAINS.

Let us consider the material to be made up of equal nonconducting spheres, and assume, furthermore, that the pores between the spheres are filled with a liquid of resistivity ρ. Let us assume, in the first place, that we have n^3 spheres of radius r with cubic packing, as shown in Fig. 5, and calculate the resistance R between two surfaces parallel to the plane of the paper, of which one passes through the centers of the spheres and the other represents a tangential plane at a distance r from the first. The resistance $d\Delta R$ of a layer of thickness dl at a distance l from the plane going through the centers of the spheres is:

$$d\Delta R = \frac{\rho - dl}{4n^2r^2 - n^2\pi(r^2 - \rho^2)}$$

from which

$$\Delta R = \frac{\rho}{\pi n^2}\int_0^r \frac{dl}{r^2\left(\frac{4}{\pi} - 1\right) + l^2} = \frac{\rho}{\pi n^2} \times \frac{1}{r\sqrt{\frac{4}{\pi} - 1}}\left|\tan^{-1}\frac{l}{r\sqrt{\frac{4}{\pi} - 1}}\right|_0^r = \frac{0.660\rho}{rn^2}$$

The resistance of a cube with the length of the side equal $2rn$ will be

$$R = \frac{1.32\rho}{rn}$$

from which we obtain

$$x = 2.64\rho$$

independent of grain size. As already mentioned, the pore volume in this case is 47.6 per cent, also independent of grain size.

If the packing is rhombic, as shown in Fig. 6, we obtain $\rho_x/\rho = 4.40$ in the direction perpendicular to the plane of the paper and $\rho_x/\rho = 3.38$ in the direction AB. The pore volume for this case is 39.5 per cent. The closest packing possible is hexagonal. For a direction perpendicular to the base we have $\rho_x/\rho = 5.81$. The pore volume is 25.9 per cent.

For a material consisting of grains which can be considered to be equal spheres tangent to each other ρ_x/ρ varies, therefore, between 2.6 and 5.8, while the pore volume varies between 47.6 and 25.9 per cent. Pore volumes greater than 47 per cent are seldom found in nature, and then only for certain kinds of soils. They originate in the fact that soils do not consist of equal grains but of aggregates of grains between which we can, of course, have large empty spaces.

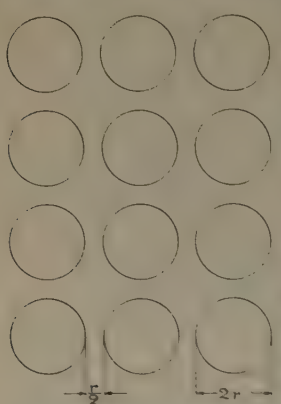

Fig. 7.—Spherical grains not touching each other in cubic arrangement.

From the data given on pore volumes of various rocks, it follows that it is in general smaller than the smallest possible for a material consisting of equal spheres. The reason for this is that grains of which a rock consists are not spherical, and also because the grains are not of equal size. The smaller grains then fill the empty spaces between the larger grains, and thereby decrease the pore volume. For igneous and metamorphic rocks, it frequently happens that the pores have an entirely different form from that discussed here.

To give an idea of the order of magnitude of ρ_x/ρ in the two extreme cases—wet soil and dense igneous rock—we shall next consider very loose cubic packing of equal grains which do not touch each other (Fig. 7). Material of exactly this construction does not occur in nature, of course, but discussion of this case is justified in order to get a figure for material with large pore volume.

For this case we obtain $\rho_x/\rho = 1.37$. The pore volume is 73.2 per cent.

For a material with a pore volume less than 26 per cent we can assume the pores to be cylindrical tubes of constant radius b, which are parallel to each other. We assume, furthermore, for the sake of simplicity, that these tubes do not touch each other. Material of exactly this construction does not exist in nature, but the case illustrates what ρ_x/ρ factor is to be expected for material with small pore volume. If we have

n tubes per square centimeter of the surface of a cube the pore volume $v = 3\pi a^2 n$ and in the direction of the tubes we have $\rho_x = \rho/\pi a^2 n$ from which it follows that

$$\frac{\rho_x}{\rho} = \frac{3}{v}.$$

Therefore, when $v = 20$ per cent, $\rho_x/\rho = 15$; when $v = 10$ per cent, $\rho_x/\rho = 30$; when $v = 5$ per cent, $\rho_x/\rho = 60$; when $v = 2$ per cent, $\rho_x/\rho = 150$.

In Fig. 8 will be found these values of ρ_x/ρ plotted as a function of v, and they allow the construction of a curve from which we can determine the specific resistance ρ_x if the pore volume and the conductivity of the liquids filling the pores are known. Experimentally determined values of ρ_x/ρ shown in the figure indicate that the constructed curve gives results close enough for practical purposes.

From the values for pore volumes given on page 371 we obtain the approximate values of ρ_x/ρ for soils and rocks saturated with water given in Table 3.

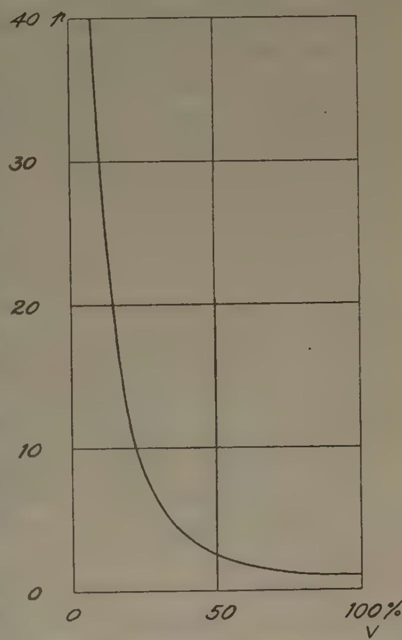

FIG. 8.—RELATION BETWEEN RESISTANCE FACTOR $p = \frac{\rho_x}{\rho}$ AND PERCENTUAL VOLUME V OF WATER IN ROCKS.

The method described here for the determination of the specific electrical resistance of soils and rocks is thus based on the determination of the specific resistance of liquids filling the pores, and on the determination of the water content (specific retention in aeration zone, pore volume in saturation zone). We shall consider the methods for such determinations.

TABLE 3.—*Approximate Value of* ρ_x/ρ

MATERIAL	ρ_x/ρ
Igneous rocks and crystalline schists	100
Dense phyllite, lime, sandstone	50 –100
Clays and sand in general	20 – 40
Porous clays, sands, sandstone, limestone and dolomite	3 – 20
Marl, loess, clay and sandy soil	1.5– 4
Turf and infusorial earth	1.0– 1.5

Determination of the Specific Resistance of Liquids

Liquids that come into consideration are electrolytes and methods for the experimental determination of their specific resistance are well known.

It happens frequently that samples of waters that one would like to have cannot be obtained; but in spite of that, their specific resistance can be determined if their chemical composition is known. The problem of determining the specific resistance of a solution of a known chemical composition is only incompletely solved. This question, however, is of great importance, and we shall consider it in detail. In the first place, it is necessary to make sure that a given analysis has a sufficient number of determinations, and is calculated in the right way. As far as the first point is concerned, a complete analysis is most desirable but is very seldom to be obtained. As a rule we must be satisfied with the total percentage of dissolved substances, the alkali content, the calcium and magnesium content, the chlorine content and SO_4 content. As will be shown, it is very important to know the CO_2 content in certain cases. The more determinations we have, the better. Strictly speaking, a complete analysis should show what ions are contained in the solution and what is the amount and dissociation of these ions. Such data will give the right understanding of the composition of the solution, and will make it possible to calculate its specific resistance very simply. In order, however, to get these simple data, we must first investigate a given analysis in order to determine what ions are present, what their amounts are in solution, and their degree of dissociation. According to J. König[26] we can proceed in the following manner. All amounts obtained from the analysis are converted into milligram-equivalents per liter, except for CO_2 and SiO_2, which are given in terms of milligram-molecules per liter of solution.[27]

If the total content of kations $= K$, of anions $= A$, milligram-equivalent per liter, while the content of $CO_2 = C$ milligram-molecules per liter, and if we write $K - A = d$, the following cases are possible:

I. d is positive
 α) $d < C$
 β) $d \geq C$
II. d is negative

(Solutions of group I are alkaline for methylorange, those of group II are acid. Iα is acid for phenolphthalein, Iβ is neutral or alkaline).

In case Iα there are in solution d milligram-equivalents of HCO_3 ions per liter, C-d milligram-molecules of free CO_2 and no CO_3 ions.

For the case $d \geq C$ there is no free carbonic acid, but there are CO_3 ions and HCO_3, and also OH ions which are generated by hydrolysis.

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[26] J. König: Chemie der menschlichen Nahrungs- und Genussmittel, **3**, 712.

[27] We assume that H_2S, SiO_2, TiO_2, HBO_2 are not present in appreciable quantities. For these cases reference may be made to König.

Their quantities can be obtained from the following equations:

$$CO_3 \text{ ions} = d + 0.1 - \sqrt{(d + 0.1)^2 - 4c(d - c)} \text{ mg-equ./l.}$$

$$HCO_3 \text{ ions} = C - 0.5 \times CO_3 \text{ mg-equ./l.}$$

$$OH \text{ ions} = d - C - 0.5 \times CO_3 \text{ mg-equ./l.}$$

In case II, which does not occur frequently, there are in solution some H ions and also HSO_4 ions, each of these groups to the amount of $0.5d$ milligram-equivalents. The content of SO_4 ions found from analysis must therefore be decreased by the quantity d.

It follows that the amount of carbonic acid in solution is very important for the right interpretation of the results of analyses. It is, therefore,

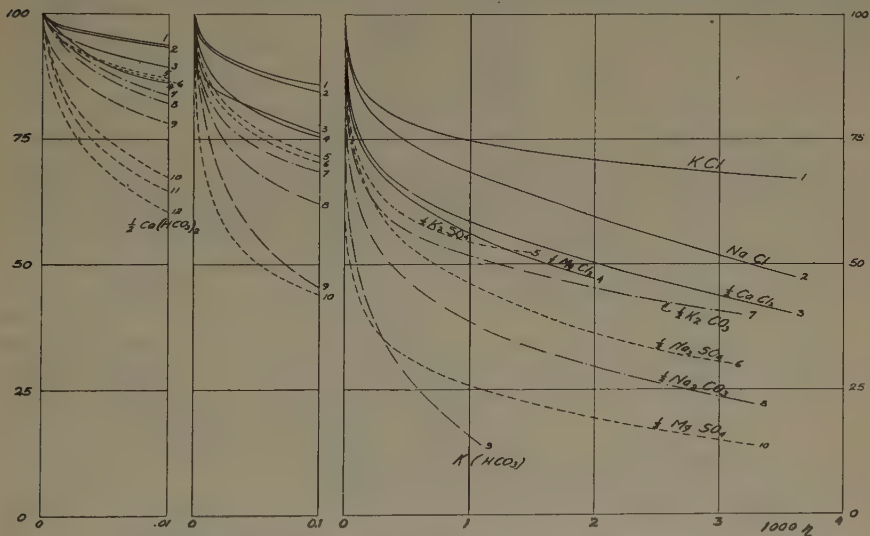

FIG. 9.—RELATION BETWEEN CONCENTRATION AND DISSOCIATION FOR DIFFERENT SOLUTIONS.

necessary to know how far a certain quantity of carbonate ions given in an analysis corresponds to carbonic acid and is not merely a number calculated to make the analysis check, and the content of carbonic acid given should not be considered unless it is stated explicitly that it is the result of an analytic determination. If that is not true, we can assume the quantity of HCO_3 ions equal to d according to case I, because in most cases free carbonic acid is present in the solution.

We now come to the calculation of the degree of dissociation and of the conductivity. This must be done in various ways according to whether the solution (1) contains a single salt as its principal constituent; (2) consists of a number of salts with common anions or kations; (3) contains a number of anions and a number of kations.

Case 1 is the simplest, because we can easily determine the degree of dissociation if the concentration is known. This can be obtained,

for instance, from tables of Landolt-Börnstein.[28] From the degree of dissociation α we obtain the resistivity ρ in ohm-centimeters according

TABLE 4.—*Mobility of Ions*

Ion	l_k	Ion	l_A
K	64.6	I	66.5
Na	43.5	Br	67.0
NH_4	64.0	Cl	65.6
½Ca	51.0	NO_3	61.7
½Ba	55.0	OH	17.4
½Mg	45.0	H	318.0
½Zn	46.0	HCO_3	38.0
½Cu	46.0	½SO_4	68.0
½Fe	47.0	½CO_3	74.0
½Mn	46.0	½HPO_4	53.0
		½$HAsO_4$	53.0

to $\rho = 1/\alpha\lambda_\infty$ where λ_∞ is the equivalent conductivity for infinite dilution in reciprocal ohm-centimeters $\lambda_\infty = l_k + l_A$ for any salt and can be obtained from Table 4, which gives the values of the mobility of ions l_k and l_A.

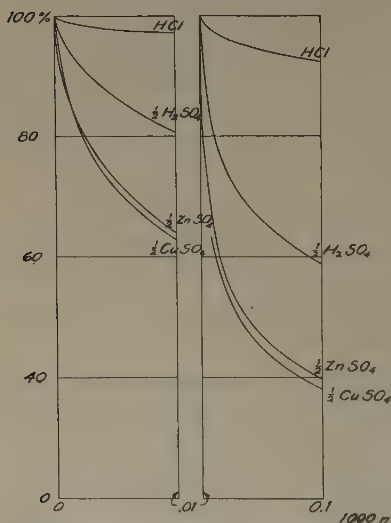

FIG. 10.—RELATION BETWEEN CONCENTRATION AND DISSOCIATION FOR DIFFERENT SOLUTIONS.

The degree of dissociation is obtained for a given concentration from Figs. 9 and 10. (These are drawn for the following salts: KCl, $CaCl_2$, K_2SO_4, $KHCO_3$, $Ca(HCO_3)_2$, $ZnSO_4$, HCl, H_2SO_4.)

The values for bicarbonate solutions are very uncertain, as only very few determinations have been made.

As the content of NaCl frequently is very high, the specific resistance of NaCl solutions of various concentration is given in Fig. 11.

Solutions containing a number of salts with the same anions or kations are "isohydric," that is, characterized by the fact that the common ion comes in the same amount from each of the dissociated salts. Arrhenius,[29] MacGregor[30] and Barmwater[31] gave

[28] Landolt-Bornstein: Physikalisch-chemische Tabellen.

[29] S. Arrhenius: Leitung von Mischungen. *Ann. d. Phys., u.Chem.* (1887) **30,** 51.

[30] J. C. MacGregor: Ueber die Bestimmung der Dissoziation von Zusammengesetzten u. s. w. *Ztsch. f. phys. Chem.* (1900) **33,** 529, and also *Phil. Mag.* (1896) **41,** 276.

[31] F. Barmwater: *Ztsch. f. phys. Chem.* (1899) **28,** 424; **45,** 557; **56,** 225.

various methods for the calculation of the specific resistance of such solutions. The simplest and best is that given by MacGregor. According to MacGregor we can consider that each of the salts takes up a separate part of the solution in which it has a certain definite concentration which we shall call "partial concentration." Let us deal with the case of four salts, 1, 2, 3 and 4, and let their concentration in gram equivalents per liter be N_1, N_2, N_3 and N_4 and the corresponding degree of dissociation

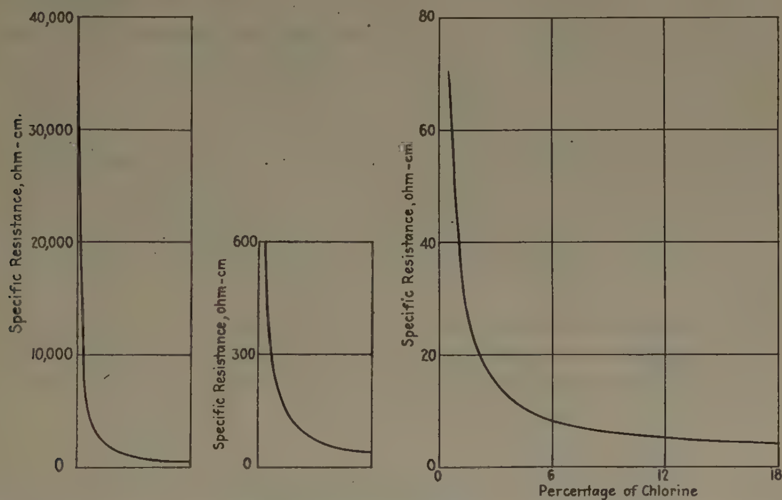

FIG. 11.—RELATION BETWEEN CHLORINE CONTENT OF WATERS AND THEIR SPECIFIC ELECTRICAL RESISTANCE.

a_1, a_2, a_3 and a_4. Furthermore let their partial concentration be V_1, V_2, V_3 and V_4. We have the following equations as shown by MacGregor:

$$\frac{a_1}{V_1} = \frac{a_2}{V_2} = \frac{a_3}{V_3} = \frac{a_4}{V_4} \quad [1]$$

$$N_1V_1 + N_2V_2 + N_3V_3 + N_4V_4 = 1 \quad [2]$$

$$\frac{a_1}{V_1} = f_1(V_1); \; \frac{a_2}{V_2} = f_2(V_2) \text{ etc.} \quad [3]$$

where f_1, f_2, etc. are known functions.

$$1000\kappa_1 = \frac{a_1}{V_1}.\lambda^1{}_\infty \quad 1000\kappa_2 = \frac{a_2}{V_2}.\lambda_\infty{}^2 \text{ etc.} \quad [4]$$

$$\rho = \frac{V_1N_1}{\kappa_1} + \frac{V_2N_2}{\kappa_2} + \frac{V_3N_3}{\kappa_3} + \frac{V_4N_4}{\kappa_4} \quad [5]$$

λ_∞ = equivalent conductivity of the partial solution for infinite dilution, κ = its resistivity, ρ = resistivity of the solution. The calculation of ρ from these equations can best be done graphically in the way shown in Fig. 12.

Functions corresponding to equation 3 are represented graphically by the curves 1, 2, 3 and 4, which are obtained by plotting V as a function of $\frac{a}{V}$ or salts 1, 2, 3 and 4. We then draw curve 5, whose ordinates are the sum $N_1V_1 + N_2V_2 + N_3V_3 + N_4V_4$ (Compare equation 2). Let a line PQ parallel to the ordinate axis through the point Q for which $V = 1$, intersect 1 in A, 2 in B, 3 in C, 4 in D, and the $\frac{a}{V}$ axis in P. OP is then the common value of $\frac{a}{V}$ and $AP = V_1$, $BP = V_2$, etc. From equation 4 we then obtain the κ value of partial solutions and then calculate ρ from equation 5.

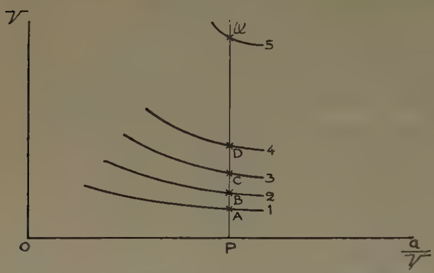

FIG. 12.—CALCULATION OF SPECIFIC RESISTIVITY FOR "ISOHYDRIC" SOLUTIONS.

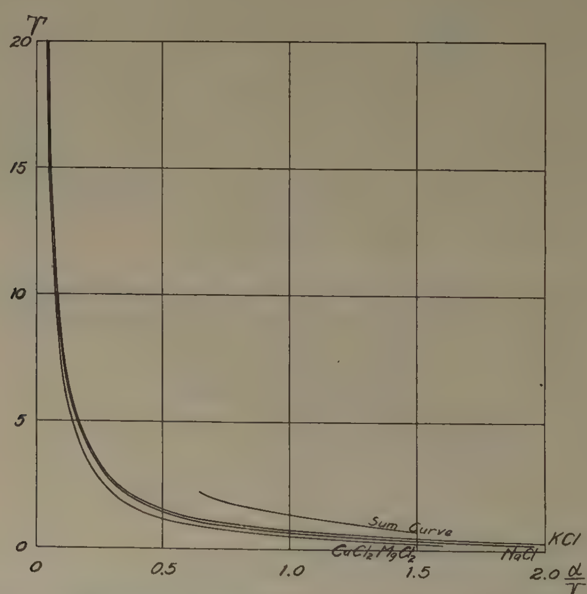

FIG. 13.—RELATION BETWEEN DISSOCIATION AND PARTIAL CONCENTRATION FOR "ISOHYDRIC" SOLUTIONS.

The method given above can be applied to salt waters found in connection with oil, because these waters contain almost exclusively chlorides of K, NA, Ca and Mg. Fig. 13 gives V as a function of $\frac{a}{V}$ for these salts. (The curves for $MgCl_2$ and $CaCl_2$ are practically identical.)

As an example, let us calculate the resistivity of a solution containing 0.5 per cent KCl, 6.5 per cent NaCl, 4 per cent $CaCl_2$, 1 per cent $MgCl_2$, which corresponds to the composition of typical salt waters from oil fields.

The concentration in gram-equivalents per liter is

KCl	$N_1 = 0.07$
NaCl	$N_2 = 1.11$
½$CaCl_2$	$N_3 = 0.72$
½$MgCl_2$	$N_4 = 0.21$

From the curve shown in Fig. 13, it follows that $a/V = 1.2$, $V_1N_1 = 0.04$, $V_2N_2 = 0.60$, $V_3N_3 = 0.09$, $V_4N_4 = 0.27$, and $\kappa_1 = 0.127$, $\kappa_2 = 0.139$, $\kappa_3 = 0.156$, $\kappa_4 = 0.131$, from which we obtain $\rho = 7.5$ ohm-centimeters.

As far as case 3, mentioned on page 381, is concerned, where the solution contains salts with several anions and several kations, there are at present no general methods for the calculation of the degree of dissociation. Barmwater[32] investigated this problem, both theoretically and experimentally. His studies refer to very dilute solutions of any number of salts (total concentration up to $\frac{1}{10}$ gram-equivalents per liter). Barmwater showed both theoretically and experimentally that the degree of dissociation for a certain salt A, in a dilute solution, which contains any number of salts with a total concentration ΣN, is the same as for a solution, which contains only salt A with the concentration ΣN. It follows that the resistivity of such a solution can be calculated in the following way: We combine anions and kations in an arbitrary way to a number of salts with concentration N_1, N_2, etc., gram-equivalents per liter. We then determine the degree of dissociation for the salts in a solution of total concentration ΣN. We then have

$$\rho = \frac{1000}{\Sigma \cdot a \cdot N \cdot \lambda_\infty} \tag{A}$$

For the special case where the solutions are so dilute that all salts are completely dissociated ($\Sigma n < 0.5$ milligram-equivalents per liter) we have

$$\rho = \frac{1000}{\Sigma n(l_k \div l_A)} \tag{B}$$

(It should be mentioned that the calculation of ρ according to equation A can be carried out only under the assumption that the solution contains no acid.) As an example, let us calculate ρ for the hypothetical salt solution above.

The result calculated according to the method of MacGregor is 7.5 ohm-cm. The method given here can, therefore, apparently be applied to solutions which are quite concentrated.

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[32] F. Barmwater: *Op. cit.*

Salts N G-equ/1		α in 2.11n Solution According to Fig. 14.
KCl	0.07	0.71
NaCl	1.11	0.586
$CaCl_2$	0.72	0.494
$MgCl_2$	0.21	0.46
$N = 2.11$		
KCl	130	6.45
NaCl	110	71.50
$CaCl_2$	116.5	41.50
$MgCl_2$	110.5	10.70
		130.15

$$= \frac{1000}{130.15} \text{ ohm-centimeters} = 7.70$$

An investigation of the conductivity on waters from different sources shows that for salty waters, say with a specific resistance of less than 1000 ohm-cm., the specific resistance of the water generally can be determined closely enough for practical purposes by the chlorine content alone and assuming only NaCl present in the solution.

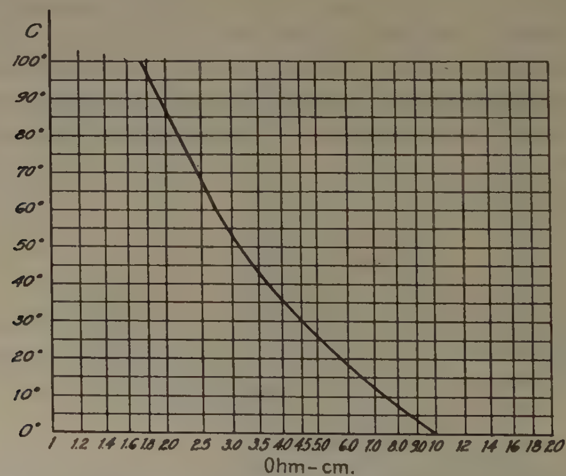

Fig. 14.—Electrical resistivity of electrolytes as function of temperature.

The resistivity of an electrolyte depends on temperature and decreases for an increase in temperature. Otto Stalhane showed that for a dilute solution the function which represents the resistivity as a function of temperature is independent of the kind of electrolyte. Fig. 14, after Stalhane, shows the specific resistance as a function of temperature. It is seen that the resistivity of an electrolyte at 0° C. is twice as great as its resistivity at 25° C.

Determination of Water Content of Soils and Rocks

The water content of soils and rocks in the zone of aeration may be determined by ascertaining the initial weight of a fresh sample and the dry weight after drying at 100° C. The difference in weight represents the retained water, and the specific retention is obtained by division with the initial volume of the sample.

In rocks below the water table the water content equals the pore volume which may be determined by different methods. In general, the following method is used, worked out by Melcher, of the United States Geological Survey.

This method is based on the principle that the volume of a sample of the sand (stone) minus the volume of its individual grains equals the volume of the pore space. The volume of the pore space divided by the volume of the sample gives the percentage of pore space by volume.

The piece that is to be used for finding the volume of the sample is weighed and then dipped into paraffin, heated to a temperature a little above its melting point. The sample with the coating of paraffin is then suspended in distilled water and weighed.

From a previous determination of the density of paraffin, and the weight of the paraffin covering the sample, its volume is obtained. Subtracting this volume from the total volume of the sample plus the volume of the paraffin gives the volume of the sample used.

A second part of the sample is weighed and crushed in an agate mortar into its separate particles, or in the case of a very fine sand, until it will pass through a 100-mesh sieve. The volume of the grains is then determined with the help of a pycnometer.

By proportion, the total volume of grains in the sample dipped in paraffin is determined. The volume of the sample dipped in paraffin minus the volume of its grains is equal to the pore volume.

4.—Order of Magnitude of the Conductivity of Soils and Rocks

In applying both the potential and electromagnetic method of geophysical exploration, it is generally easily noticeable whenever good conducting soils or rocks are present. In such cases currents are induced in the soils or rocks and rotating fields are observed. These are the cause of the well-known dull minima encountered in electrical prospecting with older alternating current methods.

According to our experience, there are, in areas with volcanic rocks and crystalline schists (areas of type 1), few regions where these currents have to be taken into account, but in areas of young sedimentary beds (areas of type 2) the opposite is true.

These observations obviously are explained by the fact that in regions of type 1 the rocks are attacked very little by flowing waters and that these waters therefore are relatively pure, and thus have a high electrical resistance. Another circumstance causing high resistance is the small pore volume of these rocks. For young sedimentary rocks the conditions are exactly opposite. Easily soluble rocks make all the waters conducting, and besides, connate waters may be present which frequently are concentrated solutions of a very low specific resistance. The pore volume is also great.

It would not be reasonable to give any average values of specific resistance for rocks and soils, because they vary inside very wide limits, but the order of magnitude can be obtained from the considerations given in this paper. The resistivity of surface waters in regions of type 1 seem to vary roughly between 3,000 and 50,000 ohm-cm. and that of ground waters between 3,000 and 15,000 ohm-cm. It follows that the specific resistance of soils in these regions can be as small as 3,000 ohm-cm. but that in general it is much higher. The specific resistance of volcanic rocks and crystalline schists close to the surface should under all circumstances considerably exceed 100,000 ohm-centimeters.

TABLE 5.—*Magnitude of Specific Resistance*

	Specific Resistance, Ohm-cm.	
	Saturated with Surface Waters	Saturated with Connate Waters
Normal limestone and sandstone	100,000–1,000,000	500–1,000
Clays and sands	40,000– 400,000	200– 400
Porous clays, sand, sandstones	6,000– 200,000	30– 200
Limestones and dolomites, marl, loess	3,000– 20,000	15– 40

In regions of type 2 the specific resistance of surface waters varies perhaps between 1,000 and 10,000 ohm-cm. Ground waters in such regions may have as low specific resistance as 100 ohm-cm. and connate waters as low as 10 ohm-cm. or less. We obtain Table 5 for magnitude of the specific resistance of rocks saturated with surface waters and connate waters.

SUMMARY

This paper is a study of the factors which mainly determine the electrical conductivity of soils and rocks. These factors are the conductivity and content of water. It is of considerable practical importance to study the electrical conductivity of soils and rocks, because

different strata may be identified by their electrical conductivity. Consequently both stratigraphic and structural studies may be carried out by electrical methods.

Conductivity of Waters in Soils and Rocks

Waters in soils and rocks can be divided into:

Surface	a. *Surface waters*, confined to the actual surface; the resistivity varies in magnitude from around 300,000 ohm-cm. to 10.
Zone of Aeration	b. *Soil water*, which may be discharged into the atmosphere by evaporation; chemical composition, not much known; resistivity from the magnitude of 10 ohm-cm. upwards, generally of magnitude 10,000 ohm-centimeters. c. *Intermediate vadose water* between the soil-water and ground-water belts (the narrow belt of capillary fringe water immediately above the water table, not taken into consideration); chemical composition, not much known; *conductivity* probably generally slightly higher than that of soil water.
Zone of Saturation	d. *Superficial ground water* from the ground-water table to a depth where water of different chemical composition is encountered; the resistivity varies in magnitude from around 100,000 ohm-cm. to 10. e. *Deep ground waters* below the superficial; the chemical composition is known from studies in mines and deep wells, especially in oil fields, where "connate waters" residual in marine sediments are often encountered. The resistivity may go as low as a few ohm-centimeters.

Conductivity of these waters can be determined either directly, if samples are available, or indirectly from the analysis according to methods described in the paper. The most important chemical constituents in waters for judging their electrical conductivity is chlorine, it being, as a matter of fact, possible to determine roughly the specific electric resistance of waters, excepting the purest, from their chlorine content alone.

It is important, from the viewpoint of applicability of electrical methods to structural studies, that only the deep ground waters parallel the geological structure.

Water Content in Soils and Rocks

Soils and rocks in the zone of aeration only contain water retained through adhesion. The water content is called the specific retention and is smaller than the pore volume. The specific retention is higher, the finer the grains, and varies from a few per cent for coarse material to the magnitude of 40 per cent for very fine material.

In the zone of saturation, all rock pores are filled with water and the water content consequently equals the pore volume, which varies from a fraction of 1 per cent for old igneous rocks and crystalline schists to 25 per cent and more for porous rocks.

Electrical Conductivity of Soils and Rocks

Examples are given of direct determination of the electrical conductivity of rocks by different methods. It is generally difficult, or impossible, to determine the electrical conductivity directly, however, and an indirect method to study the conductivity of rocks has, therefore, been developed, the principle of which is the following:

The relation between a resistivity factor p and the water content v of rocks or any porous material is given, the specific electrical resistance of a rock containing v_x per cent water of specific electrical resistance ρ being $pv_x \times \rho$; pv_x indicating the value of p for $v = v_x$. Examples of experimentally determined p, are given and found to agree well with theoretically found values. The resistivity factor, for practical purposes, can be considered to depend upon the water content only and thus does not seem to change much with different configuration of the pores. Practical experience shows that fine-grained material generally has lower resistivity than coarse-grained, even for the same water content, indicating that the waters are more salty in the former case. This is probably explained by the larger aggregate surface for retaining water by capillarity in a practically stagnant condition in the case of fine-grained material, by which there is more opportunity for the water to dissolve material and become a good electrolyte. It is possible that this circumstance will sometimes make results of conductivity determinations on fine-grained material misleading, as the retained water might be a stronger electrolyte than the recovered gravity water, available for analysis or determination of resistivity. The resistivity factor is of the magnitude 100 for igneous rocks and crystalline schists, 50 to 100 for dense lime and sandstone, 20 to 40 for clays and sands, and 2 to 20 for porous clays, sands, limes, sandstones and soils.

The specific electrical resistance of igneous rocks and dense insoluble rocks generally is very high, due to the small pore volume and the purity of the water in the pores. The magnitude of the specific resistance for such rocks is 10^5 ohm-centimeters.

The specific electrical resistance of young sediments varies tremendously and is generally low; owing to high porosity and salt water in the pores, values as low as 100 ohm-centimeters and less occur.

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A Theoretical Study of Apparent Resistivity in Surface Potential Methods*

By J. N. Hummel,† Göttingen, Germany

(New York Meeting, February, 1930)

The methods of electrical prospecting, which employ contact electrodes to produce an electric field in the ground, furnish information concerning the constitution of the material beneath the surface, as indicated by the distortion of this artificially produced field and anomalies therein.

To reveal the distortion of such field of known character, it is only necessary to measure qualitatively the direction of the field, using sensitive compensation methods. In such a case, the survey is relatively simple, and that is an important factor from a practical point of view. The distortion of the normal field thus determined permits an easy interpretation and gives approximate information as to the subsurface conditions, without the necessity of a thorough theoretical knowledge of the electrical field.

The true field of application of the equipotential lines and search-coil methods, which are based on this principle of directional determination of the electrical field in the ground, is that of search for deposits of economic minerals for the mining industry, although they can also be used in solution of other problems; for example, in the determination of the strike of anisotropic beds.

The distortion of the electric field as represented by the picture of equipotential lines or other measured curves is striking only when bodies having conductivities different from that of the country rock are embedded in the underground, or when the underground is anisotropic.

It is obvious that methods based on such a simple principle and capable of solving directly many of the practical problems of mining geology have found a widespread application. However, these methods are unable to furnish any information as to the presence, depth and physical properties of layers in the underground, which have different conductivities, but are parallel, or only slightly inclined, to the surface. Where such conditions exist in the underground, the picture of the field

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* Translated by J. A. Malkovsky, Assistant Professor, Department of Geophysics, Colorado School of Mines, Golden, Colo. and corrected and arranged by T. A. Manhart, Department of Geophysics, Colorado School of Mines. This paper is a translation of two papers published in Germany by the author, as follows: Der scheinbare spezifische Widerstand. *Ztsch. f. Geophysik* (1929) **5**, 89, 228.

† Assistant Geophysicist, Geophysical Institute.

obtained from the directional measurements is that of a normal, undisturbed field, even when the differences of electrical conductivities of the layers are very great. Yet the nature of the artificial electrical field produced over such an area is greatly affected by its subsurface structure and the distribution of the field should afford a good means for the investigation of the subsurface conditions.

Several methods have been devised to solve this special problem. They have, with their peculiar advantages and adaptabilities, each a field of application in which they are indispensable, but as they require the determination of the absolute values of the elements of the electric field, they necessitate less simple measurements and a more thorough understanding of the theory of the electric field.

The determination of the field can be made by measuring the absolute values of the potential along a finite line on the surface; which give potentials that are particularly affected by underground layers. This is not to imply that the determination of the distribution of potential on the surface could not be used for the location of conductive bodies in the ground. In fact, these methods have a wide field of application, although they are best suited and indispensable for the exploration of bedded structures.

The field methods based on the measurement of the absolute magnitudes of the potential belong to the most elegant, and in their principles, to the oldest methods of electric prospecting.[1] They have been used with great success,[2] according to the most recent publications. Outside

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[2] O. H. Gish and W. J. Rooney: Measurement of the Resistivity of Large Volumes of Undisturbed Earth. *Terrestrial Magnetism* (1925) **30**, 161–188 and *Phys. Rev.* (1925) **25**, 254.

O. H. Gish: Improved Equipment for Measuring Earth-current Potentials and Earth Resistivity. Nat. Research Council *Bull.* 11 (1926) 86–91.

W. J. Rooney and O. H. Gish: Results of Earth-resistivity Surveys near Watheroo, Western Australia and at Ebro, Spain. *Terrestrial Magnetism* (1927) **32**, 49–63.

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F. W. Lee: Measuring the Variation of Ground Resistivity with a Megger. U. S. Bur. Mines *Tech. Paper* 440 (1928).

W. A. Hotchkiss, W. J. Rooney and J. Fisher: Earth-resistivity Measurement in the Lake Superior Copper Country. Geophysical Prospecting, A. I. M. E. (1929) 51.

E. G. Leonardon and S. F. Kelly: Some Applications of Potential Methods to Structural Studies. Geophysical Prospecting, A. I. M. E. (1929) 180.

W. Weaver: Certain Applications of the Surface Potential Method. Geophysical Prospecting, A. I. M. E. (1929) 68.

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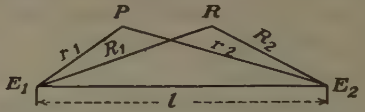
of some general statements, nothing has been published about their theory, although their whole development depends upon the theoretical study of the electric field. A thorough knowledge of the electric field is also necessary to a successful interpretation of the results.

The practical importance of these methods certainly justifies the author's work and the mathematical analysis of at least a few of the simplest problems.

The theory of the most important method belonging to this group is based upon the determination of the average resistivity of a volume of the underground, which, from the macroscopic point of view, may be considered as homogeneous. The determination of this resistivity, which can be made in many different ways, is based on the following considerations: At the surface of the ground an electric current I is introduced into a homogeneous and isotropic underground by means of two point electrodes E_1 and E_2. If the current flows from E_1 to E_2 and if the conductivity of the underground is σ, the potential at any point P in space is

$$V_P = \frac{I}{2\pi\rho}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$



where r_1 and r_2 are the distances of the point P from the electrodes. The potential difference between two points P and R in space, which have from the electrodes the distances r_1, r_2 and R_1 and R_2 respectively, is:

$$V_P - V_R = \frac{I}{2\pi\rho}\left(\frac{1}{r_1} - \frac{1}{r_2} - \frac{1}{R_1} + \frac{1}{R_2}\right)$$

This equation solved for the reciprocal value of σ gives the equation

$$\frac{1}{\sigma} = \rho = 2\pi\frac{V}{I}\frac{1}{\left(\frac{1}{r_1} - \frac{1}{r_2} - \frac{1}{R_1} + \frac{1}{R_2}\right)}$$

which is true for any position of the current electrodes E_1 and E_2 and the search electrodes P and R, where $V = V_P - V_R$.

This expression contains the magnitudes V, I, r_1, r_2, R_1 and R_2, which are measured on the surface, therefore the reciprocal conductivity, or

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[2] (*Continued.*) I. B. Crosby and E. G. Leonardon: Electrical Prospecting Applied to Foundation Problems. Geophysical Prospecting, A. I. M. E. (1929) 199.

H. Hunkel: Uber den angeblichen geophysikalischen Nachweis von Salzdomen im Oberelsass. *Ztsch. f.d. Kali und Steinsalzindustrie, sowie das Salinenwesen* (1928) 366–368, 383–385; (1929) 7–10.

I. B. Crosby and S. F. Kelly: Electrical Subsoil Exploration and the Civil Engineer. *Eng. News Rec.* (1929) 270–273.

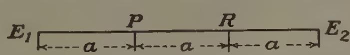
C. A. Heiland: Geophysical Methods of Prospecting. Colo. School of Mines *Quarterly* (1929) **24**, 106–111.

the resistivity of the ground, can be found experimentally. The contact resistance between the search electrodes and the ground is absent from the formula. This contact resistance varies greatly with the different properties of the electrodes themselves, as well as those of the ground in their immediate neighborhood, and may vary appreciably from point to point even on a uniform surface of a homogeneous ground. As the section of the ground that is dealt with here cannot be considered as strictly homogeneous, except when having great dimensions, this procedure, in which the underground as a whole is taken into consideration, is of great importance for the determination of the correct average resistivity.

The position of the two search electrodes with respect to the current electrodes is arbitrary. It is possible, by selecting the position of the electrodes, either to simplify the field procedure or to give the expression for resistivity a form that will simplify the interpretation of the results. Five different procedures will be given and described.

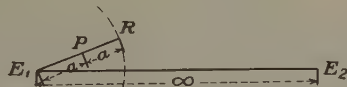
1. According to Wenner, who was the first to develop and use this method, the two search electrodes are placed in line with the two current electrodes, so that all the four points lie in the same line and at the same distance from each other. When the distance between the electrodes is a, r_1 and R_2 equal a and r_2 and R_1 equal $2a$. The expression for the resistivity changes to one admirably suited for its computation; that is,

$$\rho = 2\pi a \frac{V}{I}$$



2. Another method consists in placing the two current electrodes E_1 and E_2 a great distance apart and measuring the potential in the neighborhood of either one of them. The search electrodes are kept in a line passing through the near-by current electrode, the direction of this line being arbitrary. One of the search electrodes can again be put at a distance twice that of the other from the near current electrode. In the case of great distance between E_1 and E_2, it can be safely assumed that the farther current electrode has no influence upon the potential in the surveyed area and serves only to transport the current. Under this assumption, r_2 and R_2 in the expression for ρ become infinite, and the resistivity is given by the formula

$$\rho = 4\pi a \frac{V}{I}$$

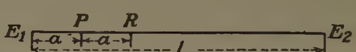


The advantage of this method lies in the fact that it is not necessary to change the position of the current electrodes when the distance a, between the search electrodes, is varied. It is necessary to move the search electrodes only, which results in a simplification of the field procedure. The disadvantageous features of this method are the great distance between the current electrodes, which necessitate long current

leads, and the possible error created by the assumption that r_2 and R_2 are infinite.

3. If the disadvantages of the long current leads and the possible error introduced in the last method are to be eliminated, and the advantage of the stationary current electrodes retained, the current electrodes can be placed closer together and the influence of the second one upon the potential in the surveyed area taken into consideration. This arrangement represents the third procedure; in which, however, the search electrodes must be kept on the line connecting the current electrodes. The distance between them being l, and $-l \leqq 2a \leqq +l$, the less convenient expression for ρ becomes:

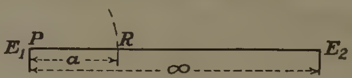
$$\rho = 2\pi \frac{V}{I} \frac{2a(l-a)(l-2a)}{(l-2a)^2 + al}$$



which for $l = 3a$ goes over to Wenner's formula.

4. It is also feasible to measure the potential difference between one of the two current electrodes and a single search electrode, which is placed anywhere in the neighborhood of this current electrode. If the influence of the second electrode is neglected, the formula for ρ becomes

$$\rho = 2\pi a \frac{V_e - V}{I}$$



where V_e is one-half the total potential difference between the current electrodes.

5. If in the last procedure the influence of the second current electrode is taken into consideration, and the search electrode moved along the line connecting the current electrodes, the equation for ρ will be

$$\rho = 2\pi \frac{V_e - V}{I} \frac{a(l-a)}{(l-2a)}$$

With any of these five methods, when applied over an underground which is homogeneous and isotropic throughout, the surveyed area must furnish a constant magnitude of ρ for any distance a between the electrodes. The variation of the distance a serves only as a control, and actually there exists a uniform resistivity in the surveyed area. However, if the computed value of ρ does change when a is varied, it is a proof that the underground cannot be, even from the macroscopic point of view, considered as homogeneous, and that there are large discontinuities of its physical properties. It is evident that the measurements resulting in the determination of change of ρ can be used to draw conclusions concerning the nature of these inhomogeneities and that these measurements constitute an electrical method of prospecting. Naturally, in this method, the formulas given above lose their validity, as they are developed under the assumption of a homogeneous medium and do not satisfy the conditions in inhomogeneous ground.

Even though the underground is known to be inhomogeneous, these formulas may be used to compute the values of ρ in the same way as for homogeneous ground. In this case, however, the quantity ρ varies with a and loses its meaning as a strictly defined resistivity. It now has no physical meaning and will be called the "apparent resistivity." Its magnitude, which cannot always be computed, lies between the two or more actual resistivities present in the ground. The apparent resistivity is then like the average resistivity, a certain mean value, the magnitude of which, however, does not remain constant and is a function of the actual or the average resistivity. This function usually assumes a complicated form but can be expressed mathematically in some special cases. Its determination is the basis for interpretation of results of practical surveys and for the determination of the applicability of methods belonging to this type of prospecting.

It should be possible to compare directly the differences of potential as actually measured with those computed, thus eliminating the resistivity, and from these differences to draw conclusions about the underground. This is possible and has been done,[3] but it is necessary to keep in mind that the drop of potential in a normal field does not have a constant value and is a function not only of the position of the search electrodes but also of the distance between the current electrodes and their potential difference. It is more instructive to compare magnitudes that are independent of the particular conditions of the experiment. Such magnitudes, which depend on the nature of the underground in relation to the arrangement of search electrodes, are the apparent resistivity and the average and actual resistivities.

The connecting link between the apparent resistivity and the average resistivities is the potential function of the inhomogeneous underground. If this function is known, the apparent resistivity can be expressed as a function of the average resistances. This will be illustrated by two typical examples; that of a spherical body of low resistivity in homogeneous and isotropic underground and that of two or three parallel homogeneous and isotropic layers of different resistivities below the surface.

Location of Disturbing Bodies

In the earlier investigations, the author succeeded in developing in simple terms the potential functions of a number of examples of inhomogeneous ground.[4] The inhomogeneity was caused by massive

[3] W. Weaver: *Op. cit.*

[4] J. N. Hummel: Uber die Tiefenwirkung bei geoelektrischen Potentiallinienmethoden. *Ztsch. f. Geophys.* (1928) 22–27.

Untersuchung der Potentialverteilung für einen speziellen Fall im Hinblick auf geoelektrische Potentiallinienverfahren. *Idem.*, 67–76.

Beiträge zur geoelektrischen Methode. *Idem.*, 179–203.

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bodies of different resistivities embedded in a homogeneous and isotropic country rock. The methods of computation previously used for the different shaped bodies and the results (provided that a finite distance between the current electrodes be assumed) can be applied to the present investigation. In order to enable the reader easily to draw a comparison, one case of potential distribution on the surface will be recomputed.[5]

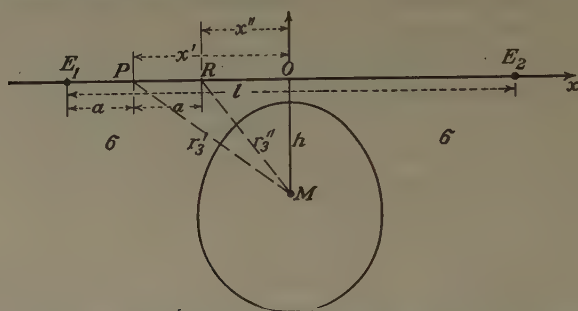

FIG. 1.—BODY OF VERY GOOD CONDUCTOR IN HOMOGENEOUS AND ISOTROPIC COUNTRY ROCK.

A body of very good conductivity is embedded in a homogeneous and isotropic ground (Fig. 1). The current is introduced into the ground and the potential differences measured as described in method 3; that is, the current electrodes are stationary and the search electrodes are moved along the line connecting the current electrodes. The respective positions of the conducting body and the current electrodes are the same as in the case for which the potential function was computed in the former investigation; namely, the midpoint of the line connecting the two current electrodes lies directly above the center of the spherical body and the distance l separating these two electrodes is four times the depth to the center of the sphere. In this special case the potential function at a point on the surface is

$$V\rho = \frac{I}{2\pi}\rho\left[\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{0.1lx^2}{r_3{}^3}\right]$$

where r_3 is the distance of the point on the surface from the center of the sphere and x is the projection of r_3 in the direction of the current. The difference of potential between two search electrodes arranged in this particular way is

[4] (*Continued.*) Untersuchung der Potentialverteilung um verschiedene Störungskörper, die sich in einem an und für sich homogenen Stromfelde befinden. *Gerlands Beiträge* (1929) **21**, 204–214.

[5] J. N. Hummel: Theoretische Grundlagen fuer die Auffindung von Stoerungskoerpern mittels solcher geoelektrischer Methoden, bei denen zwei punktfoermige Elektroden zur Erzeugung eines künstlichen Feldes verwandt werden. *Gerlands Beiträge* (1928) **20**, 281–287.

$$V = V_P - V_E = \frac{I}{2\pi}\rho \left\{ \frac{1}{2a} - \frac{1}{l-a} + \frac{1}{l-2a} + \frac{0.1l\left(\frac{l}{2}-a\right)}{\left[\left(\frac{l}{4}\right)^2 + \left(\frac{l}{2}-a\right)^2\right]^{3/2}} - \frac{0.1l\left(\frac{l}{2}-2a\right)}{\left[\left(\frac{l}{4}\right)^2 + \left(\frac{l}{2}-2a\right)^2\right]^{3/2}} \right\}$$

and the apparent resistivity for $-l \leqq 2a \leqq +l$ is

$$\rho_s = \rho\left(1 + 0.1l\left\{ \frac{\left(\frac{l}{2}-a\right)}{\left[\left(\frac{l}{4}\right)^2 + \left(\frac{l}{2}-a\right)^2\right]^{3/2}} - \frac{\left(\frac{l}{2}-2a\right)}{\left[\left(\frac{l}{4}\right)^2 + \left(\frac{l}{2}-2a\right)^2\right]^{3/2}} \right\} \cdot \frac{2a(l-a)(l-2a)}{(l-2a)^2 + al}\right)$$

The ratio $\rho_s/\rho \equiv u$ as a function of $a/l \equiv v$ is represented graphically in Fig. 2, which shows a pronounced change in u.

This is interesting because the potential curve along the same line shows much less of the disturbing effect of the sphere. The smallest value of the apparent resistivity is only about 57 per cent. of the actual resistivity of the ground.

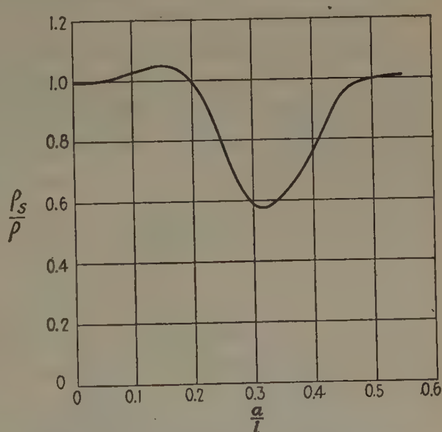

FIG. 2.—CURVE OF APPARENT RESISTIVITY OVER DISTURBING BODY SHOWN IN FIG. 1.

It can easily be seen that this method can be used very well for the location of disturbing bodies, especially those which have a shape that does not readily permit the use of either equipotential line or search-coil methods. Yet, even with a great departure of apparent resistivity from the average resistivity of the overburden, interpretation of the results of measurements obtained over disturbing bodies in the ground is not simple because of the many-valued determining factors. As long as no limiting assumptions can be made about the nature of the underground, and also as long as the shape, size, conductivity and position of the body are entirely unknown, it will not be possible even from several

surveyed and computed curves to gain much more information than that some inhomogeneity is present in the ground. To that, also, comes the fact that the deviation of the apparent resistivity produced by an inhomogeneity varies with the volume of the disturbing body, being greater with a large body than with a small one. Inasmuch as the natural orebodies most frequently have no great volumes (veins, ore-shoots), the information of their presence in the ground will be more readily gained from either the equipotential line or search-coil methods, which will be preferred because of their simpler field procedures.

Determination of Depth and Conductivity of Parallel Layers

The real field of application of methods based on the determination of apparent resistivity is that of the investigation of bedded underground structures. There the anomalies of the apparent resistivity are especially great because of the great volume of the inhomogeneity, and their interpretation is relatively simple because of the limited number of assumptions to be made.

In the investigation of bedded structures, almost all other electrical methods fail. Since these tectonic conditions frequently occur in nature, their investigation is important, and it is only natural that pure geology as well as the mining industry is much interested in methods based on the determination of the apparent resistivity and its interpretation.

The analysis of curves of measured quantities is relatively simple, because if the assumption of a succession of horizontal layers is geologically justified relatively few unknowns remain to be determined. The average resistivity under these conditions is a function of the depth alone, and this depth needs to be determined. A small dip of layers does not complicate the conditions to any great extent. For the computation of the apparent resistivity, a knowledge of the potential function is necessary and the expression for the potential must be derived first. This does not have, in this case, such a simple form as that for disturbing bodies of a geometrical shape in a homogeneous and isotropic ground, and must be represented by an infinite series, even if the conditions of the underground have been idealized to a great extent. In the derivation of the series, the physical point of view will be emphasized rather than the formal mathematical treatment.[6]

The simplest case of two parallel layers in the underground will be discussed. The upper layer is delimited by the surface of the ground; the lower layer has an infinite thickness. Three layers are actually present, as the air above the surface represents a third layer with an infinite resistivity. For the purpose of generalization these three layers are assumed to be homogeneous, isotropic and of given resistivities. The layers are bounded by two parallel planes and extend throughout

[6] Another derivation can be found in F. Ollendorff: Erdstroeme, 69. Berlin, 1928.

the whole space. A current source is placed within the middle layer, which has the thickness h. Through this source flows a current of density $I/4\pi$ and the current source lies at a distance d from the lower boundary of the upper layer. The value of the potential at any point in the three layers is to be determined. To distinguish between the different magnitudes, all the symbols related to the lower layer have the index one prime. Those related to the upper layer are designated by the double prime. Those related to the middle layer remain without index. The potential functions on the boundary planes have to fulfill the equations

$$\sigma_r \frac{\partial V_r}{\partial n} = \sigma_s \frac{\partial V_s}{\partial n} \text{ and } V_r = V_s$$

in which n is the direction normal to the plane. This equation can be satisfied only when both of the boundary planes are planes of symmetry to the current source. When there is only one boundary plane, the potential function is found by using images of the real current source on the opposite side of the boundary plane. These images are then used in the computation of the potential as imaginary current sources. With two boundary planes, the potential function is computed from an infinite succession of images of the real current source and its images formed with respect to both of the boundary planes. From the infinite number of the true and imaginary current sources the potential is then computed from

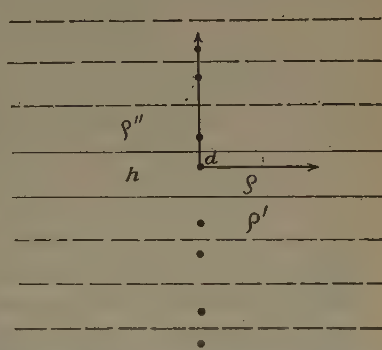

FIG. 3.—CONSTRUCTION OF IMAGES OF TRUE CURRENT SOURCE WITH RESPECT TO TWO BEDDING PLANES.

$$V = \frac{\rho}{4\pi} \sum^{m} \frac{I_m}{r_m}$$

where r_m signifies distances given by the positions of the real as well as imaginary current sources.

If the origin of the system of coordinates is placed at the real current source with the x axis parallel to the boundary line of the layers and the y axis normal to it; the r^2_m is either $x^2 + (y \pm 2nh)^2 + z^2$ or, $x^2 + (y \pm 2nh - 2d)^2 + z^2$, where n is any of the succession of whole numbers between 0 and ∞. The current of the images $I_m/4\pi$ is not yet given, but can be computed from the limiting conditions.

Using the symbols

$$\frac{\rho' - \rho}{\rho' + \rho} = k_1, \text{ and } \frac{\rho'' - \rho}{\rho'' + \rho} = k_2,$$

the equations for the potentials are

$$V = \frac{I}{4\pi}\rho\left\{\sum_{n=0}^{\infty}\left[\frac{k_1^n \cdot k_2^n}{\sqrt{x^2 + (y + 2nh)^2 + z^2}} + \frac{k_1^n \cdot k_2^{n+1}}{\sqrt{x^2 + (y - 2nh - 2d)^2 + z^2}}\right] + \sum_{n=1}^{\infty}\left[\frac{k_1^n \cdot k_2^n}{\sqrt{x^2 + (y - 2nh)^2 + z^2}} + \frac{k_1^n \cdot k_2^{n-1}}{\sqrt{x^2 + (y + 2nh - 2d)^2 + z^2}}\right]\right\},$$

$$V' = \frac{I}{4\pi}\rho(1 + k_1)\sum_{n=0}^{\infty}\left[\frac{k_1^n \cdot k_2^n}{\sqrt{x^2 + (y - 2nh)^2 + z^2}} + \frac{k_1^n \cdot k_2^{n+1}}{\sqrt{x^2 + (y - 2nh - 2d)^2 + z^2}}\right],$$

$$V'' = \frac{I}{4\pi}\rho(1 + k_2)\left[\sum_{n=0}^{\infty}\frac{k_1^n \cdot k_2^n}{\sqrt{x^2 + (y + 2nh)^2 + z^2}} + \sum_{n=1}^{\infty}\frac{k_1^n \cdot k_2^{n-1}}{\sqrt{x^2 + (y + 2nh - 2d)^2 + z^2}}\right].$$

The resistivity ρ'' being infinity, as is always the case, the k_2 becomes equal to $+1$, and, under the assumption of $d = 0$ which is permissible when the distance between the electrodes is large, these equations assume a much simpler form. The potential on the boundary plane of the non-conducting layer is found by putting $y = 0$ and is

$$V_{y=0} \equiv V''_{y=0} = \frac{I}{2\pi}\rho\left[\frac{1}{\sqrt{x^2 + z^2}} + 2\sum_{n=1}^{\infty}\frac{k_1^n}{\sqrt{x^2 + (2nh)^2 + z^2}}\right].$$

This equation, which represents the solution of the geophysical problems, will be analyzed first.

Considering the procedure of Wenner, the potential difference between the two search electrodes is

$$V = \frac{I}{2\pi a}\rho\left\{1 + 4\sum_{n=1}^{\infty}k_1^n\left[\frac{1}{\sqrt{1 + \left(2n\frac{h}{a}\right)^2}} - \frac{1}{\sqrt{4 + \left(2n\frac{h}{a}\right)^2}}\right]\right\},$$

and from this equation the apparent resistivity is

$$\rho_s = \rho\left\{1 + 4\sum_{n=1}^{\infty}k_1^n\left[\frac{1}{\sqrt{1 + \left(2n\frac{h}{a}\right)^2}} - \frac{1}{\sqrt{4 + \left(2n\frac{h}{a}\right)^2}}\right]\right\}.$$

This expression for apparent resistivity remains unchanged if one of the current electrodes is moved to infinity, as in the field procedure of method 2.

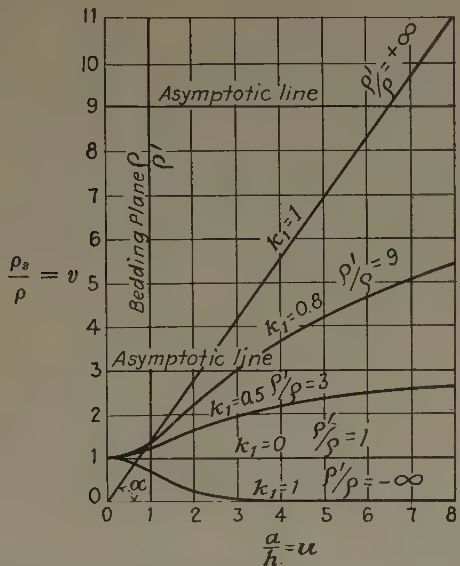

FIG. 4.—CURVES OF APPARENT RESISTIVITY OVER SURFACE LAYER OF THICKNESS h FOR DIFFERENT RATIOS OF RESISTIVITIES OF SURFACE LAYER AND INFINITE LOWER LAYER.

TABLE 1.—*Values of Magnitudes in Bracket*

$2n:u$	Bracket Value	$2n:u$	Bracket Value	$2n:u$	Bracket Value
0.125	0.493 252	4.75	0.011 982	9.75	0.001 557
0.25	0.474 159	5	0.010 421	10	0.001 446
0.375	0.444 893	5.25	0.009 115	10.25	0.001 345
0.5	0.409 356	5.5	0.008 014	10.5	0.001 253
0.75	0.331 835	5.75	0.007 081	10.75	0.001 167
1	0.259 993	6	0.006 285	11	0.001 093
1.25	0.200 696	6.25	0.005 602	11.25	0.001 023
1.5	0.154 700	6.5	0.005 014	11.5	0.000 959
1.75	0.119 851	6.75	0.004 505	11.75	0.000 900
2	0.093 660	7	0.004 061	12	0.000 846
2.25	0.073 957	7.25	0.003 673	13	0.000 668
2.5	0.059 043	7.5	0.003 332	14	0.000 530
2.75	0.047 657	7.75	0.003 032	15	0.000 437
3	0.038 878	8	0.002 767	16	0.000 361
3.25	0.032 037	8.25	0.002 531	17	0.000 301
3.5	0.026 652	8.5	0.002 322	18	0.000 254
3.75	0.022 369	8.75	0.002 134	19	0.000 216
4	0.018 929	9	0.001 966	20	0.000 186
4.25	0.016 141	9.25	0.001 815	22	0.000 140
4.5	0.013 861	9.5	0.001 680	24	0.000 108

The apparent resistivity as a function of a under these conditions is entirely independent of the position of the current-generating and measuring apparatus and is given by the change of the average resistance with the depth only. The ratio ρ_s/ρ plotted as a function of a/h for different values of a (or h) gives curves that, by selection of these coordinates, assume a universal character and are valid for any resistivity of the upper layer of the ground and for any depth of the lower layer.

Fig. 4 represents several of these curves. In order to simplify the computation of more of them, the repeatedly used values of the magnitudes in the bracket for different ratios $2n:u$ are given in Table 1, where $u = a/h$. These magnitudes are then used for the computation with different values of k_1, and also will be of service in the computation when more than one surface layer is present.

In the analysis of the curves, the unknown factor, the depth h, which must be taken from a universal scale, is to be determined. The other two unknowns—that is, the resistance of the upper layer ρ and that of the lower layer ρ'—must be taken from the characteristic shape of the curves. These possess five characteristic points: one point for $u = 0$, two points of maximum curvature, the inflection point between the two points of maximum curvature, and last and finally, the point at infinity.

The slope of the curve is given by the first differential quotient, the inflection point by the second differential quotient, and the points of maximum curvatures by the third differential quotient. These three differential quotients are:

$$\frac{\partial v}{\partial u} \equiv v' = 16\frac{1}{u^3}\sum_{n=1}^{\infty} k_1^n \cdot n^2 \left\{ \frac{1}{\left[1 + \left(\frac{2n}{u}\right)^2\right]^{3/2}} - \frac{1}{\left[4 + \left(\frac{2n}{u}\right)^2\right]^{3/2}} \right\},$$

$$\frac{\partial^2 v}{\partial u^2} \equiv v'' = 48\frac{1}{u^4}\sum_{n=1}^{\infty} k_1^n \cdot n^2 \left\{ \frac{4}{\left[4 + \left(\frac{2n}{u}\right)^2\right]^{5/2}} - \frac{1}{\left[1 + \left(\frac{2n}{u}\right)^2\right]^{5/2}} \right\},$$

$$\frac{\partial^3 v}{\partial u^3} \equiv v''' = 192\frac{1}{u^5}\sum_{n=1}^{\infty} k_1^n \cdot n^2 \left\{ \frac{1 - \left(\frac{n}{u}\right)^2}{\left[1 + \left(\frac{2n}{u}\right)^2\right]^{7/2}} - \frac{16\left[1 - \left(\frac{2n}{u}\right)^2\right]}{\left[4 + \left(\frac{2n}{u}\right)^2\right]^{7/2}} \right\}.$$

All curves have their origin at the point $u = 0$ and $v = 1$. The apparent resistivity, therefore, is the resistivity of the upper layer as long as the distance a is very small. Because the first differential quotient at the point $u = 0$ and $v = 1$ is also zero, the curve will be parallel to the axis at that point, so that the apparent resistivity differs very little from the resistivity of the upper layer, even if the distance a has a finite value.

For $u = \infty$, the equation of the curve assumes the form

$$\lim_{u \to \infty} v = 1 + 2\sum_{n=1}^{\infty} k_1^n$$

$$\sum_{n=1}^{\infty} k_1^n = \frac{k_1}{1 - k_1} \text{ for } k_1^2 < +1,$$

so that the ρ_s is equal to ρ' for a point in infinity, which means that for a distance a very much greater than the depth h the influence of the upper layer completely disappears. The first differential quotient is here also equal to zero, and the lines asymptotic to the curves are parallel to the u axis.

The coefficient k_1 varies between $+1$ and -1. If k_1 is positive, that is, if the resistance of the lower layer is higher than that of the upper, the curve ascends. With k_1 negative, or with the lower layer of a lower resistivity than the upper one, the curve descends. For $k_1 = +1$, which represents the case when the resistivity of the lower layer is infinitely large, the ascending curve is the steepest, and for $k_1 = -1$ (the zero resistivity of lower layer) the curve descends the most rapidly. The relative deviation of the curve from the normal values for a negative k_1 is always greater than for the same value of positive k_1. This shows, as has already been pointed out by Weaver,[7] that a bed of better conducting material in the depth can be detected more readily than a poorly conducting one. Nevertheless, the curves in Fig. 4 show that even the layers of poor conductors in the ground have a decided influence upon the apparent resistivity. This fact assures the method of a very good field of application, because the few other electrical methods that can be used for investigation of bedded underground can be used advantageously only in the case of conductors.

The case of $k_1 = +1$ is particularly interesting, as the most important parts of the curves approach this limiting curve as soon as the conductivities reach a certain ratio, about 1:10. The curve equation for this limiting case of $k_1 = +1$ can be further changed by using the expression,

$$\sum_{n=1}^{2i}\left[\frac{1}{\sqrt{1+\left(\frac{2n}{u}\right)^2}} - \frac{1}{\sqrt{4+\left(\frac{2n}{u}\right)^2}}\right]$$

$$= \frac{1}{2}\sum_{n=1}^{i}\left[\frac{1}{\sqrt{1+\left(\frac{2n}{u}\right)^2}} - \frac{1}{\sqrt{1+\left(\frac{2n-1}{u}\right)^2}}\right] + \sum_{n=i+1}^{2i}\frac{1}{\sqrt{1+\left(\frac{2n}{u}\right)^2}},$$

This expression is formed by placing the $\sqrt{4}$ in the denominator of the second summation before the brackets and by rearranging the

[7] W. Weaver: *Op. cit.*

members in the brackets. The expression for v is then

$$v=1+2\sum_{n=1}^{\infty}\left[\frac{1}{\sqrt{1+\left(\frac{2n}{u}\right)^2}}-\frac{1}{\sqrt{1+\left(\frac{2n-1}{u}\right)^2}}\right]+4\lim_{i\to\infty}\sum_{n=i+1}^{2i}\frac{1}{\sqrt{1+\left(\frac{2n}{u}\right)^2}}$$

because further

$$\lim_{i\to\infty}\sum_{n=i+1}^{2i}\frac{1}{\sqrt{1+\left(\frac{2n}{u}\right)^2}}=\lim_{i\to\infty}\sum_{n=i+1}^{2i}\frac{u}{2n}=\frac{u}{2}\sum_{n=1}^{\infty}\frac{1}{2n(2n-1)}$$

$$=\frac{u}{2}\log 2=\frac{u}{2}\cdot 0.6931475\cdots$$

the curve of the equation for $k = +1$ can be written as

$$v=1+2\sum_{n=1}^{\infty}\left[\frac{1}{\sqrt{1+\left(\frac{2n}{u}\right)^2}}-\frac{1}{\sqrt{1+\left(\frac{2n-1}{u}\right)^2}}\right]+2u\cdot\log 2.$$

For large values of u the sum in this expression is $-\frac{1}{2}$ and the equation changes to

$$\lim_{u\to\infty} v = 2u \log 2 = u\cdot 1.386295\cdots$$

Fig. 4 shows that this limiting value will practically be reached when $u = 1.5$. From that point the curve practically coincides with its asymptotic line, which is a straight line passing through the origin of coordinates and having the inclination $\tan\alpha = 1.386295\cdots$ As all of the other asymptotic lines are parallel to the u axis, their direction has a discontinuity at $k_1 = +1$. However, the corresponding curves change into each other continuously.

All the series are convergent for all finite values of u, with the exception of the series for $k_1 = +1$ and simultaneously $u = \infty$, which is divergent. This represents the case of a current supplied through two electrodes to a parallel layer of a finite thickness and of infinite horizontal extent, in which case the current will be reduced to zero when the distance between the current electrodes becomes infinite. This is in contrast to the conditions in the conductive medium having an infinite extent in all directions. When two current electrodes are applied to such a medium at two points of infinite distance apart, the current has everywhere a finite value, given by Ohm's law from the potential difference between the two electrodes and their contact resistances. This latter fact should also be proved with the help of the curve equation.

For $u = 1$ and $k_1 = +1$, the v is very nearly 1.5. This indicates that when the distance a is equal to the depth of the layer, the apparent resistivity is $1\frac{1}{2}$ times the resistivity of the upper layer. Inversely, this relation can be used in analysis of the surveyed curves to find their scale and, consequently, the depth of the lower bedding plane of the layer.

According to practice,[8] the line drawn parallel to v in Fig. 4 is then to be considered as the boundary line of the layers.

When using the procedure described in method 4—that is, when only one current and one search electrode are employed to measure the potential drop—the apparent resistivity is

$$\frac{\rho_s}{\rho} \equiv v = 1 + 2\sum_{n=1}^{\infty} k_1{}^n \frac{1}{\sqrt{1 + \left(\frac{2n}{u}\right)^2}}$$

Here, the shape of the curves is very different from those previously given. They correspond with the curves in Fig. 4 in two points only—in the point $u = 0$ and the point $u = \infty$. Between these, the curve is concave with respect to the u axis and already ascending at the point $u = 0$. This immediate steep ascent of the curve at $u = 0$ is due to the fact that the potential is influenced by the lower layer, even before the distance between the search electrodes reaches a certain value. The series converges for the values of $k_1 < +1$ only, and diverges for any values of u if the value $k = +1$. As this modification of the procedure has not yet found much application in practice, it will not be discussed further.

The similarity of all the methods discussed here with those used in seismic prospecting is apparent. The current electrodes correspond to the explosion points and the search electrodes to seismographs. The increase in potential difference of the current used is equivalent to the increase of the dynamite charge and the relative conductivities correspond to the relative rigidity of the material in underground. In both electric and seismic methods, curves are determined for different distances of the base points. Their shape affords several analogies and their analysis serves the same purpose; that is, that of the description of the bedded underground.

Apparent Resistivity When There Are Four Parallel Beds

The actual conditions of bedded structure are seldom so simple as to permit the assumption of two layers, and for practical purposes it is desirable to analyze the next simplest case of two surface layers over a semi-infinite one. In this type of tectonic condition, four layers actually have to be taken into consideration (Fig. 5). The upper one is represented by the layer of air, which has an infinite resistivity and an infinite vertical extent. The two inner layers have a finite thickness. The lowest one occupies the rest of the space. For the general solution of the problem it will be assumed that all the four layers possess any given resistivities. For the outside layer, which represents the atmosphere, the index 0 will be used. The symbol for the adjoining upper surface

[8] E. G. Leonardon, S. F. Kelly and I. B. Crosby: *Op. cit.*

layer has no index; the one relating to the next surface layer has index 1; and the symbol relating to the lowest unlimited layer has the index 2.

The actual resistivity of each layer will always be ρ, its thickness h and the potential V. For computation of the apparent resistivity, the determination of the potential function again is necessary. The procedure requires the use of images of the real current source with respect to the three boundary planes of the layer, and a multiple infinite number of imaginary current sources will then be obtained. The computation

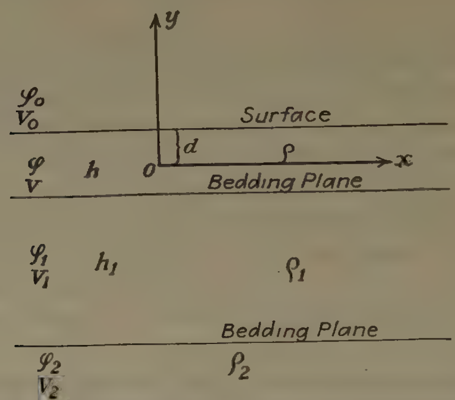

FIG. 5.—ASSUMED TECTONIC CONDITIONS. TWO SURFACE LAYERS OF GIVEN RESISTIVITIES AND THICKNESS, OVERLYING A HOMOGENEOUS UNDERGROUND.

becomes much more complicated, yet still remains elementary. The coefficients which occur in the formulas for potential are denoted by,

$$\frac{\rho_0 - \rho}{\rho_0 + \rho} = k_0, \frac{\rho_1 - \rho}{\rho_1 + \rho} = k, \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} = k_1.$$

The real current source and also the starting point for the computation lies in one of the two inner layers and, for a concrete example, this current source is assumed to be in the upper surface layer. The distance of the current source from the surface is d, and at it lies the origin of coordinates. The x axis is parallel and the y axis normal to the bedding planes. By the same procedure as before, the sum of potentials is formed and its summations arranged as follows:

$$
\begin{aligned}
V_0 = \frac{I}{4\pi}\rho(1+k_0)\sum_{n=0}^{\infty}\Bigg\{ & \frac{k^n \cdot k_0^n}{\sqrt{x^2+(y+2nh)^2+z^2}} + \frac{k^{n+1} \cdot k_0^n}{\sqrt{x^2+[y+2(n+1)h-2d]^2+z^2}}\Bigg\} \\
+ \frac{I}{4\pi}\rho(1+k_0)(1-k^2)k_1\sum_{n=0}^{\infty}\Bigg\{ & \frac{(n+1)k^n \cdot k_0^n}{\sqrt{x^2+[y+2h_1+2(n+1)h-2d]^2+z^2}} \\
& + \frac{(n+1)k^n \cdot k_0^{n+1}}{\sqrt{x^2+[y+2h_1+2(n+1)h]^2+z^2}}\Bigg\} \\
+ \frac{I}{4\pi}\rho(1+k_0)(1-k^2)k_1^2\sum_{n=0}^{\infty}\Bigg\{ & \frac{\left[\frac{n(n+1)}{2}(1-k^2)-(n+1)k^2\right]k^{n-1} \cdot k_0^n}{\sqrt{x^2+[y+4h_1+2(n+1)h-2d]^2+z^2}} \\
& + \frac{\left[\frac{(n+1)n}{2}(1-k^2)-(n+1)k^2\right]k^{n-1} \cdot k_0^{n+1}}{\sqrt{x^2+[y+4h_1+2(n+1)h]^2+z^2}}\Bigg\} \\
+ \frac{I}{4\pi}\rho(1+k_0)(1-k^2)k_1^3\sum_{n=0}^{\infty}\Bigg\{ & \frac{\left[(1-k^2)^2\frac{1}{6}(n-1)n(n+1)-(1-k^2)n(n+1)k^2+(n+1)k^4\right]k^{n-2} \cdot k_0^n}{\sqrt{x^2+[y+6h_1+2(n+1)h-2d]^2+z^2}} \\
& + \frac{\left[(1-k^2)^2\frac{1}{6}(n-1)n(n+1)-(1-k^2)n(n+1)k^2+(n+1)k^4\right]k^{n-2} \cdot k_0^{n+1}}{\sqrt{x^2+[y+6h_1+2(n+1)h]^2+z^2}}\Bigg\} + \cdots
\end{aligned}
$$

$$
\begin{aligned}
V = \frac{I}{4\pi}\rho\sum_{n=0}^{\infty}&\left\{\frac{k^{n+1}\cdot k_0^{\,n}}{\sqrt{x^2+[y+2(n+1)h-2d]^2+z^2}}+\frac{k^n\cdot k_0^{\,n}}{\sqrt{x^2+(y+2nh)^2+z^2}}\right\}\\
+\frac{I}{4\pi}\rho\sum_{n=0}^{\infty}&\left\{\frac{k^{n+1}\cdot k_0^{\,n+1}}{\sqrt{x^2+[y-2(n+1)h]^2+z^2}}+\frac{k^n\cdot k_0^{\,n+1}}{\sqrt{x^2+(y-2nh-2d)^2+z^2}}\right\}\\
+\frac{I}{4\pi}\rho(1-k^2)k_1\sum_{n=0}^{\infty}&\left\{\frac{(n+1)k^n\cdot k_0^{\,n}}{\sqrt{x^2+[y+2h_1+2(n+1)h-2d]^2+z^2}}+\frac{(n+1)k^n\cdot k_0^{\,n+1}}{\sqrt{x^2+[y+2h_1+2(n+1)h]^2+z^2}}\right\}\\
+\frac{I}{4\pi}\rho(1-k^2)k_1\sum_{n=0}^{\infty}&\left\{\frac{(n+1)k^n\cdot k_0^{\,n+1}}{\sqrt{x^2+[y-2h_1-2(n+1)h]^2+z^2}}+\frac{(n+1)k^n\cdot k_0^{\,n+2}}{\sqrt{x^2+[y-2h_1-2(n+1)h-2d]^2+z^2}}\right\}\\
+\frac{I}{4\pi}\rho(1-k^2)k_1^{\,2}\sum_{n=0}^{\infty}&\left\{\frac{\left[(1-k^2)\frac{n(n+1)}{2}-(n+1)k^2\right]k^{n-1}\cdot k_0^{\,n}}{\sqrt{x^2+[y+4h_1+2(n+1)h-2d]^2+z^2}}\right.\\
&\qquad\left.+\frac{\left[(1-k^2)\frac{n(n+1)}{2}-(n+1)k^2\right]k^{n-1}\cdot k_0^{\,n+1}}{\sqrt{x^2+[y+4h_1+2(n+1)h]^2+z^2}}\right\}\\
+\frac{I}{4\pi}\rho(1-k^2)k_1^{\,2}\sum_{n=0}^{\infty}&\left\{\frac{\left[(1-k^2)\frac{n(n+1)}{2}-(n+1)k^2\right]k^{n-1}\cdot k_0^{\,n+1}}{\sqrt{x^2+[y-4h_1-2(n+1)h]^2+z^2}}\right.\\
&\qquad\left.+\frac{\left[(1-k^2)\frac{n(n+1)}{2}-(n+1)k^2\right]k^{n-1}\cdot k_0^{\,n+2}}{\sqrt{x^2+(y-4h_1-2(n+1)h-2d]^2+z^2}}\right\}
\end{aligned}
$$

$$+ \frac{I}{4\pi}\rho(1 - k^2)k_1^3 \sum_{n=0}^{\infty} \left\{ \frac{\left[(1-k^2)^2 \frac{1}{6}(n-1)n(n+1) - (1-k^2)n(n+1)k^2 + (n+1)k^4\right]k^{n-2} \cdot k_0^{n}}{\sqrt{x^2 + [y + 6h_1 + 2(n+1)h - 2d]^2 + z^2}} \right.$$

$$\left. + \frac{\left[(1-k^2)^2 \frac{1}{6}(n-1)n(n+1) - (1-k^2)n(n+1)k^2 + (n+1)k^4\right]k^{n-2} \cdot k_0^{n+1}}{\sqrt{x^2 + [y + 6h_1 + 2(n+1)h]^2 + z^2}} \right\}$$

$$+ \frac{I}{4\pi}\rho(1 - k^2)k_1^3 \sum_{n=0}^{\infty} \left\{ \frac{\left[(1-k^2)^2 \frac{1}{6}(n-1)n(n+1) - (1-k^2)\, n(n+1)k^2 + (n+1)k^4\right]k^{n-2} \cdot k_0^{n+1}}{\sqrt{x^2 + [y - 6h_1 - 2(n+1)h]^2 + z^2}} \right.$$

$$\left. + \frac{\left[(1-k^2)^2 \frac{1}{6}(n-1)n(n+1) - (1-k^2)n(n+1)k^2 + (n+1)k^4\right]k^{n-2} \cdot k_0^{n+2}}{\sqrt{x^2 + [y - 6h_1 - 2(n+1)h - 2d]^2 + z^2}} \right\} + \cdots\cdots$$

$$V_1 = \frac{I}{4\pi}\rho(1 + k) \sum_{n=0}^{\infty} \left\{ \frac{k^n \cdot k_0^{n}}{\sqrt{x^2 + (y - 2nh)^2 + z^2}} + \frac{k^n \cdot k_0^{n+1}}{\sqrt{x^2 + (y - 2nh - 2d)^2 + z^2}} \right\}$$

$$+ \frac{I}{4\pi}\rho(1 + k)k_1 \sum_{n=0}^{\infty} \left\{ \frac{k^n \cdot k_0^{n}}{\sqrt{x^2 + [y + 2h_1 + 2(n+1)h - 2d]^2 + z^2}} + \frac{k^n \cdot k_0^{n+1}}{\sqrt{x^2 + [y + 2h_1 + 2(n+1)h]^2 + z^2}} \right\}$$

$$+ \frac{I}{4\pi}\rho(1+k)k_1 \sum_{n=0}^{\infty} \left\{ \frac{[(1-k^2)n - k^2]k^{n-1} \cdot k_0^{\,n}}{\sqrt{x^2 + [y - 2h_1 - 2nh]^2 + z^2}} + \frac{[(1-k^2)n - k^2]k^{n-1} \cdot k_0^{\,n+1}}{\sqrt{x^2 + [y - 2h_1 - 2nh - 2d]^2 + z^2}} \right\}$$

$$+ \frac{I}{4\pi}\rho(1+k)k_1^{\,2} \sum_{n=0}^{\infty} \left\{ \frac{[(1-k^2)n - k^2]k^{n-1} \cdot k_0^{\,n}}{\sqrt{x^2 + [y + 4h_1 + 2(n+1)h - 2d]^2 + z^2}} + \frac{[(1-k^2)n - k^2]k^{n-1} \cdot k_0^{\,n+1}}{\sqrt{x^2 + [y + 4h_1 + 2(n+1)h]^2 + z^2}} \right\}$$

$$+ \frac{I}{4\pi}\rho(1+k)k_1^{\,2} \sum_{n=0}^{\infty} \left\{ \frac{\left[(1-k^2)^2\frac{(n-1)n}{2} - (1-k^2)2nk^2 + k^4\right]k^{n-2} \cdot k_0^{\,n}}{\sqrt{x^2 + (y - 4h_1 - 2nh)^2 + z^2}} \right.$$

$$\left. + \frac{\left[(1-k^2)^2\frac{(n-1)n}{2} - (1-k^2)2nk^2 + k^4\right]k^{n-2} \cdot k_0^{\,n+1}}{\sqrt{x^2 + (y - 4h_1 - 2nh - 2d)^2 + z^2}} \right\}$$

$$+ \frac{I}{4\pi}\rho(1+k)k_1^{\,3} \sum_{n=0}^{\infty} \left\{ \frac{\left[(1-k^2)^2\frac{(n-1)n}{2} - (1-k^2)2nk^2 + k^4\right]k^{n-2} \cdot k_0^{\,n}}{\sqrt{x^2 + [y + 6h_1 + 2(n+1)h - 2d]^2 + z^2}} \right.$$

$$\left. + \frac{\left[(1-k^2)^2\frac{(n-1)n}{2} - (1-k^2)2nk^2 + k^4\right]k^{n-2} \cdot k_0^{\,n+1}}{\sqrt{x^2 + [y + 6h_1 + 2(n+1)h]^2 + z^2}} \right\}$$

$$+ \frac{I}{4\pi}\rho(1+k)k_1^{3}\sum_{n=0}^{\infty}\left\{\frac{\left[(1-k^2)^3\frac{1}{6}(n-2)(n-1)n - 3(1-k^2)^2\frac{(n-1)n}{2}k^2 + 3n(1-k^2)k^4 - k^6\right]k^{n-3}\cdot k_0^{\,n}}{\sqrt{x^2+(y-6h_1-2nh)^2+z^2}}\right.$$

$$\left. + \frac{\left[(1-k^2)^3\frac{1}{6}(n-2)(n-1)n - 3(1-k^2)^2\frac{n(n-1)}{2}k^2 + 3n(1-k^2)k^4 - k^6\right]k^{n-3}\cdot k_0^{\,n+1}}{\sqrt{x^2+(y-6h_1-2nh-2d)^2+z^2}}\right\}$$

$$+ \frac{I}{4\pi}\rho(1+k)k_1^{4}\sum_{n=0}^{\infty}\left\{\frac{\left[(1-k^2)^3\frac{1}{6}(n-2)(n-1)n - 3(1-k^2)^2\frac{(n-1)n}{2}k^2 + 3n(1-k^2)k^4 - k^6\right]k^{n-3}\cdot k_0^{\,n}}{\sqrt{x^2+[y+8h_1+2(n+1)h-2d]^2+z^2}}\right.$$

$$\left. + \frac{\left[(1-k^2)^3\frac{1}{6}(n-2)(n-1)n - 3(1-k^2)^2\frac{(n-1)n}{2}k^2 + 3n(1-k^2)k^4 - k^6\right]k^{n-3}\cdot k_0^{\,n+1}}{\sqrt{x^2+[y+8h_1+2(n+1)h]^2+z^2}}\right\} + \cdots$$

$$V_2 = \frac{I}{4\pi}\rho(1+k)(1+k_1)\sum_{n=0}^{\infty}\left\{\frac{k^n\cdot k_0^{\,n}}{\sqrt{x^2+(y-2nh)^2+z^2}} + \frac{k^n\cdot k_0^{\,n+1}}{\sqrt{x^2+(y-2nh-2d)^2+z^2}}\right\}$$

$$+ \frac{I}{4\pi}\rho(1+k)(1+k_1)k_1\sum_{n=0}^{\infty}\left\{\frac{[(1-k^2)n-k^2]k^{n-1}\cdot k_0^{\,n}}{\sqrt{x^2+(y-2h_1-2nh)^2+z^2}} + \frac{[(1-k^2)n-k^2]k^{n-1}\cdot k_0^{\,n+1}}{\sqrt{x^2+(y-2h_1-2nh-2d)^2+z^2}}\right\}$$

$$+ \frac{I}{4\pi}\rho(1+k)(1+k_1)k_1^2 \sum_{n=0}^{\infty} \left\{ \frac{\left[(1-k^2)^2\frac{(n-1)n}{2} - (1-k^2)2nk^2 + k^4\right]k^{n-2}\cdot k_0^n}{\sqrt{x^2+(y-4h_1-2nh)^2+z^2}} \right.$$

$$\left. + \frac{\left[(1-k^2)^2\frac{(n-1)n}{2} - (1-k^2)2nk^2 + k^4\right]k^{n-2}\cdot k_0^{n+1}}{\sqrt{x^2+(y-4h_1-2nh-2d)^2+z^2}} \right\}$$

$$+ \frac{I}{4\pi}\rho(1+k)(1+k_1)k_1^3 \sum_{n=0}^{\infty} \left\{ \frac{\left[(1-k^2)^3\frac{1}{6}(n-2)(n-1)n - 3(1-k^2)^2\frac{(n-1)n}{2}k^2 + 3n(1-k^2)k^4 - k^6\right]k^{n-3}\cdot k_0^n}{\sqrt{x^2+(y-6h_1-2nh)^2+z^2}} \right.$$

$$\left. + \frac{\left[(1-k^2)^3\frac{1}{6}(n-2)(n-1)n - 3(1-k^2)^2\frac{(n-1)n}{2}k^2 + 3n(1-k^2)k^4 - k^6\right]k^{n-3}\cdot k_0^{n+1}}{\sqrt{x^2+(y-6h_1-2nh-2d)^2+z^2}} \right\} +$$

. . . .

These four equations satisfy Laplace's equation, as well as the equations for the conditions on the boundary planes. This can best be proved by their differentiation

$$V_r = V_s, \quad \frac{1}{\rho_r}\cdot\frac{\partial V_r}{\partial y} = \frac{1}{\rho_s}\cdot\frac{\partial V_s}{\partial y}$$

For the determination of the apparent resistivity of the ground, from the surface measurements only, the expression of V for $y = d$ must be taken into consideration. This can be made simpler if, in agreement with the actual conditions, k_0 is put equal to $+1$ and d equal to 0.

Then

$$V_{y=0} = \frac{I}{\pi}\rho\left(\frac{1}{2\sqrt{x^2+z^2}} + \sum_{n=0}^{\infty}\left\{\frac{k^{n+1}}{\sqrt{x^2+[2(n+1)h]^2+z^2}}\right.\right.$$

$$+ (1-k^2)k_1\frac{(n+1)k^n}{\sqrt{x^2+[2h_1+2(n+1)h]^2+z^2}} + (1-k^2)k_1{}^2\frac{\left[(1-k^2)\frac{n(n+1)}{2}-(n+1)k^2\right]k^{n-1}}{\sqrt{x^2+[4h_1+2(n+1)h]^2+z^2}}$$

$$\left.\left.+ (1-k^2)k_1{}^3\frac{\left[(1-k^2)^2\frac{1}{6}(n-1)n(n+1)-(1-k^2)n(n+1)k^2+(n+1)k^4\right]k^{n-2}}{\sqrt{x^2+[6h_1+2(n+1)h]^2+z^2}} + \dots\right\}\right).$$

If Wenner's arrangement of current and search electrodes is used for the generating and measurement of the potential in the ground, the expression for the apparent resistivity relative to the actual resistivity of the upper layer is:

$$\frac{\rho_s}{\rho} = 1 + 4\sum_{n=0}^{\infty}k^{n+1}\left\{\frac{1}{\sqrt{1+\left[2(n+1)\frac{h}{a}\right]^2}} - \frac{1}{\sqrt{4+\left[2(n+1)\frac{h}{a}\right]^2}}\right\}$$

$$+ 4(1-k^2)k_1\sum_{n=0}^{\infty}(n+1)k^n\left\{\frac{1}{\sqrt{1+\left[2\frac{h_1}{a}+2(n+1)\frac{h}{a}\right]^2}} - \frac{1}{\sqrt{4+\left[2\frac{h_1}{a}+2(n+1)\frac{h}{a}\right]^2}}\right\}$$

$$+ 4(1 - k^2)k_1^2 \sum_{n=0}^{\infty} \left[\frac{n(n+1)}{2}(1 - k^2) - (n+1)k^2 \right] k^{n-1}$$

$$\left\{ \frac{1}{\sqrt{1 + \left[4\frac{h_1}{a} + 2(n+1)\frac{h}{a} \right]^2}} - \frac{1}{\sqrt{4 + \left[4\frac{h_1}{a} + 2(n+1)\frac{h}{a} \right]^2}} \right\}$$

$$+ 4(1 - k^2)k_1^3 \sum_{n=0}^{\infty} \left[\frac{1}{6}(n-1)n(n+1)(1 - k^2)^2 - n(n+1)(1 - k^2)k^2 + (n+1)k^4 \right] k^{n-2}$$

$$\left\{ \frac{1}{\sqrt{1 + \left[6\frac{h_1}{a} + 2(n+1)\frac{h}{a} \right]^2}} - \frac{1}{\sqrt{4 + \left[6\frac{h_1}{a} + 2(n+1)\frac{h}{a} \right]^2}} \right\}$$

. . . .

This is the basic formula for all further special investigation. In its numerical evaluation the values given in Table 1 can be used.

The first analysis will be made for the special case that the conductivity of the lowest layer is very poor and that both of the surface layers have the same thickness. Then $k_1 = +1$ and $h_1 = h$.

These values substituted in the last equation give the expression for apparent resistivity in an especially simple form

$$\frac{\rho_s}{\rho} = 1 + 4k\sum_{n=0}^{\infty}\left\{\frac{1}{\sqrt{1+\left[2(2n+1)\frac{h}{a}\right]^2}} - \frac{1}{\sqrt{4+\left[2(2n+1)\frac{h}{a}\right]^2}}\right\}$$
$$+4\sum_{n=0}^{\infty}\left\{\frac{1}{\sqrt{1+\left[4(n+1)\frac{h}{a}\right]^2}} - \frac{1}{\sqrt{4+\left[4(n+1)\frac{h}{a}\right]^2}}\right\},$$

which contains only two summations. It can be checked easily either by putting $\rho_1 = \rho$ and $k = 0$, or by putting $\rho_1 = \infty$ and $k = +1$. In the first case the formula gives the apparent resistivity over a single surface layer of the thickness $2h$, and in the second case the apparent resistivity over a single layer of the thickness h.

Putting in $\frac{\rho_s}{\rho} = v$ and $a/h = u$, and rearranging it, we have

$$v = 1 + 4k\sum_{n=0}^{\infty}\left\{\frac{1}{\sqrt{1+\left[\frac{2(2n+1)}{u}\right]^2}} - \frac{1}{\sqrt{4+\left[\frac{2(2n+1)}{u}\right]^2}}\right\}$$
$$+2\sum_{n=0}^{\infty}\left\{\frac{1}{\sqrt{1+\left[\frac{4(n+1)}{u}\right]^2}} - \frac{1}{\sqrt{1+\left[\frac{2(2n+1)}{u}\right]^2}}\right\} + 2u\cdot\log_e 2.$$

The first sum converges to $\frac{1}{2}u \log_e 2$, and the second to $-\frac{1}{2}$, when u assumes great values. In this limiting case

$$\lim_{u\to\infty} v = 1 + 4k\tfrac{1}{2}u\cdot\log_e 2 - 1 + 2u\cdot\log_e 2$$

or also

$$\lim_{u\to\infty} \rho_s = \frac{2}{\frac{1}{\rho}+\frac{1}{\rho'}}\,2u\cdot\log_e 2$$

which shows that the asymptotes of all curves must pass through the zero point.

In this form the equation represents another interesting fact: When ρ'_s is the apparent resistivity over a single layer of the thickness h and

resistivity ρ, and ρ''_s the apparent resistivity over a similar layer which has the resistivity ρ', the above equation for the apparent resistivity of both of the layers assumes the form

$$\frac{2}{\rho_s} = \frac{1}{\rho_s'} + \frac{1}{\rho_s''}.$$

This equation indicates that with a great distance separating the current and search electrodes, the apparent resistances follow Kirchhoff's law

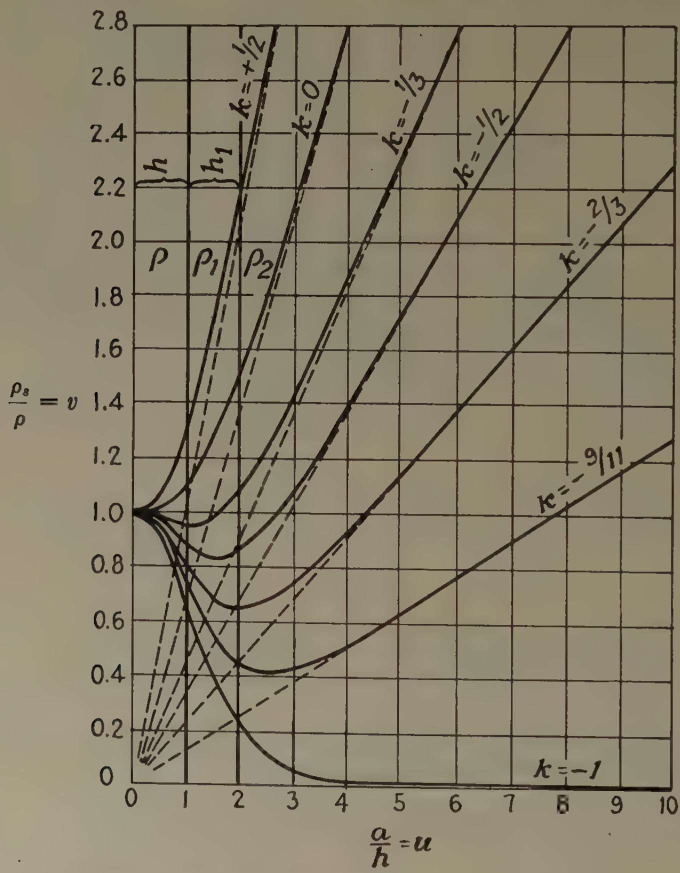

FIG. 6.—CURVES OF APPARENT RESISTIVITY FOR CASE OF $h = h_1$ AND INFINITE RESISTIVITY OF THE UNDERGROUND ($\rho_2 = \infty$), AND FOR DIFFERENT RATIOS OF RESISTIVITIES OF SURFACE LAYERS.

for two resistances connected in parallel. This fact can be generalized for the case of more than two layers of any thicknesses, because it will always be possible to represent any number of the surface layers by a single one, the resistance of which can be computed from the individual resistances of each of the layers with the help of the Kirchhoff's law for

resistances in parallel. The apparent resistances computed under the assumption of such a surface layer covering an infinite layer in the underground will give, plotted in the diagram, an approximation curve which approaches the true one, and for a large a is asymptotic to it. The computation of the true apparent resistivity of several layers for the parts of the curve practically coinciding with those of the approximation curve is much more difficult than the computation of the approximation curve. For that reason the determination of the true values will be

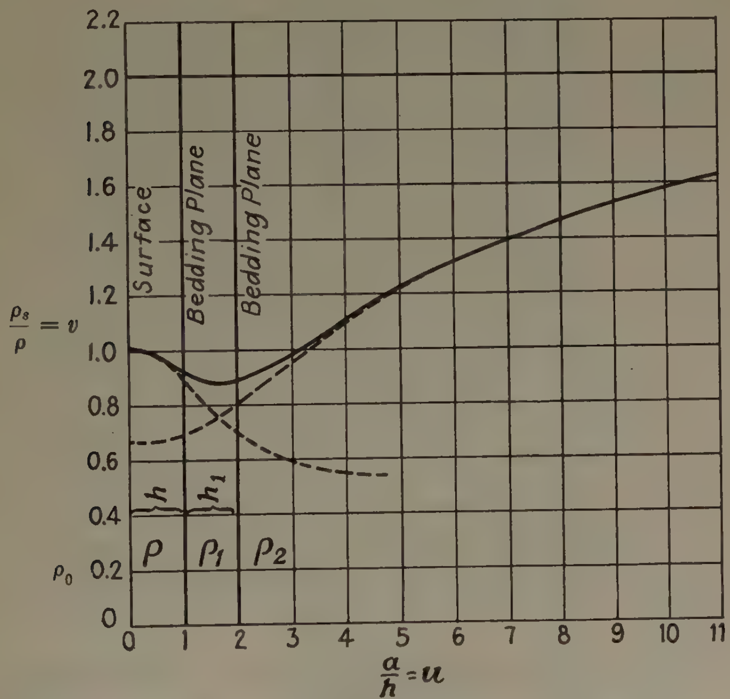

FIG. 7.—CURVE OF APPARENT RESISTIVITY FOR CASE $h = h_1$ AND RATIO $\rho_0 : \rho : \rho_1 : \rho_2 = \infty : 1 : 0.5 : 2$.

made easier when the approximation curve is drawn first. This also makes it possible to replace the very difficult computation of the curve for a larger number of layers by graphic interpolation from the approximation curves, computed on the above principle. Similar procedure can also be used in the case of continuous change of the resistivity of the underground, which may, as an example, be a function of the depth.

As the continuation of curves belonging to several surface layers can be computed from a single surface layer which has the corresponding average resistivity, the shape of the curves at a great enough distance from the origin of coordinates is almost independent of the particular

properties of these upper layers. Its shape is then determined by the average physical properties of the succession of the upper layers only. This fact gives the method a practically unlimited depth penetration. The shielding of the electric field either by conductors or insulators near the surface, which is an important factor determining the range of the depth penetration of all other electrical methods, is in this method entirely eliminated.

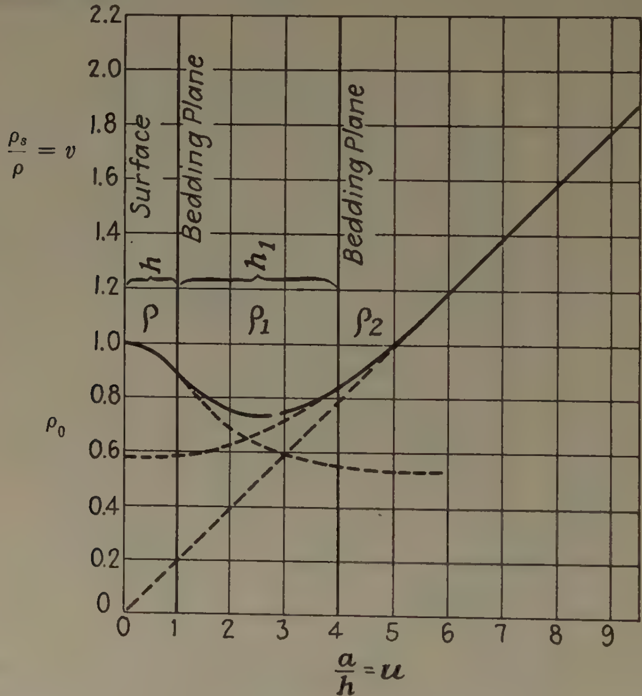

FIG. 8.—CURVE OF APPARENT RESISTIVITY FOR CASE OF $h_1 = 3h$ AND RATIO $\rho_0 : \rho : \rho_1 : \rho_2 = \infty : 1 : 0.5 : \infty$.

The approximation curves of apparent resistivity computed under this assumption of simplified conditions in the underground are given in Fig. 6, for various resistivities of the two upper layers; that is, for various values of k. The diagram gives asymptotic lines of the curves. The distances representing the points, where the distance a is equal to the depth of one of the bedding planes, are given by lines parallel to the v axis. The position of the minimum of the curves with respect to these points is important for the analysis of the curves. The diagram shows that this lowest point of the curve changes its position and that there is no simple relation between the minimum and the depths of the layers. An accurate analysis of the curve can be accomplished only by examination of its entire course.

The general case will be considered again. It requires a long and somewhat tedious computation but some special cases were computed and illustrated in diagrams.

First, the apparent resistivity for two surface layers of equal thicknesses will be analyzed. The resistivity of the lowest layer is this time finite. If, for example, $k_1 = 3/5$, $k = -1/3$, $k_0 = +1$, and $h_1 = h$, the curve assumes the shape given in Fig. 7.

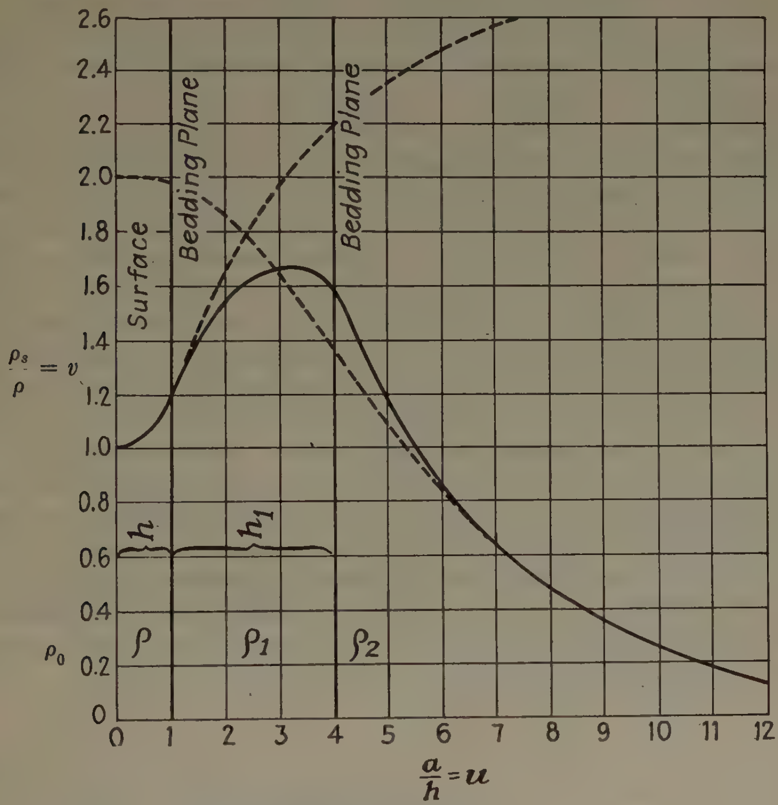

FIG. 9.—CURVE OF APPARENT RESISTIVITY FOR CASE OF $h_1 = 3h$ AND RATIO $\rho_0 : \rho : \rho_1 : \rho_2 = \infty : 1 : 3 : 0$.

The asymptotic line to it is here parallel to the u axis. The approximation curves are drawn in dotted lines, and the figure shows how they can be used in the construction of the true curve. In turn, the approximation curves which aid in the analysis of the problem can be constructed from the true curves.

Two more curves are shown in the Figs. 8 and 9. They represent the case of two surface layers, the lower one being three times as thick as the upper one. Here h_1 equals $3h$ and k_0 is again $+1$. The k and k_1 are respectively $-1/3$ and $+1$; then $+1/2$ and -1. The curves corresponding to

these conditions are given in the figures together with the dotted approximation curves, computed with the help of Kirchhoff's Law.

The different examples show that the apparent resistivity is an excellent indicator of the conditions in the bedded ground. However, its interpretation requires a great deal of experience and a thorough theoretical knowledge. It is desirable to know the exact solution of the problems presented by such tectonic conditions. The generalization of the given mathematical analysis for one or two surface layers (or more than two layers) offers no great difficulties, but involves a much more complicated computation. The method of graphic approximation should then be preferred.

The field of application of all the methods, based either on the determination of the apparent resistivity or on the survey of the absolute potential differences, is entirely different from the field of application of the qualitative methods which employ the directional survey of the potential, or current, in the ground. The equipotential surfaces and their picture on the surface of the ground can be greatly disturbed by long conductors in the direction of the flow of current, or by sheets of insulating material embedded in the ground normal to the direction of the current, even if the volume of such formations is very small. The picture of the equipotential surfaces, if their density is not considered, remains, on the other hand, totally undisturbed by great volumes of horizontal layers. They do, however, produce great changes in the distribution of the absolute potential; that is, in the density of the equipotential surfaces. In conclusion, it could be said that the successful use of equipotential lines and search-coil methods depends on the shape and position of the inhomogeneity, while the use of methods based on apparent resistivity depends on their volume. In both cases, and under extremely favorable conditions the ratio of conductivity governs the magnitude of the anomaly.

Mathematical Theory of Electrical Flow in Stratified Media with Horizontal, Homogeneous and Isotropic Layers

By D. O. Ehrenburg* and R. J. Watson,* Boulder, Colo.

(New York Meeting, February, 1931)

During the earlier period of electrical prospecting, the search for orebodies was by far the most important application of this method of geophysical prospecting. In the past few years, however, increasing attention has been paid to the possibilities of this method in the investigation of subsurface structures. Not only have fault zones been located, but attempts have been made to determine the depth to certain strata and thus contouring on some bed which proves to be a good "electrical marker."

Many workers have published the results of field tests showing how their particular interpretations have agreed with the actual field conditions, but very few have stated their method of interpretation. Weaver, Hummel and Stefanesco and Schlumberger[1] have given a more theoretical discussion of the resistivity methods. Hummel and Stefanesco and Schlumberger have published mathematical solutions for the flow of electricity in a medium involving four layers. In this paper the authors present a solution for the flow of electricity in a medium of any number of horizontal strata. By means of this solution, curves showing the relationship between the apparent resistivity and the electrode spacing can be computed for ideal cases.

Point-source in an Infinite Homogeneous Medium

In resistivity measurements the current is conveyed to the earth by means of two electrodes, which can be mathematically treated as point-sources of equal and opposite strength.

A point-source of strength I is defined as a point at which electricity is being generated at the constant rate of I coulombs per second, or I

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[1] W. Weaver: Some Applications of the Surface Potential Methods to Structural Studies. Geophysical Prospecting, A. I. M. E. (1929) 68.

J. N. Hummel: The Apparent Specific Resistance. *Ztsch. f. Geophysik* (1929) 5, 89–104.

The Apparent Specific Resistance in the Case of Four Parallel Beds. *Ibid.*, 228–238.

J. Stefanesco in collaboration with C. and M. Schlumberger: On the Distribution of Electric Potential in Stratified Earth with Horizontal, Homogeneous and Isotropic Layers. *Jnl. de Physique et le Radium* (1930) 1, 132–140.

amperes. Consider a point-source of strength I in an infinite homogeneous medium of resistivity ρ. After the steady-state condition has been reached, there can be no accumulation of charge at any point. Consequently, if we surround the point-source by a closed surface, the quantity of electricity flowing outward across this surface must be I coulombs per second, or I amperes. It will be convenient to assume for such a surface a sphere of arbitrary radius R, with center at the point-source. By symmetry, the current density at any point on the surface of the sphere must be the same; and since the total current flowing across the sphere is I and the surface of the sphere $4\pi R^2$,

$$i = \frac{I}{4\pi R^2} \qquad [1]$$

where i is the current density at steady state. To obtain the potential at any point a distance R from the source, we shall make use of Ohm's law written in differential notation; namely,

$$\frac{dv}{dn} = -\rho i \qquad [2]$$

where dv/dn is the Normal Potential Gradient. In our particular case

$$\frac{dv}{dn} = \frac{dv}{dR} \qquad [3]$$

since the flow is radial. By virtue of equations 1, 2 and 3, we have

$$\frac{dv}{dR} = -\frac{\rho I}{4\pi R^2} \qquad [4]$$

or, integrating,

$$v = \frac{\rho I}{4\pi R} + C \qquad [4.1]$$

To evaluate the constant of integration C, note that the potential must vanish at infinity. This necessitates

$$C = 0$$

hence

$$v = \frac{\rho I}{4\pi R} \qquad [4.11]$$

In Cartesian coordinates, with origin at the point-source, equation 4.11 takes the form

$$v = \frac{\rho I}{4\pi} \cdot \frac{1}{\sqrt{x^2 + y^2 + z^2}} \qquad [4.12]$$

The expression vanishes at infinity (to the first order in terms of distance), remains finite everywhere except at the point-source (0, 0, 0), and is a solution of the steady-state equation of electric flow, namely

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = 0 \qquad [5]$$

Maxwell's Generalized Method of Images

In dealing with a *bounded* medium, it is convenient to extend it into an *unbounded* or infinite medium, so as to make the solution come under the type of equation 4.12. We imagine the bounded medium extended to infinity in *all* directions, and place sources in the fictitious part of this assumed infinite medium in such a manner as to *leave the flow in the actually existing part unaffected.* The problem then reduces to finding a suitable distribution of sources in the fictitious part so as to fulfill this condition. Often this can be executed by a simple optical analogy.

Suppose we wish to find the distribution of potential in a *semi-infinite* medium (half-space) of resistivity ρ_1, which borders on another half-space, of resistivity ρ_2; the flow being due to a single point-source I in the first medium at a distance h from the bounding plane. We can choose the XY plane coinciding with the boundary, so that the source will be $(0, 0, h)$, and the two half-spaces can be represented by

$z > 0$, (ρ_1)
$z < 0$, (ρ_2) respectively.

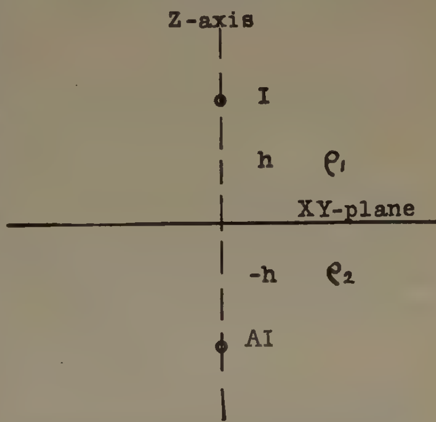

Fig. 1.—Image in second medium.

Continuing our optical analogy, we imagine that the boundary plane $z = 0$ acts as a mirror. In this case, to an observer within the medium ρ_1, an image of the source I will appear at the point $(0, 0, -h)$ (Fig. 1). If the reflection coefficient of the mirror is not unity, part of the light being transmitted through the mirror, an observer in medium ρ_2 will dimly see the source I, but no image thereof. Hence, making use of equation 4.12 modified for a source at $(0, 0, h)$ instead of $(0, 0, 0)$, we obtain our tentative solutions in the form

$$v_1 = \frac{\rho_1 I}{4\pi}\left\{\frac{1}{\sqrt{x^2 + y^2 + (z - h)^2}} + \frac{A}{\sqrt{x^2 + y^2 + (z + h)^2}}\right\} \qquad z > 0 \qquad [6]$$

$$v_2 = \frac{\rho_2 I}{4\pi}\left\{\frac{(1 - A)}{\sqrt{x^2 + y^2 + (z - h)^2}}\right\} \qquad z < 0 \qquad [7]$$

where A is the fraction of light *reflected* and $1 - A$ the fraction *transmitted.* Solution 6 is based on the previous assumption that medium ρ_2 is extended infinitely in the negative direction, and that an additional source is placed at $(0, 0, -h)$; and a corresponding assumption is made in equation 7.

To be correct, these tentative solutions must satisfy the differential equation 5 and the following conditions:

1. The potential must suffer no discontinuity at the boundary plane, or

$$\text{when } z = 0, \qquad v_1 = v_2 \qquad [8]$$

2. At the boundary plane, the component of the current density normal to this plane must be the same computed from either side. That is

$$\text{when } z = 0, \qquad (i_1)_z = (i_2)_z \qquad [9]$$

But by Ohm's law

$$i_z = -\frac{1}{\rho}\frac{\partial v}{\partial z} \qquad [2.1]$$

Hence equation 2.1 can be written in the form.

$$\frac{1}{\rho_1}\frac{\partial v_1}{\partial z} = \frac{1}{\rho_2}\frac{\partial v_2}{\partial z} \qquad [9.1]$$

3. Each solution must remain finite in the limits within which it is supposed to hold, everywhere except at the point-source $(0, 0, h)$; that term of v_1 which becomes infinite at $(0, 0, h)$ must by reference to equation 4.12, be

$$\frac{\rho_1 I}{4\pi}\,\frac{1}{\sqrt{x^2 + y^2 + (z - h)^2}}$$

Now, equation 5 is seen to be satisfied by virtue of the properties of the Newtonian potential, or by direct substitution. Conditions 1 and 2 can be easily shown to be satisfied, by applying equations 8 and 9.1 to the tentative solutions 6 and 7, provided that

$$A = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} \qquad [10]$$

Remembering that v_1 holds only for $z \geqq 0$ and v_2 for $z \leqq 0$, we immediately see that condition 3 also is satisfied.

The striking feature about this analogy is that whereas in optics the reflection coefficient A varies from 0 to 1, in this case its range is from -1 to $+1$. In other words, we have positive and negative mirrors: insulators correspond to positive mirrors and conductors to negative ones. A perfect insulator is a totally reflecting positive mirror ($A = 1$) and a perfect conductor a totally reflecting negative mirror ($A = -1$).

Point-source in a Semi-infinite Homogeneous Medium

In practical applications to earth-resistivity measurements, we seldom deal with infinite bodies, but usually with semi-infinite ones

since the earth is generally treated as a half-space. Therefore we will now develop the formula for a point-source in a semi-infinite homogeneous medium bounded by an insulating plane. Apparently, equation 6 is directly applicable; in this case

$$\rho_2 = \infty \text{ and } A = 1$$

Dropping the subscripts, we can write equation 6 for this special case as

$$v = \frac{\rho I}{4\pi}\left\{\frac{1}{\sqrt{x^2+y^2+(z-h)^2}} + \frac{1}{\sqrt{x^2+y^2+(z+h)^2}}\right\} \qquad z \geqq 0 \quad [6.1]$$

Since in field practice the electrode is usually placed at the surface of the earth, the case where the point-source is located at (0, 0, 0) is of special interest. Setting $h = 0$ in equation 6.1, we obtain for this particular case,

$$v = \frac{\rho I}{2\pi} \cdot \frac{1}{\sqrt{x^2+y^2+z^2}} \qquad z \geqq 0 \quad [6.11]$$

For greater compactness, we will rewrite this in cylindrical space coordinates; that is, letting $x^2 + y^2 = R^2$.

$$v = \frac{\rho I}{2\pi} \cdot \frac{1}{\sqrt{R^2+z^2}} \qquad z \geqq 0 \quad [6.12]$$

Point-source in a Stratified Semi-infinite Medium

Statement of Problem

Whereas the cases treated above are of prime importance in establishing fundamental principles and fixing ideas, the only case directly applicable to practical problems of electrical exploration by the resistivity methods is that of a stratified semi-infinite medium. The simplest and most important problem of this type consists of an arbitrary number of *homogeneous* layers, of arbitrary resistivity and arbitrary thickness. Whereas simple particular cases can be treated readily by the method of images, obtaining a general solution is a much harder task.

The general solution must hold for any number of layers and for all possible combinations of resistivities and thicknesses. In addition, it should be sufficiently simple to yield numerical data upon direct substitution for any arbitrary case. In the method of attack which follows, these conditions are met completely, except for the purely theoretical restriction that the thicknesses of the layers must be chosen as rational commeasurable numbers.

We divide the half-space into arbitrary layers of thickness $m/2$, and resistivities $\rho_1, \rho_2, \rho_3, \rho_4, \rho_5 \ldots$ etc. The resistivities of two adjacent layers may be either different from or equal to each other. Suppose that we want to obtain the solution for four layers whose resistivities are in the ratio of 1: 19: 141: 9, and whose thicknesses are in the ratio of

15: 10: 25: ∞. Let the thicknesses be $3m/2$, m, $5m/2$, ∞, respectively. The resistivities will be

$$\rho_1 = \rho_2 = \rho_3 = \rho_1 \qquad \tfrac{3m}{2}$$

$$\rho_4 = \rho_5 = 19\rho_1 \qquad m$$

$$\rho_6 = \rho_7 = \rho_8 = \rho_9 = \rho_{10} = 141\rho_1 \qquad \tfrac{5m}{2}$$

$$\rho_{11} = \rho_{12} = \rho_{13} = \rho_{14} = \rho_n = 9\rho_1 \qquad \infty$$

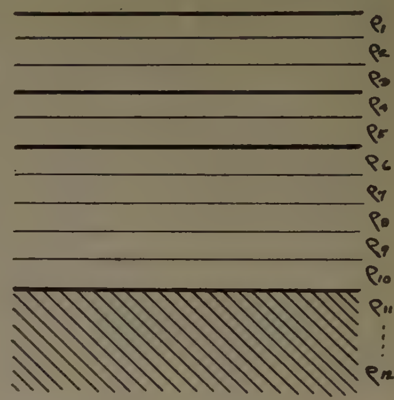

FIG. 2.—FOUR-LAYER PROBLEM.

If we define the general reflection coefficient as

$$A_n = \frac{\rho_{n+1} - \rho_n}{\rho_{n+1} + \rho_n} \qquad [10.1]$$

we have the following relations in terms of the reflection coefficients:

$$\begin{array}{lll} A_1 = 0 & A_6 = 0 & A_{11} = 0 \\ A_2 = 0 & A_7 = 0 & A_{12} = 0 \\ A_3 = 0.9000 & A_8 = 0 & \cdots\cdots \\ A_4 = 0 & A_9 = 0 & A_n = 0 \\ A_5 = 0.7625 & A_{10} = -0.8800 & \end{array}$$

Method of Attack

Theoretically, Maxwell's generalized method of images is applicable to any number of layers, but in practice it will be found that with more than three layers this method becomes awkward. (Hummel's solution[2] actually deals with a three-layer earth only, the fourth "layer" being the air.)

The method presented here is analytic rather than synthetic, although certain principles of the image theory are made use of indirectly in building up the tentative solution. The differential equations finally lead to a set of algebraic equations; these result in formulas that can be arranged in a rather simple form, so that particular solutions for arbitrary cases may be built up and carried out numerically by one unfamiliar with both the method of images and the calculus.

[2] J. N. Hummel: *Op. cit.*

The Tentative Solution

Let the XY plane, or polar plane, coincide with the insulating boundary of the half-space, and let the origin of coordinates be at the point-source. The positive direction of the Z axis will be downward; that is, *into* the stratified medium. We will now attempt to satisfy the equation of conduction (5) and the boundary conditions called for at the beginning of this section, by assuming a solution of the form

$$v_1 = \frac{I}{2\pi}\left\{\frac{S_{11}}{\sqrt{R^2+z^2}} + \frac{S_{12}}{\sqrt{R^2+(z-m)^2}} + \frac{S_{12}}{\sqrt{R^2+(z+m)^2}} + \frac{S_{13}}{\sqrt{R^2+(z-2m)^2}} + \frac{S_{13}}{\sqrt{R^2+(z+2m)^2}} \right. \quad \ldots \; 0 \leqq z \leqq \frac{m}{2}$$

$$v_2 = \frac{I}{2\pi}\left\{\frac{S_{21}}{\sqrt{R^2+z^2}} + \frac{S'_{22}}{\sqrt{R^2+(z+m)^2}} + \frac{S_{23}}{\sqrt{R^2+(z-2m)^2}} + \frac{S'_{23}}{\sqrt{R^2+(z+2m)^2}} \right. \quad \ldots \; \frac{m}{2} \leqq z \leqq m$$

$$v_3 = \frac{I}{2\pi}\left\{\frac{S_{31}}{\sqrt{R^2+z^2}} + \frac{S'_{32}}{\sqrt{R^2+(z+m)^2}} + \frac{S_{33}}{\sqrt{R^2+(z-2m)^2}} + \frac{S'_{33}}{\sqrt{R^2+(z+2m)^2}} \right. \quad \ldots \; m \leqq z \leqq \frac{3m}{2}$$

$$v_4 = \frac{I}{2\pi}\left\{\frac{S_{41}}{\sqrt{R^2+z^2}} + \frac{S_{42}}{\sqrt{R^2+(z-m)^2}} + \frac{S'_{42}}{\sqrt{R^2+(z+m)^2}} + \frac{S'_{43}}{\sqrt{R^2(+(z+2m)^2}} \right. \quad \ldots \; \frac{3m}{2} \leqq z \leqq 2m$$

$$v_5 = \frac{I}{2\pi}\left\{\frac{S_{51}}{\sqrt{R^2+z^2}} + \frac{S_{52}}{\sqrt{R^2+(z-m)^2}} + \frac{S'_{52}}{\sqrt{R^2+(z+m)^2}} + \frac{S'_{53}}{\sqrt{R^2+(z+2m)^2}} \right. \quad \ldots \; 2m \leqq z \leqq 5\frac{m}{2}$$

etc.

That the solution satisfies the equation of conduction can be seen either from the properties of the Newtonian potential, or by direct substitution into equation 5. It only remains to show that it also satisfies all boundary conditions; namely:

1. There shall be no flow across the insulating plane $Z = 0$. This is at once seen to hold true from the symmetrical construction of the solution v_1 (the primes being omitted); or it can be proved by showing that $\partial v_1/\partial z$ vanishes for $Z = 0$.

2. Each solution must remain finite within the limits in which it is supposed to hold; except that v_1 must become infinite at the point source (0, 0, 0); that term of v_1 which becomes infinite at (0, 0, 0) must, by reference to equation 6.12, be

$$v = \frac{\rho_1 I}{2\pi\sqrt{R^2+z^2}}$$

The first part of this condition is satisfied by virtue of the fact that the terms which would contain the coefficients S_{22}, S_{32}, S_{43}, S_{53} . . . $S_{n,\frac{1}{2}n+1}$, $S_{n+1,\frac{1}{2}n+1}$, have been omitted. The second part is satisfied provided that

$$S_{11} = \rho_1 \tag{11}$$

3. At the nth boundary

$$v_n = v_{n+1}; \quad \frac{1}{\rho_n}\frac{\partial v_n}{\partial z} = \frac{1}{\rho_{n+1}}\frac{\partial v_{n+1}}{\partial z}$$

Substituting and carrying out the indicated partial differentiations, we find that these conditions are also satisfied, provided that

$$\begin{array}{ll}
S_{11} + S_{12} = S_{21} & \rho_2(S_{11} - S_{12}) = \rho_1(S_{21}) \\
S_{12} + S_{13} = S'_{22} + S_{23} & \rho_2(S_{12} - S_{13}) = \rho_1(S'_{22} - S_{23}) \\
S_{13} + S_{14} = S'_{23} + S_{24} & \rho_2(S_{13} - S_{14}) = \rho_1(S'_{23} - S_{24}) \\
S_{14} + S_{15} = S'_{24} + S_{25} & \rho_2(S_{14} - S_{15}) = \rho_1(S'_{24} - S_{25}) \\
\cdot\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\ \cdot \\
\cdot\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\ \cdot \\
S_{21} + S_{23} = S_{31} + S_{33} & \rho_3(S_{21} - S_{23}) = \rho_2(S_{31} - S_{33}) \\
S'_{22} + S_{24} = S'_{32} + S_{34} & \rho_3(S'_{22} - S_{24}) = \rho_2(S'_{32} - S_{34}) \\
S'_{23} + S_{25} = S'_{33} + S_{35} & \rho_3(S'_{23} - S_{25}) = \rho_2(S'_{33} - S_{35}) \\
S'_{24} + S_{26} = S'_{34} + S_{36} & \rho_3(S'_{24} - S_{26}(= \rho_2(S'_{34} - S_{36}) \\
\cdot\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\ \cdot \\
\cdot\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\ \cdot \\
S_{33} = S_{42} & -\rho_4 S_{33} = \rho_3 S_{42} \\
S_{31} + S_{34} = S_{41} + S_{44} & \rho_4(S_{31} - S_{34}) = \rho_3(S_{41} - S_{44}) \\
S'_{32} + S_{35} = S'_{42} + S_{45} & \rho_4(S'_{32} - S_{35}) = \rho_3(S'_{42} - S_{45}) \\
S'_{33} + S_{36} = S'_{43} + S_{46} & \rho_4(S'_{33} - S_{36}) = \rho_3(S'_{43} - S_{46}) \\
S'_{34} + S_{37} = S'_{44} + S_{47} & \rho_4(S'_{34} - S_{37}) = \rho_3(S'_{44} - S_{47}) \\
\cdot\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\ \cdot \\
\cdot\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\ \cdot \\
S_{42} + S_{44} = S_{52} + S_{54} & \rho_5(S_{42} - S_{44}) = \rho_4(S_{52} - S_{54}) \\
S_{41} + S_{45} = S_{51} + S_{55} & \rho_5(S_{41} - S_{45}) = \rho_4(S_{51} - S_{55}) \\
S'_{42} + S_{46} = S'_{52} + S_{56} & \rho_5(S'_{42} - S_{46}) = \rho_4(S'_{52} - S_{56}) \\
S'_{43} + S_{47} = S'_{53} + S_{57} & \rho_5(S'_{43} - S_{47}) = \rho_4(S'_{53} - S_{57}) \\
\cdot\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\ \cdot \\
\cdot\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\ \cdot \\
S_{54} = S_{63} & -\rho_6 S_{54} = \rho_5 S_{63} \\
S_{52} + S_{55} = S_{62} + S_{65} & \rho_6(S_{52} - S_{55}) = \rho_5(S_{62} - S_{65}) \\
S_{51} + S_{56} = S_{61} + S_{66} & \rho_6(S_{51} - S_{56}) = \rho_5(S_{61} - S_{66}) \\
S'_{52} + S_{57} = S'_{62} + S_{67} & \rho_6(S'_{52} - S_{57}) = \rho_5(S'_{62} - S_{67}) \\
S'_{53} + S_{58} = S'_{63} + S_{68} & \rho_6(S'_{53} - S_{58}) = \rho_5(S'_{63} - S_{68}) \\
\cdot\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\ \cdot \\
\cdot\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\ \cdot
\end{array}$$

etc. etc. etc. etc.

Validity of the Solution

In order to demonstrate the validity of the solution assumed, we must merely show that the algebraic equations in the S coefficients are consistent with each other.

In the first place, notice that of necessity

$$S_{33} = S_{42} = 0; \quad S_{54} = S_{63} = 0, \text{ etc.}$$

Making use of these relations we find that

$$S_{44} = S_{52} = 0; \quad S_{65} = S_{73} = 0, \text{ etc.}$$

After all possible cancellations of the type just indicated have been made, the S equations can be rewritten in the following form

Group A

$S_{11} + S_{12} = S_{21};\ \rho_2(S_{11} - S_{12}) = \rho_1 S_{21}$

$S_{21} + S_{23} = S_{31};\ \rho_3(S_{21} - S_{23}) = \rho_2 S_{31}$, etc.

Group B

$S_{12} + S_{13} = S'_{22} + S_{23};\ \rho_2(S_{12} - S_{13}) = \rho_1(S'_{22} - S_{23})$

$S_{13} + S_{14} = S'_{23} + S_{24};\ \rho_2(S_{13} - S_{14}) = \rho_1(S'_{23} - S_{24})$

$S_{14} + S_{15} = S'_{24} + S_{25};\ \rho_2(S_{14} - S_{15}) = \rho_1(S'_{24} - S_{25})$

etc. etc.

$S'_{22} + S_{24} = S'_{32} + S_{34};\ \rho_3(S'_{22} - S_{24}) = \rho_2(S'_{32} - S_{34})$, etc.

$S'_{23} + S_{25} = S'_{33} + S_{35};\ \rho_3(S'_{23} - S_{25}) = \rho_2(S'_{33} - S_{35})$, etc.

$S'_{24} + S_{26} = S'_{34} + S_{36};\ \rho_3(S'_{24} - S_{26}) = \rho_2(S'_{34} - S_{36})$, etc.

etc. etc.

Rewriting by interchanging rows and columns we have for group A

$$\begin{aligned} S_{11} + S_{12} &= S_{21} \\ \rho_2(S_{11} - S_{12}) &= \rho_1 S_{21} \\ S_{21} + S_{23} &= S_{31} \\ \rho_3(S_{21} - S_{23}) &= \rho_2 S_{31} \\ \text{etc.} & \quad \text{etc.} \end{aligned}$$

For group B,

Column I

$$\begin{aligned} S_{12} + S_{13} &= S'_{22} + S_{23} \\ \rho_2(S_{12} - S_{13}) &= \rho_1(S'_{22} - S_{23}) \\ S'_{22} + S_{24} &= S'_{32} + S_{34} \\ \rho_3(S'_{22} - S_{24}) &= \rho_2(S'_{32} - S_{34}) \\ S'_{32} + S_{35} &= S'_{42} + S_{45} \\ \rho_4(S'_{32} - S_{35}) &= \rho_3(S'_{42} - S_{45}) \\ \text{etc.} & \quad \text{etc.} \end{aligned}$$

Column II

$$\begin{aligned} S_{13} + S_{14} &= S'_{23} + S_{24} \\ \rho_2(S_{13} - S_{14}) &= \rho_1(S'_{23} - S_{24}) \\ S'_{23} + S_{25} &= S'_{33} + S_{35} \\ \rho_3(S'_{23} - S_{25}) &= \rho_2(S'_{33} - S_{35}) \\ S'_{33} + S_{36} &= S'_{43} + S_{46} \\ \rho_4(S'_{33} - S_{36}) &= \rho_3(S'_{43} - S_{46}) \\ \text{etc.} & \quad \text{etc.} \end{aligned}$$

Column III

$$S_{14} + S_{15} = S'_{24} + S_{25} \qquad \text{etc.}$$
$$\rho_2(S_{14} - S_{15}) = \rho_1(S'_{24} - S_{25})$$
$$S'_{24} + S_{26} = S'_{34} + S_{36} \qquad \text{etc.}$$
$$\rho_3(S'_{24} - S_{26}) = \rho_2(S'_{34} - S_{36})$$
$$S'_{34} + S_{37} = S'_{44} + S_{47} \qquad \text{etc.}$$
$$\rho_4(S'_{34} - S_{37}) = \rho_3(S'_{44} - S_{47})$$
$$\text{etc.} \qquad \text{etc.}$$

It will be noted that the value of S_{11} is fixed by equation 11. This means that the first two rows in group A present a pair of linear equations in two unknowns, S_{12} and S_{21}; it is easy to see that the two equations are consistent, and the unknowns can be found.

Passing to the next pair in group A, we observe that again there are two equations in two unknowns, S_{23} and S_{31} (S_{21} being known from and uniquely determined by the pair above). In this manner we can solve all equations in group A. The complete solution of the equations constituting group A gives two sequences of S:

first $S_{11}, S_{21}, S_{31}, S_{41}, S_{51} \ldots$ etc.
second $S_{12}, S_{23}, S_{34}, S_{45}, S_{56} \ldots$ etc.

All members of the second sequence and *none* of the first will appear in group B.

Now, considering the first pair in column 1 of group B, we find that S_{12} and S_{23} are known from group A (second sequence), and that the pair can be solved for S_{13} and S'_{22}, whose value has not been fixed as yet. Passing to the second pair in the same column, we find that S'_{22} is known from the pair above and S_{34} is known from group A (second sequence); and that we can solve for S_{24} and S'_{32}. In this manner we can solve all equations in column 1 of group B, working diagonally downward.

In column 2, group B, in the first pair of equations, S_{13} and S_{24} are known from column 1, group B. Hence we can solve for S_{14} and S'_{23}, the values of which have not been fixed as yet.

Passing to the second pair in the same column, we find that S'_{23} is known from the first pair, and S_{35} from column 1, group B.

Continuing this reasoning we can show that all the equations of group B are consistent and can be solved, determining uniquely every S except those already determined by group A.

Solution for Group A

The equations of group A can be worked out as explained above but are most easily solved by inspection.

The first sequence of S resulting from group A is of the type $S_{n,1}$, which optically corresponds (according to the analogy given in an earlier section) to the original point-source as seen by an observer in the nth

medium. The fraction of light transmitted through $n - 1$ mirrors can easily be calculated and our tentative solution immediately presents itself in the form:

$$S_{n,1} = (1 - A_1)(1 - A_2)(1 - A_3) \ . \ . \ . \ (1 - A_{n-1})\rho_n \qquad [12]$$

(Compare with equation 7). By virtue of equation 10.1,

$$1 + A_{p-1} = \frac{2\rho_p}{\rho_p + \rho_{p-1}} \qquad [10\ 11]$$

and

$$1 - A_{p-1} = \frac{2\rho_{p-1}}{\rho_p + \rho_{p-1}} \qquad [10.12]$$

Hence $\rho_p(1 - A_{p-1}) = \rho_{p-1}(1 + A_{p-1})$

Applying a transformation of this type to equation 12,

$$\begin{aligned} &\rho_n(1 - A_{n-1})(1 - A_{n-2}) \cdots (1 - A_1) = \\ &\rho_{n-1}(1 + A_{n-1})(1 - A_{n-2}) \cdots (1 - A_1) = \\ &\rho_{n-2}(1 + A_{n-1})(1 + A_{n-2})(1 - A_{n-3}) \cdots (1 - A_1) = \\ &\quad . \quad . \quad . \quad . \quad . \quad . \\ &\rho_1(1 + A_{n-1})(1 + A_{n-2})(1 + A_{n-3}) \cdots (1 + A_1) \end{aligned}$$

Hence equation 12 can be replaced by

$$S_{n,1} = \rho_1(1 + A_1)(1 + A_2)(1 + A_3) \cdots (1 + A_{n-1}) \qquad [12.1]$$

It must now be shown that equation 12.1 satisfies the type form of group A; namely

$$S_{n,1} + S_{n,n+1} = S_{n+1,1}$$
$$\rho_{n+1}(S_{n,1} - S_{n,n+1}) = \rho_n S_{n+1,1}$$

Eliminating $S_{n,n+1}$, we obtain

$$2\rho_{n+1}S_{n,1} = S_{n+1,1}(\rho_{n+1} + \rho_n)$$

or, dividing by $(\rho_{n+1} + \rho_n)$ and making use of equation 10.11,

$$(1 + A_n)S_{n,1} = S_{n+1,1}$$

Evidently, this is satisfied by equation 12.1; hence our tentative solution is correct.

Solving the above type form of group A for the second sequence of S we obtain

$$(\rho_{n+1} + \rho_n)S_{n,n+1} = (\rho_{n+1} - \rho_n)S_{n,1}$$

and making use of equation 10.1

$$S_{n,n+1} = A_n S_{n,1}$$

Thus it is seen that corresponding terms in the two sequences of the solutions of group A differ only by the factor A_n.

$S_{11} = \rho_1$
$S_{21} = (1 + A_1)\rho_1$
$S_{31} = (1 + A_1)(1 + A_2)\rho_1$
$S_{41} = (1 + A_1)(1 + A_2)(1 + A_3)\rho_1$, etc.

$S_{12} = A_1\rho_1$
$S_{23} = (1 + A_1)A_2\rho_1$
$S_{34} = (1 + A_1)(1 + A_2)A_3\rho_1$
$S_{45} = (1 + A_1)(1 + A_2)(1 + A_3)A_4\rho_1$, etc.

Solution for Group B

Since the ordinary methods of electrical prospecting are limited to the measurement of surface potentials, we are interested only in obtaining the potential v_1 for the uppermost layer, and shall solve group B only for the sequence

$$S_{11},\ S_{12},\ S_{13},\ S_{14},\ S_{15},\ S_{16}\ \ldots\ \text{etc.}$$

From the section on validity of solution, the general terms of group B, provided that we agree to drop the primes when the first subscript is unity, are

$$S'_{n,m} + S_{n,m+n} = S'_{n+1,m} + S_{n+1,m+n} \qquad [13]$$
$$\rho_{n+1}(S'_{n,m} - S_{n,m+n}) = \rho_n(S'_{n+1,m} - S_{n+1,m+n}) \qquad [14]$$

where $m \geqq 2$, $n \geqq 1$.

By treating equations 13 and 14 simultaneously and making use of equations 10.1 and 10.12, we obtain

$$-A_nS'_{n,m} + S_{n,m+n} = (1 - A_n)S_{n+1,m+n} \qquad [15]$$
$$S'_{n,m} - A_nS_{n,m+n} = (1 - A_n)S'_{n+1,m} \qquad [16]$$

S_{11} and S_{12} are already known, so we shall start by solving for S_{13}.

Determination of S_{13}.—Let $m = 2$ and replace n by $n - 1$ in equation 15. We obtain

$$-A_{n-1}S'_{n-1,2} + S_{n-1,n+1} = (1 - A_{n-1})S_{n,n+1} \qquad [15.1]$$

Let $n = 2$ in equation 15.1. Transposing, we obtain

$$S_{13} = (1 - A_1)S_{23} + A_1S_{12}$$

Taking the value of S_{23} from the table at the end of the section on solution for group A, we obtain

$$S_{13} = A_1S_{12} + (1 - A_1^2)A_2\rho_1$$

Determination of S_{14}.—Let $m = 3$ and replace n by $n - 2$ in equation 15. Then

$$-A_{n-2}S'_{n-2,3} + S_{n-2,n+1} = (1 - A_{n-2})S_{n-1,n+1} \qquad [15.2]$$

In equation 16 replace n by $n - 2$ and m by 2.

Then $$S'_{n-2,2} - A_{n-2}S_{n-2,n} = (1 - A_{n-2})S'_{n-1,2} \qquad [16.2]$$

Multiply equation 15.1 by $(1 - A_{u-2})$ and substitute the values of equations 15.2 and 16.2. This will give

$$-A_{n-2}S'_{n-2,2} + A_{n-1}A_{n-2}S_{n-2,n} - A_{n-2}S'_{n-2,3} + S_{n-2,n+1} = (1 - A_{n-1})(1 - A_{n-2})S_{n,n+1} \quad [17]$$

Collecting and transposing terms and letting $n = 3$, and substituting the value of S_{34} obtained from the table at the end of the section on solution for group A, we obtain

$$S_{14} = (1 - A_1^2)(1 - A_2^2)A_3\rho_1 + S_{13}(A_1 - A_1A_2) + S_{12}A_2$$

Determination of S_{15}.—Reduce the four terms of the left-hand side of equation 17. In equation 15, replacing n by $n - 3$, and m by 4,

$$-A_{n-3}S'_{n-3,4} + S_{n-3,n+1} = (1 - A_{n-3})S_{n-2,n+1} \quad [15.3]$$

In equation 16, replacing n by $n - 3$, and m by 3

$$S'_{n-3,3} - A_{n-3}S_{n-3,n} = (1 - A_{n-3})S'_{n-2,3} \quad [16.3]$$

In equation 15, replacing n by $n - 3$, and m by 3

$$-A_{n-3}S'_{n-3,3} + S_{n-3,n} = (1 - A_{n-3})S_{n-2,n} \quad [15.4]$$

In equation 16, replacing n by $n - 3$, and m by 2

$$S'_{n-3,2} - A_{n-3}S_{n-3,n-1} = (1 - A_{n-3})S'_{n-2,2} \quad [16.4]$$

Multiplying equation 17 by the factor $(1 - A_{n-3})$ and substituting into it equations 15.3, 16.3, 15.4 and 16.4, we obtain

$$\begin{aligned}-A_{n-1}S'_{n-3,2} + A_{n-1}A_{n-3}S_{n-3,n-1} - A_{n-1}A_{n-2}A_{n-3}S'_{n-3,3} \\ + A_{n-1}A_{n-2}S_{n-3,n} \\ -A_{n-2}S'_{n-3,3} + A_{n-2}A_{n-3}S_{n-3,n} - A_{n-3}S'_{n-3,4} + S_{n-3,n+1} \\ = (1 - A_{n-1})(1 - A_{n-2})(1 - A_{n-3})S_{n,n+1} \quad [18]\end{aligned}$$

Letting $n = 4$, transposing and collecting terms, and substituting for S_{45}, we obtain

$$\begin{aligned}S_{15} = (1 - A_1^2)(1 - A_2^2)(1 - A_3^2)A_4\rho_1 &+ S_{14}(A_1 - A_1A_2 - A_2A_3) \\ &+ S_{13}(A_2 - A_1A_3 + A_1A_2A_3) \\ &+ S_{12}(A_3)\end{aligned}$$

Determination of S_{16}.—The eight terms on the left-hand side of equation 18 can be reduced to six. These six terms are then reduced in the same manner as in the determination of S_{15}. This procedure can be

carried out until as many values of S are determined as are found necessary for the calculations.

Numerical Calculations

For numerical calculations, we are interested only in the surface potentials. Thus we set $z = 0$.

Let
$$\frac{S_{1,n+1}}{S_{11}} = \frac{S_{1,n+1}}{\rho_1} = Q_n \text{ then}$$

$$(v_1)_{z=0} = V = \frac{\rho_1 I}{2\pi}\left\{\frac{1}{R} + \frac{2Q_1}{\sqrt{R^2+m^2}} + \frac{2Q_2}{\sqrt{R^2+4m^2}} + \cdots\right\}$$

Suppose for field work we use the Wenner method[3] of placing the power and pickup electrodes. This configuration is shown in Fig. 3.

Wenner's formula for the "apparent resistivity" is

$$\rho_a = 2\pi R\frac{\Delta v}{I}$$

Our expression for Δv for this configuration of electrodes is

$$\Delta v = \frac{\rho_1 I}{2\pi}\left\{\frac{1}{R} + 4Q_1\left[\frac{1}{\sqrt{R^2+m^2}} - \frac{1}{\sqrt{4R^2+m^2}}\right] + \right.$$
$$\left. 4Q_2\left[\frac{1}{\sqrt{R^2+4m^2}} - \frac{1}{\sqrt{4R^2+4m^2}}\right] + \cdots \right. \quad [19]$$

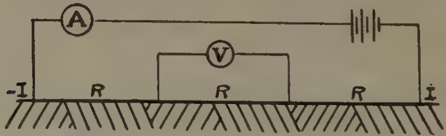

Fig. 3.—Wenner method of placing electrodes.

Substituting equation 19 in Wenner's formula, we obtain

$$\rho_a = 2\pi R\frac{\Delta v}{I} = \rho_1\left\{1 + 2RQ_1P_{R,1} + 2Q_2RP_{R,2} + 2RQ_3P_{R,3} + \cdots\right.$$

$$= \rho_1\left\{1 + 2R\sum_{n=1}^{n=\infty} Q_nP_{R,n}\right\}, \text{ where } \rho_{R1n} = 2\left[\frac{1}{\sqrt{R^2+n^2m^2}} - \frac{1}{\sqrt{4R^2+n^2m^2}}\right]$$

For computing purposes it is convenient to prepare a list of the values of P. These differ for each electrode spacing R, but are the same for all

[3] F. Wenner: A Method of Measuring Earth Resistivity. U. S. Bur. Stds. *Sci. Paper* 258 (1917), 469–478.

problems involving the Wenner configuration of electrodes, provided R and m are always chosen the same. Q, on the contrary, is the same for all electrode spacings in the same problem, but differs in different problems.

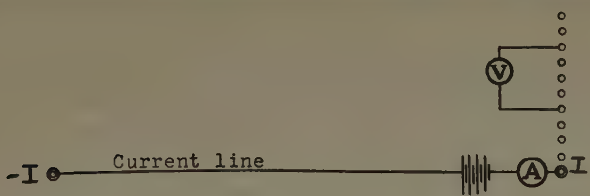

FIG. 4.—CONFIGURATION OF ELECTRODES IN A STATIONARY CURRENT ELECTRODE METHOD.

The current line should be great compared to the length of the potential line in order that $-I$ may be neglected in computations.

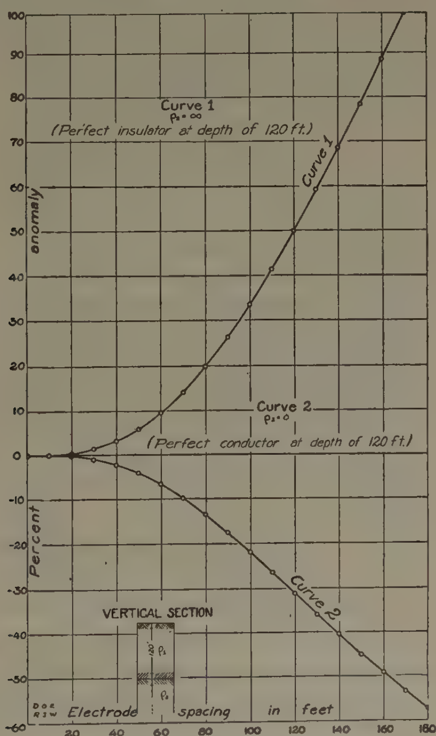

FIG. 5.—TWO-LAYER PROBLEM.

In calculations ρ_1 may be, for convenience, taken as unity or 100 per cent. The vertical anomaly of resistivity is

$$B = 2R \sum_{n=1}^{\infty} Q_n P_{R,n}$$

Field Work

Since the apparent resistivity is $\rho_a = \rho_1(1 + B)$ it is easy to see that if the values of ρ_a for the different electrode spacings are to be compared, ρ_1, the resistivity of the overburden, must remain constant. In certain terrains, ρ_1 changes from point to point considerably, and when the current electrode is moved, changes in ρ_1 occur which may overshadow the vertical anomaly B. Hence it is safer and sometimes necessary to use stationary current electrodes.

The mathematical formula for the stationary electrode method illustrated in Fig. 4 differs from the formula for the Wenner method only by the factor 2.

Figs. 5 and 6 show the variation of the percentage anomaly of resistivity with electrode spacing, using the stationary current electrode system. Curves 1 and 2 illustrate a two-layer problem, curves 3 and 4 a three-layer problem. It is interesting to note that in not one case

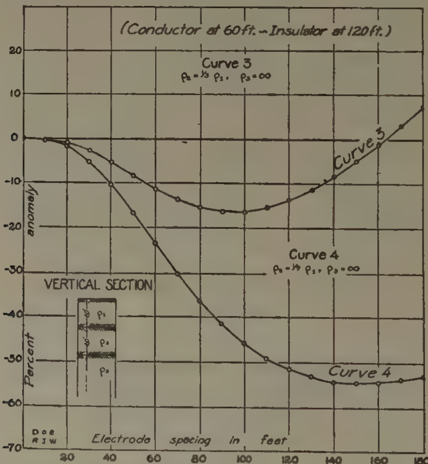

Fig. 6.— Three-layer problem.

does a maximum, minimum or point of inflection correspond in electrode spacing to the depth of an abrupt change in resistivity. Moreover, the positions of minima and points of inflection change with the ratio of resistivity, the vertical section remaining the same.

DISCUSSION

(Allen H. Rogers presiding)

S. F. Kelly, New York, N. Y.—I have one or two words to say in the nature of a slight criticism of the actual putting together of this paper. On page 429 the authors use a symbol S with numerous subscripts, and give no indication of what the S is nor what the subscripts are. In the formula given on page 436, they have given an expression for Q, but leave us rather in the dark as to the meaning of the other symbols.

L. Gilchrist, Toronto, Ont. (written discussion).—The final conclusion of the authors needs careful consideration; *viz.*, that if a potential be applied at a point A in the upper surface of the top layer of a series of homogeneous conductors of different resisting horizontal layers between which there is no discontinuity there will be no sharply defined change in distribution of potential from the upper surface downward and therefore no well-defined maxima and minima or points of inflection in the average resistivity in the interequipotential bowl sections.

This development has been presented by others for at least the case of two layers, and this is acknowledged by the authors. Similar conclusions have been reached by all of those who have developed the problem from the same foundations, and it may be taken, therefore, that the conclusions are well founded.

However, it should be pointed out that the conclusion is also inevitable that there cannot be a sharply defined change in the distribution of potential radially from A along the upper surface of the top layer. Now it is just this radially distributed potential that has been determined by investigators in the field and in several cases there are points at which there is a rapid if not sharply defined change in the distribution of potential, and in many of these cases there is a correlation between the distances from A to these rapid changes and the depth of the conducting layers. It would appear then that the foundations of the development are not those existing in general in the field; that, in fact, in the field there are changing conditions in the successive layers or at the contacts of the layers which are not comprehended in the foundations of the mathematical development.

It is a common experience of those who make investigations in the field with two electrodes A(+) and B(−) placed at points some distance apart on the upper surface of an extensive deep layer of fairly homogeneous glacial overburden that the distribution of potential about either of the electrodes for some distance is remarkably symmetrical, much more so than if the layer were a homogeneous conductor such as is contemplated in the author's paper. It is found also that one or more underlying layers may even enhance this state of symmetry.

It is suggested therefore that the dispersion of the currents due to the discontinuities in the materials of the layers themselves and the moisture content in these layers, both of which in general differ in successive layers, together with local potential differences of the contour of the layers, are factors contributing to rapid changes in the distribution of potential on the surface and should be taken account of by the authors.

It is also suggested that in field investigations where distribution of potential on the surface has been determined as a function of the distance from the point of application of the potential, the Laplace equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

be solved with $V = F(r)$ at the surface as gotten experimentally from the surface measurements as the primary and only dependable boundary condition which may be applied with confidence. And further to enable this to be done, it is suggested that the layout by which the potential is applied to the surface should always be such that symmetry would obtain for a single homogeneous continuous layer of good conductivity. This condition is obtained by the use of a central point or line electrode with symmetrically placed distant point or line electrodes in which the currents entering at the distant electrodes have been equalized. If lack of symmetry in distribution of potential about the central electrode exists, the direction in which a change in homogeneity will be indicated. If symmetry in distribution of potential exists, but a sharp change in $F(r)$ with some value of r the change in homogeneity would be indicated as existing in depth.

I. Roman, West Orange, N. J. (written discussion).—This paper, I believe, constitutes a real contribution to the theory of electric resistivity measurements. One of the outstanding parts of the paper is as follows: The paper considers several layers of arbitrary thickness, which is assumed the same for all. This assumption of equal thickness is the important feature of the paper, for it makes possible the prediction of the positions of all images which can occur. Suppose that a current source is located at some point in one of the media. This source has an image in every boundary separating two of the layers; each image has an image in every boundary; and this extension of images causes a rapid accumulation of images as the number of reflections increases. In the present analysis, the source is placed at the surface of the ground, which is the boundary between the first two media. All images must lie at distances from this surface which are multiples of twice the common thickness, so that the set is overlapping. We can arrange them in an order determined by the single quantity, which is the ratio of the distance to twice the thickness. In general, the method is not so simple.

Mr. Kelly said that he did not understand the meaning of S. It is this: (1) In each of the layers there is an expression for the potential, this expression being, in general, different for each layer; (2) there are certain boundary conditions, namely, that the potential itself and the product of the conductivity by the derivative of the potential normal to the interfaces are both continuous; (3) the first subscript of S indicates the potential expression of which it is a part—that is, the layer over which the expression is valid; (4) the second subscript of S indicates the particular image, either isolated or overlapping, which contributes that particular term to the potential expression.

Each potential expression thus has an infinite number of terms, one from each of an infinite set of images. The authors derive expressions for only the first few terms of the potential in the upper layer. In the case of a two-layer earth, which I have considered in an unpublished manuscript, it was necessary in some cases to calculate over one hundred terms of the series to obtain sufficiently accurate results. The first five terms would be of little use in such a case. If one hundred terms were needed for each calculation, the time required for a really practical analysis for, say, five layers, might easily exceed ten years.

The expression for P on page 436 is the bracket in equation 19. There seems to be no question about it, although there is no statement to that effect. The authors state that the value of P depends only on the size of the Wenner configuration used; really, it depends on the ratio of R to m, as well as on R. This is not made clear in the text. The value of Q, as the authors state, depends upon the various resistivities.

There is another point. The authors state that the interpretation is improved if the current poles are kept fixed and the potential electrodes moved. From theoretical considerations, at least in the case of two layers, the measurements made depend only upon the manner in which the four points are paired and not upon which pair is used for the current and which for the potential. I do not think it makes any difference, and I fail to agree with the statement at the top of page 438.

The authors say that the results are closed, simple and compact. I do not agree with them. An expression with an infinite number of terms can hardly be called closed. An inspection will show that the results are not simple. As for being compact, I have not proved that they are not, but my experience with similar forms leads me to believe that they are seldom compact in cases arising in practice.

S. F. Kelly.—Is it possible that I may have misunderstood the authors' suggestion about the placing of the current electrodes? As I understand it, their suggestion for maintaining the current electrodes stationary is entirely a field procedure to avoid errors induced by changing resistivity in the surface material. That is, as one moves

the current electrodes from one place to another, the resistivity of the material in which they are placed will vary, which will cause the introduction of errors that have nothing whatever to do with the vertical changes in resistivity. As I understand it, they are not saying that this is a theoretically better arrangement in the case of perfectly homogeneous media.

I. ROMAN.—The same effect enters if the position of the potential electrodes is changed. Any irregularity at the points where the four leads enter the ground will affect the readings, regardless of which two are the current poles and which are the potential electrodes.

S. F. KELLY.—Yes, but if four are moved together there are four possible changes in resistivity of surficial material that may be introduced, while the modifications they propose provide that two can be kept stationary and only the other two moved, thus cutting in half the possible causes of error.

I. ROMAN.—In practice, the system of keeping two or three of the points fixed and moving the other pair, or one, is often used. The Wenner configuration simplifies the calculations, but it seems to be more or less antiquated for exploration work.

R. J. WATSON (written discussion).—Referring to Dr. Gilchrist's remarks: the authors themselves have observed in field work some rather abrupt changes in the potential gradient around a current electrode but have felt rather skeptical about always assigning their cause to vertical changes in the section. In many cases too little was known about the electrical character of the various beds to warrant any close correlation. It has been their fortunate experience to encounter conditions that gave excellent curves of the "three-layer" type, as illustrated in Fig. 6. A study of core-drill results showed that there was present in the vertical section a thick bed of material which would undoubtedly act as an excellent resistor. In scarcely any of the results obtained by the authors did the minimum of the resistivity curve correspond to the depth of the resistor. Experience in the region allowed them to formulate a correction which made their interpretations of some value.

In order that a better method of interpretation might be developed, based on actual measurements as well as mathematical theory, a number of model experiments were performed. The layers of differing resistivity were obtained by moistening a sandy clay and rolling it out into layers of a definite thickness. Great care was exercised in this work and the results obtained gave great encouragement. To date it must be truthfully stated that only the cases in which there is a perfect resistor at the bottom of the series of layers have given really successful results. So far it has been difficult to obtain a practical perfect conductor of sufficient size for the purpose without involving great expense. As to the cases involving a perfect resistor at the base, the experiments may be said to have been extremely successful from the authors' point of view in that the resistivity curves obtained check well with the curves computed from the formulas in the paper.

Referring to the discrepancy between the conclusions of the mathematical development and the model work on the one hand, and the results of field investigations (some of which have been attended with astonishing success) on the other, the suggestion of Dr. Gilchrist may be of value. It may be that the electrical conditions actually existing in many of the localities where experiments have been made are not nearly so simple as are assumed in the mathematical development or actually realized in the model work.

Dr. Roman is too pessimistic about the difficulties in computing the curves. After constructing a suitable set of special tables the work proceeds fairly rapidly. A competent calculating machine operator could compute curves 3 and 4 in one day,

making use of these tables; 25 terms seem to be sufficient for these curves. The important point about the computation is the compiling of the special tables for a given configuration of electrodes having sufficient accuracy to be applicable to the more complex problems. Even if it were necessary to use 100 terms, and it is doubtful whether conditions would ever be so bad, a competent computor equipped with a modern calculating machine could do the work in a great deal less time than 10 years. After all, one would compute only a set of type curves and much of the interpretation could be done by interpolation. The results of one curve might be used in a number of localities.

D. O. Ehrenburg (written discussion).—Dr. Roman has refused to recognize the advantage of keeping the current electrodes stationary, in spite of Mr. Kelly's explanation. However, a simple consideration of the lines of flow shows that a minor irregularity of the medium *away* from the source (such as at a probe electrode) will distort the electric field but slightly; whereas an irregularity *at* the source will distort the field considerably. This is borne out very closely by a number of special field tests conducted by the authors.

Observed and Theoretical Electromagnetic Model Response of Conducting Spheres

By L. B. Slichter,* Madison, Wis.

(New York Meeting, February, 1930)

After statement of the principles of similitude which apply to electromagnetic modeling, charts showing the inductive response of conducting spheres as dependent upon frequency, conductivity, and size are exhibited in this paper. The shape of the curve which relates intensity of response to frequency is significant in indicating the advisability of low frequencies in the exploration for conducting bodies, when consistent with the securing of a large percentage of the ideally possible response. Comparisons of theoretical response and that observed with models are shown, and relationships with the subject of applied electromagnetic prospecting are indicated.

Maxwell Field Equations

The Maxwell field equations summarize the physical laws that govern electrical field phenomena; they are the basis for the solution of electromagnetic problems. These concise equations, with adequately formulated boundary conditions, specify the complete electrical condition of continuous media, and the development of their solution amounts merely to a more convenient re-expression of phenomena already completely described. Thus, the immediate content of the field equations themselves is sufficient and sometimes convenient for the treating of certain questions (such as those relating to electromagnetic models) which are independent of the special factors of geometry, or the nature of an impressed field.

The four Maxwell field equations are written below, in c.g.s. gaussian units:

$$\text{curl } \frac{B}{\mu} = \frac{4\pi}{c}\left(\sigma E + \frac{\epsilon \dot{E}}{4\pi}\right) \qquad [1]$$

$$\text{curl } E = -\dot{B}/c \qquad [2]$$

$$\text{div } \epsilon E = 4\pi\rho \qquad [3]$$

$$\text{div } B = 0 \qquad [4]$$

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* Mason, Slichter & Gauld. The writer wishes to acknowledge his indebtedness to his associates, particularly B. B. Gauld and D. L. Hay, for their valuable cooperation and assistance in connection with the electromagnetic model curves.

where B = magnetic vector in electromagnetic units.
E = electric vector in electrostatic units.
μ = magnetic permeability.
ϵ = dielectic constant.
σ = electrical conductivity in electrostatic units.
ρ = charge density, in electrostatic units.
c = velocity of light.

The first two of these equations may be said to be the fundamental ones, for the last two are implied by them; the last one by the second, and the third by the first equation, if one introduces the condition for conservation of electrical charge, div $i + \dot{\rho} = 0$. The boundary conditions concerning the discontinuities in the electromagnetic vectors at the interfaces of continuous media are also deducible from the field equations themselves.[1]

Problems to Be Solved

In the usual problems of electrodynamics, one regards as known the three constants of materials, σ, ϵ, and μ, the excitation field, and the geometry of the region in interest, and solves for the two electromagnetic field vectors E and B. In electrical prospecting, on the contrary, one measures the vectors E and B (or their equivalents) at the surface, and desires to deduce from them knowledge concerning the unknown geometrical configuration and constants of materials in the concealed subsurface. The knowns and unknowns are interchanged in these two types of problems.

The second, or geophysical type of problem, is much the more difficult of the two. The known data refer only to the source field and to boundary values of the electromagnetic field over the upper surface of the region in interest, and these data are insufficient uniquely to determine the desired unknowns. Even when deficiencies are supplied by reasonable assumptions, the mathematical difficulties are enormous; and, as yet, theory provides the geophysicist no tools of sufficient power for a direct analysis of the problem. Electromagnetic interpretations in applied geophysics are therefore necessarily made by an indirect method—that is, by the comparison and matching of field results with the solutions of problems of the first mentioned type, in which geometry and constants of materials are completely postulated. The process is one of cut and try; an effort to fit field data with solutions of known problems. Such a procedure clearly requires a large store of typical cases with which to make comparisons, interpolations and extrapolations. In a word, a comprehensive key or catalog is demanded, by the aid of which field data may be classified and studied. This need is met in part by theoretical solu-

[1] Full critical treatment of the field equations is given by M. Mason and W. Weaver: The Electromagnetic Field. Univ. of Chicago Press, 1929.

tions, but the analytical difficulties are serious except in the cases of simple geometry, and it is expedient to shift some of the burden from theory to experiment. By means of models, one may obtain some of the large amount of detailed knowledge concerning specific electromagnetic responses, which is necessary in establishing a key sufficiently comprehensive to be of value in a comparative scheme for interpreting field data. The term "interpretation" is used in a narrow sense in this paper, and refers only to deductions concerning the *electrical* structure or composition of the region. These purely electromagnetic interpretations must, of course, be further translated into their significance in terms of the local geology.

Electromagnetic Models

In an electromagnetic model which is a true geometric reduction of the original, it would clearly be desirable for the electromagnetic vectors at corresponding points to be identical, so that the model would constitute a direct reading reproduction of the field problem. It is evident from examination of the first two, or fundamental, field equations that a model of this kind is possible, provided it is constructed in accordance with certain simple rules of transformation. The appropriate transformations may be seen as follows: The constants of materials μ and ϵ in the first field equation are dimensionless in the gaussian system of units. Since c is a velocity, it is clear that the E and B vectors here have the same dimensions. The conductivity factor σ has, then, dimensions $(1/T)$. Let, now, a model of the actual field problem be made by the following procedure. Introduce small units, each reduced by the same factor, for measuring length and time as they enter throughout the model, both explicitly in the co-ordinates, and implicitly in the electrical constants of materials. Preserve equal in the model and the original the numerical results of all corresponding measurements, when these measurements refer to any of the quantities,—time, length, or electrical constants of material,—which enter the field equations. Then the conditions on the E and B vectors, as expressed by the field equations, remain identical in the two systems, and, under the same boundary conditions, the solutions for E and B must obviously be the same throughout the regions.

Conceived in terms of the standard large units, the model appears as follows: Lengths are, of course, uniformly reduced by the chosen ratio of units; time intervals are shortened by a like factor, hence frequencies are raised; the dielectric constant and permeability are unchanged, and the conductivity is multiplied by the scale factor. Referring to the third field equation, which, as has been said, is dependent upon the first, the density of charge is multiplied by the scale factor in correspondence with the increased conductivity term.

The reproduction of the large-scale boundary conditions is obviously a practical matter subject to the limitations of a model's restricted extent.

The model should provide the appropriate electrical environment about the particular features under study, over distances sufficiently large to render negligible distant effects arising from discrepancies external to the model. The reproduction of the source field usually offers no difficulties, especially since one is customarily concerned not with absolute intensities but merely with those relative to the source. In accordance with the preceding paragraph, values of current densities, charge densities and frequencies in the source should be multiplied by the scale factor.

In summary, then, identical values of E and B (all quantities now measured in standard large units) are obtained at corresponding points when the conductivity in the model is multiplied by the scale factor, the dielectric constant and permeability left unchanged, and the current, charge densities and frequencies in the sources are increased by the scale factor.

Models of the above type may be constructed to reproduce faithfully complex actual conditions of geometry and materials, but as a foundation for the general study and interpretation of field results, the most instructive cases will be simple ones. Simple models whose theoretical response can be calculated readily offer valuable opportunities for checks between theory and experiment. As such an example, the response of a conducting sphere in an inductive field will be discussed and compared with measurements obtained with models.

Inductive Response of Spherical Conductor

The general problem of the reaction of a spherical body to an impressed electromagnetic field has been treated by numerous authors; especially fully by Debye.[2] In connection with studies on light pressure, Debye develops the general form of the solution in terms of arbitrary electrical constants of material, ϵ, μ and σ, both for the sphere and its surroundings, and treats in particular the case of a plane polarized wave incident upon a sphere. Livens[3] considers the problem of a conducting sphere in a uniform oscillating magnetic field, which is a type of excitation commonly approximated in electromagnetic prospecting. March[4] recently presented a paper developing general solutions of the Debye type for the case of a dipolar source field, which is also a type of excitation often employed in practical prospecting, but this work is as yet published only in abstract.

The Livens treatment is subject to the somewhat drastic restriction that the conductivity of the material surrounding the sphere must be negligibly small. This condition, however, often finds fair approximation

[2] P. Debye: Der Lichtdruck auf Kugeln von beliebigen Material, *Ann. d. Phys.* (1909) **30**, 57.

[3] G. H. Livens: Theory of Electricity, 400. Cambridge University Press, 1918.

[4] H. W. March: The Field of a Magnetic Dipole in the Presence of a Conducting Sphere. Abs., *Bull.* Amer. Math. Soc. (1929) **35**, 455.

at low frequencies in applied geophysical work, for the inductive response of the country rock is then usually slight. Moreover, in model work, a great practical simplification is obtained if the response of the conducting model is studied in the ideally insulated case; that is, when surrounded by air. These considerations, combined with the advantage of the ease of computation of the resulting solution, recommend the specialized treatment of Livens as a sufficient theoretical foundation for the consideration of the present model results. This solution will be briefly outlined, as an introduction to the discussion which follows.

The inducing magnetic field of frequency $w/2\pi$ is assumed substantially uniform throughout the space occupied by the sphere. Inside the sphere, the conductivity σ, in the complex constant of material occurring in the field equations, is assumed to predominate to such an extent that the imaginary part is negligible. The extreme predominance of the real or conductivity term in electrical ore prospecting is generally recognized but is perhaps worth emphasizing again here. To illustrate, if σ is written in reciprocal ohms instead of electrostatic units, the above constant of material is $9 \times 10^{11}\sigma_p + i\frac{\epsilon n}{2}$ where n is the frequency and σ_p the conductivity in practical units. Hence, even at 100,000 cycles, and for ores as high as 1000 ohms in resistivity and possessing a high dielectric constant of 18, the real term is a thousand-fold the imaginary one. Externally to the sphere, both the conductivity and dielectric constant are assumed to be vanishingly small, a condition which, to be sure, is impossible of realization in the case of the dielectric constant, but is nevertheless a fair approximation in the neighborhood of the sphere for the conductivities and frequencies here in interest.[5]

The problem is formulated in the appropriate spherical coordinates, r, θ, and ϕ, and the abbreviation $\pi \equiv r \sin \theta E_\phi$ introduced. The uniform impressed field, B_0e^{iwt} is, by Faraday's law, expressed in terms of the function π through the relation

$$\pi_0 = -i\frac{wr^2}{2c} \sin^2 \theta B_0 e^{iwt}$$

The solutions for π satisfying the three boundary conditions—(1) the vanishing of the secondary field at infinity; (2) its finiteness at all points; and (3) the continuity of the tangential as well as the normal component of B across the boundary (since finite current densities only are permissible), and the continuity of the tangential component of E—are found to be, outside the sphere,

$$\pi_1 = \pi_0\left(1 + \frac{A_1}{r^3}\right) \qquad [5]$$

[5] The error in the curves of Fig. 1 incurred by assigning to ϵ the value zero, instead of unity, is within 1 per cent., as verified by comparison with the results of H. W. March.

and inside the sphere,

$$\pi_2 = \pi_0 A_2 \lambda r \frac{d(\sin \lambda r/\lambda r)}{d\lambda r} \tag{6}$$

where
$$\lambda^2 = -\frac{4\pi i w \sigma}{c^2}$$

From these solutions, the components of B are to be obtained through the relations

$$\frac{\partial \pi}{\partial \theta} = -\frac{r^2 \sin \theta}{c} \dot{B}_r$$
$$\frac{\partial \pi}{\partial r} = \frac{r \sin \theta}{c} \dot{B}_\theta$$

In ore prospecting the values of the complex quantity λr will usually be greater than unity. For example, at 1000 cycles, and for $r = 30$ meters and $1/\sigma = 8$ ohms,

$$|\lambda r| = 3\pi$$

hence an expansion in series for small λr, as used by Livens, is here unsuitable, and the computations will be modified accordingly.

The values of the constants A_1 and A_2 above as determined from the boundary conditions are:

$$A_1 = a^3\left\{\frac{3}{(\lambda a)^2} - 1 - \frac{3 \cos \lambda a}{\lambda a \sin \lambda a}\right\} = a^3(X_1 + iY_1) \tag{7}$$
$$= -a^3\left[1 + \frac{3}{2p_1}\frac{\sin p_1 \cos p_1 - \sinh p_1 \cosh p_1}{\sin^2 p_1 + \sinh^2 p_1}\right]$$
$$- i\frac{3a^3}{2p_1}\left[\frac{\sin p_1 \cos p_1 + \sinh p_1 \cosh p_1}{\sin^2 p_1 + \sinh^2 p_1} - \frac{1}{p_1}\right]$$

$$A_2 = -\frac{3a^2}{\lambda a \sin \lambda a} = \tag{8}$$
$$\frac{-3a^2\{\sin p_1 \cosh p_1 - \sinh p_1 \cos p_1 + i(\sinh p_1 \cos p_1 + \sin p_1 \cosh p_1)\}}{2p_1(\sin^2 p_1 + \sinh^2 p_1)}.$$

where
$$p_1 = \frac{a}{c}\sqrt{2\pi\sigma w} \tag{9}$$

On retaining the real part of the solution, corresponding to a real excitation $B_0 e^{iwt}$ it results that the external partial field due to the sphere alone, is,

$$B_r = -\left(\frac{a}{r}\right)^3 Q_1 \cos \theta B_0 \cos (wt + \delta_1)$$
$$B_\theta = -\frac{1}{2}\left(\frac{a}{r}\right)^3 Q_1 \sin \theta B_0 \cos (wt + \delta_1)$$

where
$$Q_1 = \sqrt{X_1{}^2 + Y_1{}^2}; \quad \tan \delta_1 = \frac{Y_1}{X_1} \tag{10}$$

Thus the secondary field has the well-known and simple form due to a dipole. The values of the amplitude factor Q_1 and the phase angle δ_1 are determined by the frequency, and the size and conductivity of the sphere, only as they occur in the parameter p_1. As the conductivity becomes very large, Q_1 approaches unity, and the phase angle zero. In this circumstance, the response corresponds to that of a *nonconducting* sphere in a uniform field of current flow. Curves for the response functions Q_1 and δ_1 will be shown subsequently, in connection with the corresponding functions for the hollow sphere.

Inductive Response of Spherical Shell

The analogous response function π for a conducting thin spherical shell is readily obtained in a similar manner. The result is that the secondary field at external points has a form identical to that previously derived, but the relations between phase and intensity are slightly altered.

In the case of the thin shell, the current flow is concentrated at the surface, leading to the following discontinuity in the tangential components of B at the two sides of the shell,

$$(B_\theta)_1 - (B_\theta)_2 = \frac{4\pi\sigma\gamma}{c} E_\phi$$

where γ is the thickness of the shell. The appropriate solutions for π are: outside the sphere,

$$\pi_1 = \pi_0\left(1 + \frac{A_1}{r^3}\right)$$

as before, and inside,

$$\pi_2 = \pi_0(1 + A_2)$$

where, from the boundary conditions,

$$\left.\begin{aligned} A_1 &= -\frac{a^3 p_2 e^{i\delta_2}}{\sqrt{1 + p_2^2}} \\ A_2 &= -\frac{p_2 e^{i\delta_2}}{\sqrt{1 + p_2^2}} \\ \tan\delta &= \frac{1}{p_2} \\ p_2 &\equiv \frac{4\pi a w \sigma\gamma}{3c^2}. \end{aligned}\right\} \qquad [11]$$

Thus the secondary magnetic field inside the sphere has the uniform value

$$\begin{aligned} B_r &= -Q_2 \cos\theta B_0 \cos(wt + \delta_2) \\ B_\theta &= -Q_2 \sin\theta B_0 \cos(wt + \delta_2) \end{aligned} \qquad [12]$$

and outside, the simple form

$$B_r = -\left(\frac{a}{r}\right)^3 \cos\theta Q_2 B_0 \cos(wt + \delta_2) \qquad [13]$$

$$B_\theta = -\frac{1}{2}\left(\frac{a}{r}\right)^3 \sin\theta Q_2 B_0 \cos(wt + \delta_2)$$

where

$$Q_2 = \cos\delta_2 = \frac{p_2}{\sqrt{1 + p_2^2}} \qquad [14]$$

Dependence of Response upon Conductivity, Frequency and Size

The dependence of the intensity and phase angle of the response field for the sphere and spherical shell upon the frequency, conductivity and diameter is shown graphically in Figs. 1 and 2 respectively. In the upper

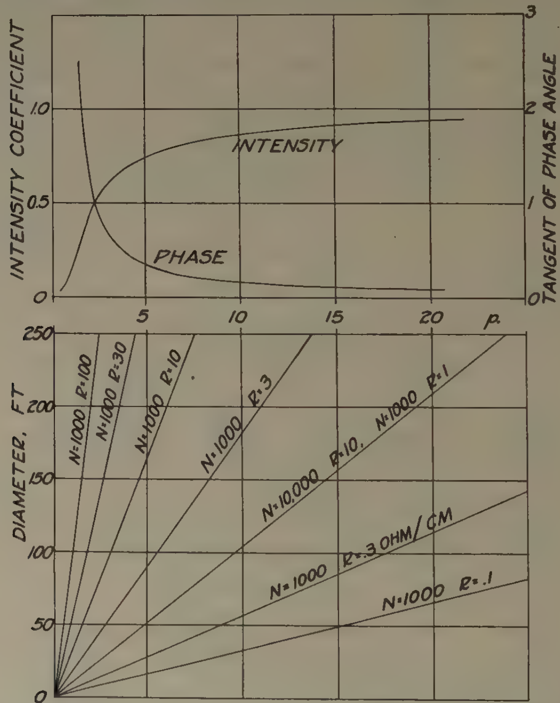

FIG. 1.—INDUCTIVE RESPONSE OF SPHERICAL CONDUCTOR AS INFLUENCED BY ITS SIZE AND RESISTIVITY AND BY FREQUENCY OF IMPRESSED FIELD.

portion of these graphs, the intensity coefficient, Q and phase angle δ are plotted against the parameter p as abscissa. In the lower graphs, values of p are shown in terms of the diameter in feet, resistivity in ohms per centimeter and frequency. It is clear that a value for either of the response coefficients Q or δ uniquely determines a corresponding value

for p, but this value may clearly result from an infinite number of different combinations of the physical quantities, resistivity, frequency and size. In particular, when frequency is known, an observed response

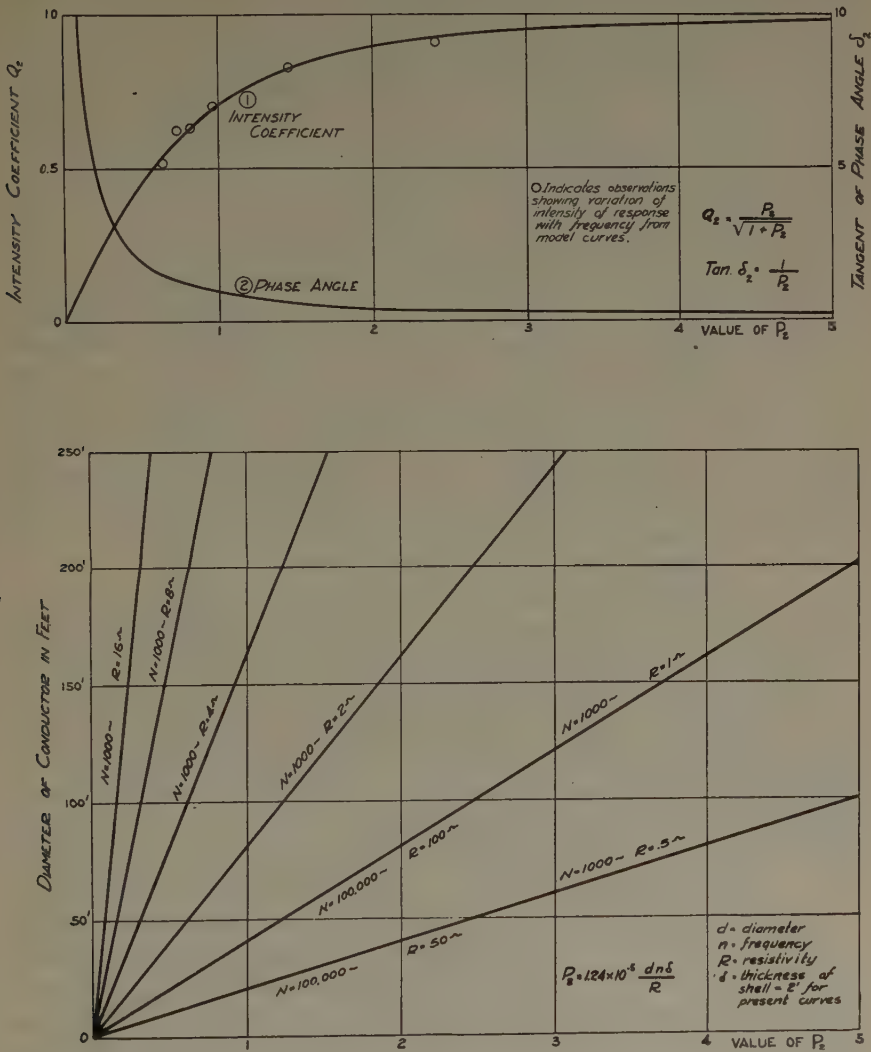

FIG. 2.—RESPONSE FUNCTIONS FOR SPHERICAL SHELL IN UNIFORM INDUCTIVE FIELD.

suffices to fix only a value for the ratio of the diameter to the resistivity, and their separate determination is impossible.

The coefficient Q_1 expresses the response intensity relative to that of a perfectly conducting sphere as unity. This coefficient has a rapid and nearly linear increase with p_1 for values of p_1 less than 3, but flattens

out for values exceeding 5, so that an insignificant gain in response is attained thereafter. Referring to Fig. 1, 75 per cent. of the possible response occurs for $p_1 = 5$, and 85 per cent. for $p_1 = 10$. This saturation effect in the response curves is significant. It is clear, for example, that one may determine the range for p_1 by choice of frequency, such that the response of poor conductors will occur at the lower part of the curve, while at the same time good conductors will respond according to the high values. To illustrate, a change in p_1 from ½ to 10 corresponding to a resistivity change from 1000 ohms to 2½ ohms (when the diameter is 165 ft. and frequency 1000 cycles), increases the intensity of response from 0.032 to 0.85, or about 26 times. The phase angle is correspondingly reduced from practically 90° to 9°. However, the excellent differentiation between the 2½-ohm and 1000-ohm conductor illustrated above is lost if the values of p_1 are greatly increased, as through the use of too high a frequency, for in such cases, the response coefficient even for bodies of widely different conductivities occur on the flat part of the curve, and are merged indistinguishably. For example, referring to the comparison made above, if the frequency is increased to 100,000 cycles, the corresponding values of p_1 are changed to 5 and 100 respectively, and the response coefficients to 0.75 and practically 1.0. Thus the differentiation between the 2½-ohm and 1000-ohm conductors is only 25 per cent. instead of 2600 per cent. as formerly.

Since the radius enters linearly in the parameter p (whereas the frequency and conductivity occur under the radical) and in addition occurs as a cube multiplying the amplitude factor, the peculiar importance of the factor of size in producing response is evident. Large structures such as topographic features, even though of low conductivity, are therefore competent to produce large response if a sufficiently high frequency is utilized. A field example of this fact is seen in Fig. 3, which shows a series of observations[6] taken under identical conditions except for a change in frequency from 1000 cycles to 60,000 cycles. The upper two curves show respectively distortions in the dip and strike directions of an impressed dipolar magnetic field of 60,000 cycles, and the lower two curves the same quantities measured for the same type of field at 1000 cycles. The traverse passed over glacial sand and gravel just north of Boucher Lake, in Falconbridge township, near Sudbury, Ontario, and across an arm of the lake as shown in the topographic profile beneath. The influence of this water is clearly seen at 60,000 cycles when it produces distortions of the order of 20°. At 1000 cycles, however, the topographic response is practically nil, amounting to about 2°.

In choosing frequencies for electromagnetic prospecting, the aim evidently should be to keep them low enough to avoid difficulties from

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[6] The data are from experiments by B. B. Gauld and D. L. Hay.

topography and high enough to obtain a fairly large fraction of the possible perfect response. For conducting ores a saturation point is soon obtained, at which the orebody reacts substantially as a perfect conductor, and beyond this point an increase in frequency can serve only to increase the disturbing influences of topography. Naturally, the question of the most effective frequency ranges is dependent upon the electrical nature of both the orebody and the surrounding structure.

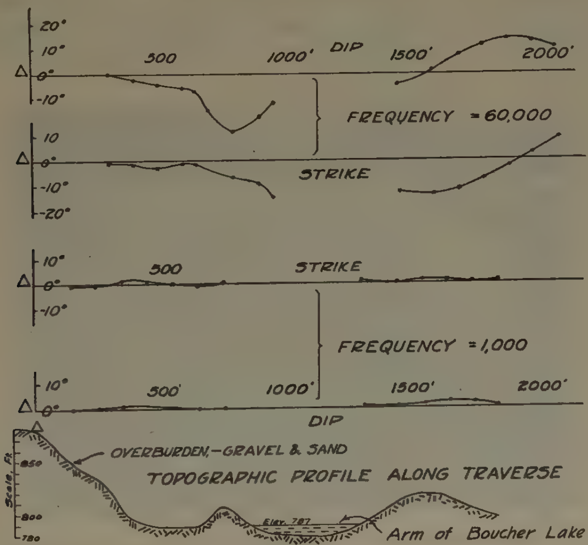

Fig. 3.—Comparison of topographic response at 60,000 and 1,000 cycles.

The response functions shown in Figs. 1 and 2 show that as the intensity coefficient becomes smaller the phase angle rapidly increases; hence, when the response parameter is small (as when the conductivity is small, the size small, or the frequency low), the utilization of the phase angle should be of far greater importance than for conductors showing a high value of the response parameter p.

Model Results

The model results which follow are intended to substantiate the computed response of spherical conductors, although one may well regard the matter in the reverse sense, and place the emphasis upon the testing of the reliability of the experimental work. In these model experiments, the source field was produced by a small loop with plane vertical, situated 30 ft. from the model in interest. Frequencies between 13,000 and 50,000 cycles were utilized, but 50,000 cycles were customary, corresponding to a scale factor of 50 to 1 on the basis of 1000 cycle field work. The quantities observed were the directions of the distorted electromagnetic

field, as observed by the positions of silence of a small test coil, connected to a heterodyne amplifier and earphones. These distortions were recorded as deflections, in degrees, from the direction of the undisturbed incident field, in the horizontal and in the vertical planes, and will be referred to hereafter as the "strike" and "dip" distortions respectively. The observations were taken at short intervals along a straight wooden track raised 4 ft. above the earth to reduce earth reactions, and passing over the model in simulation of a field traverse above a conducting ore. The traverse line lay, in the present cases, in the plane of symmetry of the source field. Errors due to the use of metal in parts of the observing

FIG. 4.—EXPERIMENTAL ARRANGEMENTS USED IN MODELING.

apparatus approaching close to the model were carefully avoided. The experimental arrangements are illustrated in Fig. 4, which shows the wooden track, the small exploring coil that is movable along it, and a 3-ft. dia. sphere that was used in some of the work. This type of apparatus was used in hundreds of the simpler model tests for illustrating the effects upon inductive response of such elementary factors as shape and orientation of the conductor and its depth of cover.

OBSERVED RESPONSE OF SPHERICAL MODEL

Although the theoretical problems of the response of the hollow and solid sphere are similar, several practical considerations favor the hollow sphere for use in modeling. A material for a solid sphere, equivalent on the reduced scale to the conductivity of an ore sulfide, was not available, but the appropriate conductivity was readily obtained by the use of a layer of tinfoil laid on a spherical shell. The response of such a sphere, 3 ft. dia., when subject to the inductive field of a loop located 30 ft. from its center, is shown in the model curves (Figs. 5 and 6). Three aspects of

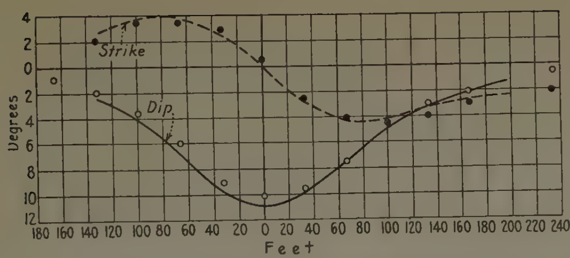

FIG. 5.—COMPARISON OF THEORETICAL AND EXPERIMENTAL DISTORTION OF ELECTROMAGNETIC FIELD DUE TO CONDUCTING SPHERE.

- • Observed strike, model.
- ° Observed dip, model.
- ——— Theoretical strike curve.
- ——— Theoretical dip curve.

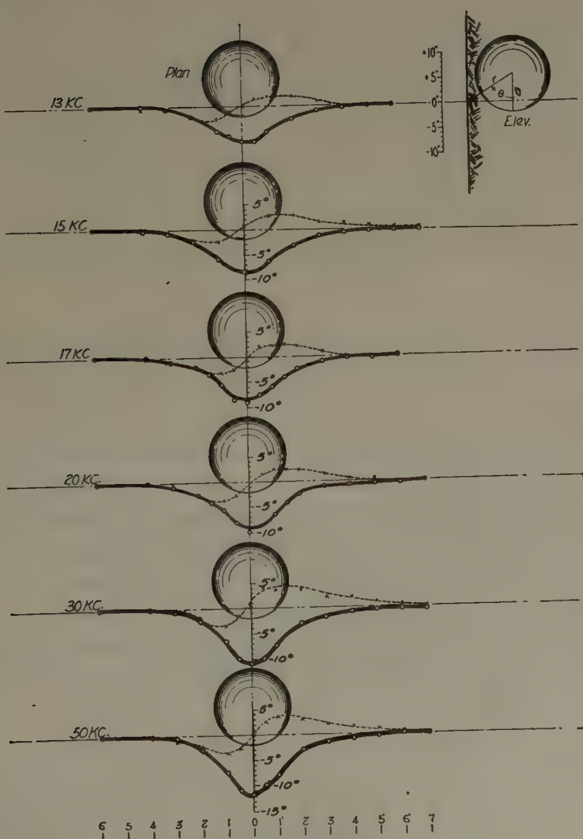

FIG. 6.—MODEL EXPERIMENT. INDUCTIVE RESPONSE OF SPHERE AT VARIOUS FREQUENCIES.

the theoretical predictions may be examined by these response curves: (1) the geometric one of the shape of the secondary field, (2) the dependence of the response intensity upon frequency, and (3) the intensity of the response.

1. The general theoretical requirement that the secondary field be equivalent to that of a dipole located at the center of the sphere may be directly tested by comparing the observed response along the traverse line with that theoretically produced at the same points by such a dipole. In making this comparison, questions concerning the intensity factor, which will be considered later, can be avoided by arbitrarily making the assumed dipolar field coincide with the observed readings at a given point. Fig. 5 illustrates such a comparison.

Here the solid points represent observed readings for the strike, and the circles, observed dip readings. The corresponding theoretical dipolar response of a sphere along the same traverse is shown by the dotted curve for strike distortions, and by the solid one for dip. The intensity factor for the theoretical curves is chosen to fit the maxima of the observed dip curves. Although the impressed field is considered uniform over the sphere, correction was made for its slightly changing value over the larger distances along the traverse line when forming the resultant field at these points. This fact accounts for the slight lack of symmetry in the computed response. The theoretical curve and experimental points are in agreement within an experimental error of the order of 1°, and it is to be concluded that a secondary field of the required dipolar type exists.

Dependence of Response Intensity upon Frequency

In accordance with equation 14, the intensity coefficient Q_2 depends upon the frequency only through p_2, which in turn is linear in the frequency. To exhibit the dependence of the observed response upon frequency, a series of Q_2 values as observed through a range of frequencies will be associated with their corresponding values of p_2 as fixed by equation 14. The question in interest is, then, the degree to which these p_2 values satisfy the requirement of linearity in frequency.

The model curves to be examined in this way are seen in Fig. 6. The position of the spherical shell with respect to the traverse line is shown in plan and elevation, and the observed distortions of the primary field in strike and dip are plotted in degrees by the dotted and solid curves, respectively. The curves illustrate a gradual increase in response amplitude with increasing frequencies. The value of Q_2 associated with each of these curves is conveniently found by use of the angle of maximum dip distortion, Δ, which is related to the response coëfficient by the geometrical condition

$$\tan \Delta = \frac{3Q(a/r)^3 \sin \theta \cos \theta}{2 + Q(a/r)^3(\sin^2 \theta - 2 \cos^2 \theta)} \quad [15]$$

where r is the distance from the observing point to the center of the sphere and θ is the angle between r and the horizontal, as indicated in Fig. 6. In the present curves $\theta = 59°$, and $a/r = 0.715$. In Table 1, column 2, are listed the measured values of Δ, opposite their corresponding frequencies, and in column 3 the value of Q_2 associated with Δ in accordance with equation 15. The values of p_2 fixed by Q_2 through equation 14 are listed in column 4, and in column 5 the values of p_2/n, which theory requires should be constant. The percentage deviation of the observed values

TABLE 1.—*Observed Variation of Response Intensity of Spherical Shell with Frequency*

1	2	3	4	5	6	7	8
n	Δ	Q_2	p_2	$p_2/n \times 10^4$	Deviation of 5 from Mean, Per Cent.	$\bar{p}_2$	$\bar{Q}_2$
13,200	7.0°	0.514	0.60	0.455	−6.6	0.642	0.537
15,100	8.5°	0.626	0.802	0.53	+8.8	0.735	0.585
16,950	8.8°	0.652	0.86	0.51	+4.7	0.825	0.635
20,000	9.5°	0.705	1.0	0.50	+2.7	0.974	0.695
30,000	11.0°	0.822	1.44	0.48	−1.4	1.46	0.825
49,500	12.0°	0.910	2.22	0.45	−7.6	2.41	0.92
				Av. 0.487			

from their mean, 0.487, is listed in column 6. These variations lie in the range 1.4 per cent. to 8.8 per cent., which is within the experimental error in observing the distortions. To this extent, then, the model response fulfills theoretical requirements as to the variation of intensity with frequency. Columns 7 and 8 list quantities $\bar{p}_2$ and $\bar{Q}_2$ which are obtained by use of the average value of p_2/n (*i. e.*, 0.487×10^{-4}) and the actual frequency values. In Fig. 2 are plotted the observed values of Q_2 (column 3) against the $\bar{p}_2$ values of column 7, which are strictly linear in the frequency; these plotted points, in comparison with the smooth curve, illustrate graphically the correspondence between the observed and theoretical variation of response intensity with frequency.

Intensity of Observed Response

The surface conductivity of the 0.0005 in. thick tinfoil coating, which formed the cover of the spherical shell, was measured, in small strips, as 81.5 ± 3 per cent. reciprocal ohms, referring to 1 sq. cm. of surface. The material was applied to the sphere in strips about 6 in. wide by 20 in. long, with tacky varnish, and poor contact at the joints undoubtedly raised the

effective resistivity considerably above that measured for the material, but as no independent measurements of the resultant resistivity of the model as a whole were made, a satisfactorily accurate value was not available upon which to base the theoretical intensity of response. The order of magnitude of the conductivity, however, is indicated by the measurements on the material itself.

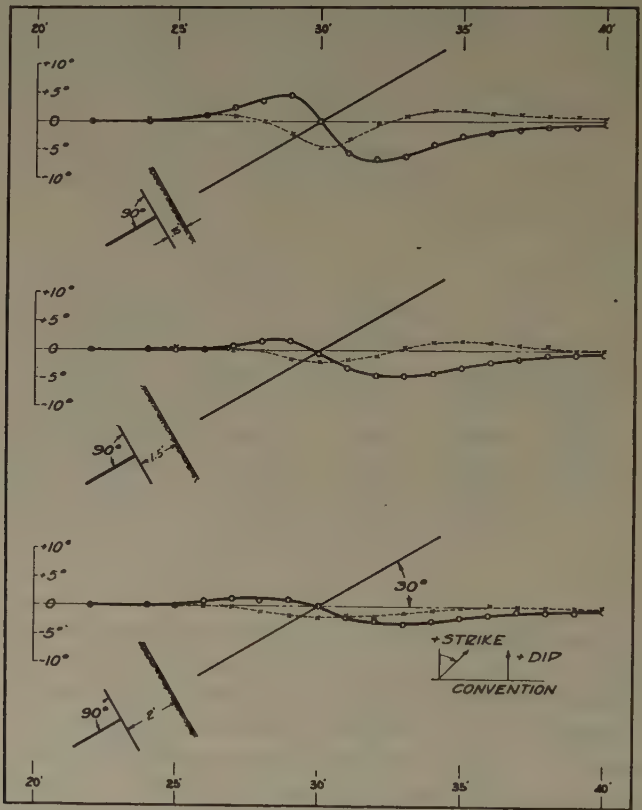

FIG. 7.—MODEL RESPONSE. INDUCTIVE RESPONSE OF RECTANGULAR SHEET AT VARIOUS DEPTHS OF COVER.

The data in Table 1 showed that the observed response curves of Fig. 6 were best represented by a mean value of 0.487×10^{-4} for the ratio $p_2/n = \frac{8\pi^2 a\delta\gamma}{3c^2}$. On substituting for the radius the value 45.6 cm., it results that $\sigma_p\gamma = 40.5$ reciprocal ohms or about one-half the measured value of 81.5 found for sample sheets of the tinfoil. The difference, as has been said, is probably caused by resistance introduced at the joints.

The higher measured values of 81.5 for $\sigma\gamma$ would yield, at 13,000 cycles, a maximum dip of 10°, and at 49,500 cycles, 13°, instead of the observed values of 7° and 12° respectively.

In summary, the model curves illustrated in Figs. 5 and 6 exhibit in a satisfactory way the dipolar shape of the response field and its variation in intensity with the frequency of the impressed field. The apparent conductivity of the model computed from the observations of response is about 50 per cent. of the measured value for the material, a difference probably due to contact resistance between adjacent strips.

Model Response of Plane Conductor at Different Depths of Cover

Fig. 7 is an example of model curves similar to those described for the sphere, taken for a plane rectangular conductor, situated as shown with respect to the traverse line. The curves refer to three different depths of cover, also indicated in the figure, and show the change in the form of the response so occasioned. These curves, together with those for the sphere, are shown merely to illustrate the possibilities of the study of inductive response by means of models. Hundreds of these experiments are required to form any adequate picture of the effect of the many variables—size, dip, depth of cover, shape, conductivity, etc.—which are the elementary factors in an actual electromagnetic prospecting problem. The utility of model work in providing a comprehensive key for aiding the interpretation of field data has, in our experience, been unquestionably established.

Least Squares in Practical Geophysics*

By Irwin Roman,† West Orange, N. J.

Part I. Introduction

The literature of geophysics as applied to the discovery of mineral deposits has been very extensive during the past few years,[1] but there seem to be few references to the use of the method of least squares for obtaining better interpretations of the observed data.[2] There seems to be a general reluctance among geologists and engineers to acquire the technique of least squares, in spite of the fact that a thorough knowledge of the theory is not necessary for the use of the method.

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* Scheduled for the New York Meeting, February, 1932.

† Consulting Mathematical Geophysicist.

[1] The reader who is not familiar with the progress made in this field in recent years may consult the following references, as typical, not exhaustive:

Ambronn: A Textbook of Practical Geophysics. (German edition translated into English by Cobb.)

Eve and Keyes: Applied Geophysics in the Search for Minerals.

Gutenberg (Editor in Chief): Lehrbuch der Geophysik. (In German.)

Gutenberg (Editor in Chief): Handbuch der Geophysik. (In German.)

U. S. Bureau of Mines *Geophysical Abstracts*. Issued monthly.

Annotated Bibliography for Economic Geology.

Publications of American Institute of Mining and Metallurgical Engineers.

Bulletin, American Association of Petroleum Geologists.

Gerland's *Beiträge zur Geophysik* and *Ergänzungshefte für angewandte Geophysik*. (In German.)

Journal, Institution of Petroleum Technologists of Great Britain.

Japanese Journal of Astronomy and Geophysics.

Zeitschrift für Geophysik. (In German.)

[2] Barton has suggested a set of standardized nets with solutions made by a specialist, and the results arranged into forms to be completed by the computer. The specialist is evidently to decide how to select the proper form and net. His method is very convenient when the survey is simple enough to justify such an interpretation, in the case of surveys made with the magnetometer or torsion balance. In his examples, the results involve only two or three decimal places. Since the last figure is almost always masked by the forcing errors of calculations, there is some question as to the reliability of the final figures, admitting the correctness of the net and the theoretical solution.

See D. C. Barton: Control and Adjustment of Surveys with the Magnetometer or the Torsion Balance. *Bull.* Amer. Assn. Petr. Geol. (1929) **13**, 1163–1186.

In the present paper, the method will be illustrated in its application to three distinct problems. Its application to other problems will be apparent. In each of the three problems, we shall consider the method of obtaining the data, the mathematical formulation of the problem, the solution of the exact case, and the details of the solution of the excess case by least squares, with numerical examples to illustrate the method and the forms of calculation.

No attempt will be made to show that the method of least squares furnishes the best interpretation, nor to define what is meant by the term "best." For the present purpose, it will be sufficient to consider the method as a satisfactory method of interpreting data when there are more than enough observations to make the determination in the usual manner.

The problems discussed have arisen in the practical work of a large geophysical prospecting organization and the theory underlying the data will be taken without explanation. It is impossible to state the precise form of the problem as presented to the writer and no specific acknowledgements can be made. In each case, the problem was presented in general form only. The writer thanks all those persons who have assisted him by suggestion or criticism.

Part II. The Method of Least Squares

In making actual measurements, there are always approximations and errors. If a single desired quantity is measured just once, that measurement must be taken as the best available value for the measured quantity. If this same quantity is measured more than once, the various measurements will disagree the more as the fineness of the measure is increased. In such cases, we define the best value as the arithmetic mean of the separate measurements. This is usually called "the average value" of the separate measures.

In many cases, the desired quantity is measured by an indirect method. The measured quantity is not one of the unknowns sought, but a known function of all or part of them. The simplest function is the linear function. If the function is not linear, we may select approximate solutions and expand the function of the unknowns in a Taylor's series about these values. Examples of this method will appear in detail.

For a linear function, the method of least squares is standard and well known.[3] Let the number of unknowns be n, and the number of distinct observations be m. If $m < n$, it will be necessary to select values for at

[3] There are so many excellent treatments of the method of least squares that no attempt will be made to select the most useful of them. Six of those available in English are: D. P. Bartlett: Method of Least Squares; W. Chauvenet: A Treatise on the Method of Least Squares; C. L. Crandall: Geodesy and Least Squares; W. W. Johnson: The Theory of Errors and the Method of Least Squares; M. Merriman: A Textbook on the Method of Least Squares; T. W. Wright and J. F. Hayford: The Adjustment of Observations.

least $(n - m)$ of the unknowns to solve for the others. If $m = n$, the solution is direct and may be made as desired. If $m > n$, there will, in general, be different solutions and values for the unknowns according to which n equations are solved simultaneously. This case $(m > n)$ will be considered by the method of least squares. In the examples to be discussed, we may use this method as furnishing the best available values for the unknowns as determined by the observations made. We reduce the observation equations to a set of n "normal" equations and solve this system simultaneously. If n is small, we may use the usual methods of algebra; if n is large, we may use the method of Gauss.[4]

Let the observation equations be

$$\sum_{j=1}^{n} a_{ij}x_j = k_i \qquad (i = 1, 2, 3, \ldots, m). \qquad [1]$$

The unknowns are the x_j and the equation 1 is the i-th observation equation. The normal system built in the usual manner is

$$\left.\begin{aligned} &\sum_{j=1}^{n} [uj]x_j = [uk] \qquad (u = 1, 2, 3, \ldots, n) \\ &\text{where} \\ &[uj] = \sum_{i=1}^{m} a_{iu}a_{ij} \\ &[uk] = \sum_{i=1}^{m} a_{iu}k_i \end{aligned}\right\}. \qquad [2]$$

This represents a system of n equations in n variables, which may be solved as convenient. The method of solution will be discussed for each problem. In particular, we have:

$$\left.\begin{aligned} [uu] &= \sum_{i=1}^{m} a_{iu}^2 \\ [uv] &= [vu] \end{aligned}\right\} \qquad [3]$$

and the system is symmetric.

To assist readers who are unfamiliar with the notation of the method of least squares, but who have a mathematical training enabling them to understand the basic concepts, we may interpret the preceding paragraph in less formal manner. The observation equations, expressed by equation 1, consist of m equations, each equation representing one of the m observations made in the investigation. In each equation, of which the i-th is written as typical, the observed quantity is the constant k_i, and underlying theory states that this constant is equal to a linear combination of the separate unknowns x_j. By a linear combination we understand that each unknown x_j is multiplied by a constant coefficient a_{ij} and

[4] Any reference of footnote 3.

the products added as indicated by the summation sign Σ. The coefficients are furnished by the theory and depend on the independent quantities of the experimental arrangement. The constant is the value of the dependent unknown which is measured by the experiment. Some, but not all, of the coefficients in each observation equation may be zero. The first subscript of a_{ij} indicates the equation and the second subscript indicates the particular unknown with which a_{ij} is associated.

The normal system, expressed by equation 2, consists of as many equations as there are unknowns, so that it has the usual form of simultaneous algebraic equations. In each equation, of which the u-th is written, the coefficients are the "normal coefficients," represented by the square brackets, and the constant is the "normal constant." To obtain the normal coefficient $[uj]$, we multiply the coefficient of x_u by the coefficient of x_j in each observation equation and add the products for all of the equations. To obtain the normal constant $[uk]$, we multiply the constant of each equation by the coefficient of x_u in that equation and add the products for all of the equations. The relations of equation 3 follow directly from the definitions of the symbols. The specific interpretations of the various unknowns will be discussed for each problem.

Part III. Sound Surveying

Section 1. *The Problem*

In the use of the seismograph for geophysical exploration, it is necessary to know the distance from the source of the disturbance to the receiver of the impulse. This source is usually a shot of explosive, and the receiver a geophone or other recording seismograph. In some cases, this distance may be measured directly, at small expense. In many cases, especially where the distances are large, the direct method is expensive and often involves difficult surveys. The explosive is often fired in locations which are accessible only with difficulty. The distances involved have been as long as ten miles and usually exceed a mile, in refraction work. Balloons and transits have been used, but the method is both expensive and cumbersome. For conditions of poor visibility, it is useless. Sound surveying has proved satisfactory, but involves inaccuracies such as errors in the equivalent speed of sound. In fact, in some cases, the sound impulse does not travel in a straight line, although it is sufficiently accurate to assume that the speed of sound may be determined sufficiently accurately from laboratory measurements, corrected for temperature and wind velocity.

The method is as follows: A shot point is located in a selected area, usually away from roads and dwellings. The receiving units, consisting of geophones, radio receivers, blastphones, etc., are located in known positions, usually on roadways. Each survey involves the exploding of

several shots, at most a few hundred feet apart. For each shot, the time of explosion is transmitted to a receiver by radio and the sound of the explosion is transmitted to a blastphone by the air. Knowing the best available speed of sound in the direction from shot to receiver, the distance may be calculated, approximately. Each determination is complete except for errors, and each two records furnish the determination of the position of the shot point. Actually, such a determination is not usually sufficiently accurate. Considering the distances involved, all the shots of a group may be considered as exploded at a single point. Then the distance from this point to each known receiving point may be calculated. By increasing the number of observations, there will be an excess of observations over the number of unknowns and we may determine the best position of the shot point by least squares. After determining the position of the shot point, we reverse the method and calculate the distance from this point to each receiver, instead of using the observed distance for each record. The method is economical and as accurate as necessary for most purposes. No additional apparatus or measurements are needed for the least square adjustment. An ordinarily good computer can reduce the values in a comparatively short time. No knowledge of the shot position is needed, and it is not necessary that the roads or fields be passable from shot to receiver as for a transit survey.

Section 2 (*Part* III). *Formulation*

Let the shots be exploded at the point $P \equiv (x, y)$, assumed the same for all the shots (Fig. 1). Let the i-th receiver be located at $P_i \equiv (x_i, y_i)$,

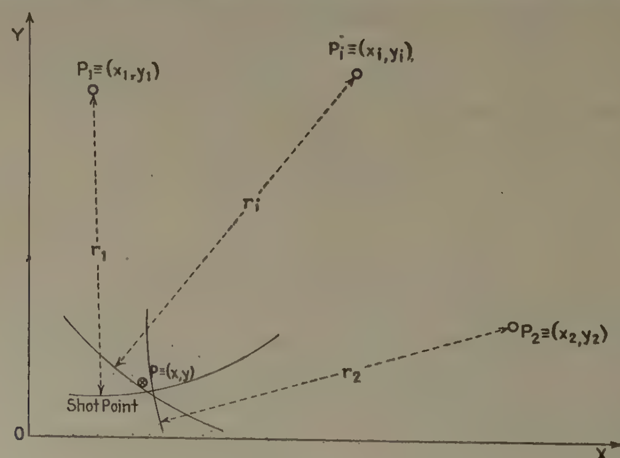

FIG. 1.

the position of which is known as accurately as needed Then the distance from P to P_i is

$$r_i = \sqrt{(x - x_i)^2 + (y - y_i)^2}. \qquad [4]$$

If there are only two observations, we may write:

$$\left.\begin{aligned} x^2 + y^2 - 2x_1x - 2y_1y &= r_1^2 - x_1^2 - y_1^2 \\ x^2 + y^2 - 2x_2x - 2y_2y &= r_2^2 - x_2^2 - y_2^2 \end{aligned}\right\}. \qquad [5]$$

The solution of these equations is direct and unique except for a choice between two solutions, the choice to be made from additional information. In numerical cases, the solution is simple. The algebraic solution is direct, but is complicated in the writing and is omitted from this discussion.

If there are more than two observations to determine the two unknowns x and y, we may use least squares. Since the measured quantity is r_i and not r_i^2, we are not justified in squaring relations (equation 4) before normalizing the system. Since the observation equations are not linear, the first step is to make them linear. To do this, let (x_0, y_0) be approximate solutions, determined in any convenient manner. One method is a graphic solution, drawing an arc from each receiver point with a radius equal, in scale, to the observed distance (Fig. 1). The approximate solution is estimated by inspection. A second method consists of selecting two of the observation equations and solving as an exact system, choosing the proper solution of the pair from additional knowledge of the survey. A third method, which has proved convenient, is to select three separate observations, to square each equation, and to subtract one equation from each of the other two, leaving two linear equations in two variables. This method involves no ambiguity. It will be illustrated in example 1.

After selecting the approximate values x_0 and y_0, we may expand by Taylor's series, retaining only the linear terms in $(x - x_0)$ and $(y - y_0)$. Then we have:

$$f_i(x, y) = f_i(x_0, y_0) + (x - x_0)\frac{\partial}{\partial x}f_i(x_0, y_0) + (y - y_0)\frac{\partial}{\partial y}f_i(x_0, y_0) + \cdots \qquad [6]$$

In this equation, $f_i(x, y)$ represents the function of the variables x and y which appears in the i-th observation equation. The symbols $\partial/\partial x$ and $\partial/\partial y$ represent partial differentiation with reference to x and y, respectively. In the present problem, we have:

$$\left.\begin{aligned} f_i(x, y) &= \sqrt{(x - x_i)^2 + (y - y_i)^2} \\ \frac{\partial}{\partial x}f_i(x, y) &= \frac{x - x_i}{f_i(x, y)} \\ \frac{\partial}{\partial y}f_i(x, y) &= \frac{y - y_i}{f_i(x, y)} \end{aligned}\right\} \qquad [7]$$

If we write

$$\left.\begin{aligned} \alpha &= x - x_0 & \beta &= y - y_0 \\ \xi_i &= x_i - x_0 & \eta_i &= y_i - y_0 \\ \rho_i &= \sqrt{\xi_i^2 + \eta_i^2} & k_i &= \rho_i - r_i \\ \lambda_i &= \xi_i/\rho_i & \mu_i &= \eta_i/\rho_i \end{aligned}\right\}, \qquad [8]$$

we have:

$$\left.\begin{aligned} f_i(x_0, y_0) &= \rho_i \\ \frac{\partial}{\partial x} f_i(x_0, y_0) &= -\xi_i/\rho_i = -\lambda_i \\ \frac{\partial}{\partial y} f_i(x_0, y_0) &= -\eta_i/\rho_i = -\mu_i \end{aligned}\right\}, \qquad [9]$$

so that equation 6 leads to the relations:

$$\lambda_i\alpha + \mu_i\beta = k_i. \qquad [10]$$

The quantities α and β represent the displacements, parallel to the coordinate axes of the approximate shot position from the position to be accepted. The quantities ξ_i and η_i represent the displacements of the approximate shot point from the i-th receiver. The quantity ρ_i represents the distance between the approximate shot position and the i-th receiver. The quantity k_i represents the excess of the approximated over the accepted distance between the shot point and the i-th receiver. The quantities λ_i and μ_i represent the direction cosines of the line from the i-th receiver to the approximated shot point.

These equations are linear in α and β, the constants and coefficients being determined from the observed data and the approximate solution selected for the expansion. They determine the corrections α and β to be added to the approximate values x_0 and y_0 to obtain the best solutions, x and y. The method is increasingly convenient as the approximation (x_0, y_0) approaches the final value (x, y).

For the observation system, equation 10, the normal system is

$$\left.\begin{aligned} (\Sigma\lambda_i^2)\alpha + (\Sigma\lambda_i\mu_i)\beta &= (\Sigma\lambda_i k_i) \\ (\Sigma\lambda_i\mu_i)\alpha + (\Sigma\mu_i^2)\beta &= (\Sigma\mu_i k_i) \end{aligned}\right\}, \qquad [11]$$

where Σ is understood to represent $\sum_{i=1}^{m}$. If we write

$$\left.\begin{aligned} \Delta &= (\Sigma\lambda_i^2)(\Sigma\mu_i^2) - (\Sigma\lambda_i\mu_i)^2 \\ \Delta_\alpha &= (\Sigma\lambda_i k_i)(\Sigma\mu_i^2) - (\Sigma\lambda_i\mu_i)(\Sigma\mu_i k_i) \\ \Delta_\beta &= (\Sigma\lambda_i^2)(\Sigma\mu_i k_i) - (\Sigma\lambda_i\mu_i)(\Sigma\lambda_i k_i) \end{aligned}\right\}, \qquad [12]$$

we have

$$\left.\begin{aligned} \alpha &= \Delta_\alpha/\Delta \qquad & x &= x_0 + \alpha \\ \beta &= \Delta_\beta/\Delta \qquad & y &= y_0 + \beta \end{aligned}\right\}. \qquad [13]$$

If this new solution is used as an approximate solution and the calculations repeated, a closer approximation is obtained. Thus, the solution may be made as accurately as the data will permit. It must be emphasized that the least square solution may be made as accurately as desired, but that this solution will furnish only the best location obtainable from the observations. It may be far from the actual shot location. Fortunately, in practice it is usually sufficiently ac curate.

Section 3 (*Part* III). *Example* 1

Let the data be those shown in Table 1. The values under x_i are the distances of the various receiving points north of a selected origin. The values under y_i are the corresponding distances east of the origin. The values under r_i are the observed distances. The values x_0 and y_0 are the approximations selected. The unit is 10,000 ft. The table is adapted to 12 records, but where forms are made for field use a sufficient number of lines should be chosen so that there will always be enough lines for the observations of the group. The counter i is used for identification and does not enter the calculations.

Before calculating, it is usually convenient to select a unit of distance such that the distances involved will be mostly in the interval between 1 and 10. Then, for most of the distances, there will be one digit ahead of the decimal point, and an agreed number of retained decimal places will correspond to a definite number of significant figures. In the present case, the distances observed vary from 26,841 to 71,862 ft. The choice of 10,000 ft. as the unit reduces the distances to the interval 2.6 to 7.2, which is convenient.

To find an approximate solution, consider observations 1, 6 and 12. Then we have

$$
\begin{array}{lll}
x_1 = 7.1862 & x_6 = 6.1627 & x_{12} = 3.6616 \\
y_1 = 2.6841 & y_6 = 5.1915 & y_{12} = 6.6651 \\
r_1 = 3.7830 & r_6 = 3.4580 & r_{12} = 3.5920
\end{array}
$$

The observation equations for these points may be written:

$$
\begin{aligned}
x^2 + y^2 - 14.3724x - 5.3682y &= -44.5348 \\
x^2 + y^2 - 12.3254x - 10.3830y &= -52.9728 \\
x^2 + y^2 - 7.3232x - 13.3302y &= -44.9284.
\end{aligned}
$$

If we subtract the third equation from each of the other two, we have the pair of equations:

$$
\begin{aligned}
-7.0492x + 7.9620y &= +0.3936 \\
-5.0022x + 2.9472y &= -8.0444
\end{aligned}
$$

The solution of this pair of equations is $x = 3.4227$ and $y = 3.0797$, which are the values used for x_0 and y_0 in Table 1. Other choices of observations and methods lead to other values for x_0 and y_0.

The calculations in the table explain themselves, the notation being given in equations 8, 12 and 13. There are several check relations, which should be applied. They are

$$\lambda_i^2 + \mu_i^2 = 1$$

TABLE 1

Approximate Solution			$x_0 = 3.4227$			$y_0 = 3.0797$			Unit = 10,000 Ft.				
i	x_i	y_i	$x_i - x_0$	$y_i - y_0$	r_i	$\sqrt{\xi_i^2 + \eta_i^2}$	$\rho_i - r_i$	ξ_i/ρ_i	η_i/ρ_i	λ_i^2	$\lambda_i\mu_i$	μ_i^2	$\lambda_i k_i$	$\mu_i k_i$
			ξ_i	η_i		ρ_i	k_i	λ_i	μ_i					
1	7.1862	2.6841	+ 3.7635	− 0.3956	3.7830	3.7843	+0.0013	+0.9945	−0.1045	0.9890	−0.1039	0.0109	+0.0013	−0.0001
2	6.6750	3.1260	+ 3.2523	+ 0.0463	3.2690	3.2526	−0.0164	+0.9999	+0.0142	0.9998	+0.0142	0.0002	−0.0164	−0.0002
3	6.6563	3.7710	+ 3.2336	+ 0.6913	3.3180	3.3067	−0.0113	+0.9779	+0.2091	0.9563	+0.2045	0.0437	−0.0111	−0.0024
4	6.5992	4.1855	+ 3.1765	+ 1.1058	3.3570	3.3635	+0.0065	+0.9444	+0.3288	0.8919	+0.3105	0.1081	+0.0061	+0.0021
5	6.3713	4.7821	+ 2.9486	+ 1.7024	3.4220	3.4048	−0.0172	+0.8660	+0.5000	0.7500	+0.4330	0.2500	−0.0149	−0.0086
6	6.1627	5.1915	+ 2.7400	+ 2.1118	3.4580	3.4594	+0.0014	+0.7920	+0.6105	0.6273	+0.4835	0.3727	+0.0011	+0.0009
7	5.9096	5.6986	+ 2.4869	+ 2.6189	3.6190	3.6116	−0.0074	+0.6886	+0.7251	0.4742	+0.4993	0.5258	−0.0051	−0.0054
8	5.5164	6.1270	+ 2.0937	+ 3.0473	3.6580	3.6972	+0.0392	+0.5663	+0.8242	0.3207	+0.4667	0.6793	+0.0222	+0.0323
9	5.0005	6.2622	+ 1.5778	+ 3.1825	3.5440	3.5521	+0.0081	+0.4442	+0.8959	0.1973	+0.3980	0.8026	+0.0036	+0.0073
10	4.6166	6.3414	+ 1.1939	+ 3.2617	3.4740	3.4733	−0.0007	+0.3437	+0.9391	0.1181	+0.3228	0.8819	−0.0002	−0.0007
11	4.0529	6.3679	+ 0.6302	+ 3.2882	3.3450	3.3480	+0.0030	+0.1882	+0.9821	0.0354	+0.1848	0.9645	+0.0006	+0.0029
12	3.6616	6.6651	+ 0.2389	+ 3.5854	3.5920	3.5934	+0.0014	+0.0665	+0.9978	0.0044	+0.0664	0.9956	+0.0001	+0.0014
	Σx_i	Σy_i	$\Sigma \xi_i$	$\Sigma \eta_i$	Σr_i	$\Sigma \rho_i$	Σk_i			$\Sigma \lambda_i^2$	$\Sigma \lambda_i\mu_i$	$\Sigma \mu_i^2$	$\Sigma \lambda_i k_i$	$\Sigma \mu_i k_i$
	68.4083	61.2024	+27.3359	+24.2460	41.8390	41.8469	+0.0079			6.3644	+3.2798	5.6353	−0.0127	+0.0295

$$6.3644\alpha + 3.2798\beta = -0.0127$$
$$3.2798\alpha + 5.6353\beta = +0.0295$$

$\alpha = -0.0067$
$\beta = +0.0091$

$x = 3.4160$
$y = 3.0888$

for each observation and the four sum relations

$$\Sigma x_i - \Sigma \xi_i = m x_0$$
$$\Sigma y_i - \Sigma \eta_i = m y_0$$
$$\Sigma \rho_i - \Sigma r_i = \Sigma k_i$$
$$\Sigma \lambda_i^2 + \Sigma \mu_i^2 = m.$$

In the particular case considered, the corrections are $\alpha = -0.0067$ and $\beta = +0.0091$ and the corrected values are $x = 3.4160$, $y = 3.0888$. If we repeat the calculations, using these values for the approximation, we find the corrections to be $\alpha = 0$ and $\beta = -0.0001$, which check the values calculated in Table 1. If α and β were not small, the calculations would have to be repeated until the corrections were small enough to be neglected. The corrections indicate a solution correct to within one foot. However, as mentioned above, these are the corrections which express the deviation of the approximated point from the best point obtainable from the data. This does not express the deviation of the approximate point from the actual point. This latter deviation is probably of the order of 50 ft., which is sufficiently accurate for most purposes. In fact, for this case, the corrections in the first calculation are only 67 and 91 ft., so that a single calculation would be sufficient. These values are of the order of 0.33 per cent. of the distances observed, so that they are well within the limits of accuracy for the problem.

The value of k_i represents the discrepancy between the observed distance r_i and the calculated distance ρ_i of the receiver from the approximate shot point. In Table 1, observation 8 has a discrepancy of 392 ft. The average absolute discrepancy is 95 ft., so that all other discrepancies are less than twice the mean discrepancy, while observation 8 has a discrepancy over four times the mean. This suggests discarding observation 8. Without this observation, the best point is $x = 3.4152$ and $y = 3.0834$. The effects of discarding this reading are 8 ft. in x and 54 ft. in y, so that the best point is displaced by only 55 ft. This illustrates the general principles of retaining all observations except those discarded for an independent reason.

Part IV. Outlining a Structure by Seismic Reflections

Section 1. *The Problem*

A second application of the method of least squares is the outlining of a buried structure by seismic reflections. Let a structure be embedded in a homogeneous medium of lower transmission speed, so that seismic disturbances sent from a point near the surface are reflected at the structure and received at a geophone of which the location is accurately known. The shot point has one or more images in the boundary of the structure. In general, the image will depend upon the position of the

geophone as well as on that of the shot. Each point of the structure that makes the time of travel of the disturbance from the shot point to the geophone an extremum, that is, either a maximum or a minimum, will be a reflecting point. Since the first impulse is the most reliable, experimentally, there is a uniquely selected image of the shot for each geophone position. For simplicity, we shall assume that the structure is essentially plane over the region involved in the reflection. In this case, the shot will have a unique image independently of the position of the geophone. When this image has been located, the position and direction of the reflecting plane is available.

There are some cases for which the method is available without the restriction of homogeneity of the embedding medium. If the reflecting plane is horizontal and the speed is a variable function of the depth alone, in the upper medium, the path will be curved but can be replaced by an idealized, equivalent rectilinear path combined with an equivalent speed. A complete study of other cases is beyond the scope of this paper. In practice, the idealized picture merely represents the general features of the actual structure. For explorative purposes, this simple picture is of great value. In most cases, it is the best picture available, so that its defects are secondary.

The method is based on the optical theorem of reflection: To an observer, a wave reflected at a surface appears to have come from a point which is the image of the source in the plane tangent to the surface at the point of reflection; the time of travel is the same as though the impulse came from the image. If the medium of transmission is homogeneous, the propagation will be rectilinear from the source to the surface and from the surface to the receiver; further, the time of travel will be the same as though the impulse came rectilinearly from the image without reflection. The image of a point in a plane is such that the plane bisects perpendicularly the line segment between the image and the source. Thus, the surface and the source may be replaced by the image, and the reflecting surface may be considered as removed.

When the image position is known, the reflecting plane is completely determined as the perpendicular bisector of the pair of points: image—source. Thus a structure which does not have abrupt changes in shape may be outlined, at least approximately, by a series of shot points with geophones properly placed.

If there is no preliminary information about the structure, or the region, there are four unknowns: the speed with which the wave travels through the upper medium and the three coordinates of position of the image point. Accordingly, four observations are needed, and these must be independent of each other. We may use one shot with four geophones or four shots with one geophone. Each independent fact known about the situation reduces the number of observations needed.

Thus, if the speed of the embedding médium is known, three geophones will suffice.

If the geophones are located in symmetrical positions, it may happen that the observations will not be independent. Thus, the four corners of a square will not furnish a solution. The details of this will appear later. In general, it is advisable to locate the geophones as unsymmetrically as possible. If the direction of dip of the structure is known, the geophones may profitably be located in that direction, but in general it is better to cover as large an area as possible and to avoid all kinds of symmetry.

Two distinct cases occur: we shall call them "exact" and "excess." In the exact case, there are as many independent observations as there are unknowns. The solution is unique, except for finite ambiguities involved in the choice of signs when extracting square roots in the process of solution. The solution is a matter of algebra and furnishes no difficulty beyond the algebraic manipulation. In the excess case, there are more independent observations than unknowns and the interpretation may be made by least squares. An approximate solution is assumed and the values adjusted so as to minimize the sum of the squares of the discrepancies between the observed and the calculated times of travel. To obtain the approximate solutions, the values may be estimated from previous experience, by selecting enough observations to solve as an exact system, or by the artifice used in example 1. We shall consider the exact case as a preliminary step in the adjustment, by least squares, of the excess case.

The time of travel as recorded by a single geophone furnishes the information that the image lies somewhere on a sphere whose center is at the geophone and whose radius is the distance traveled by the impulse in the observed time. If the speed is known, this radius is fixed. If the speed is not known, we know only that the image lies on one of the spheres of the family with centers at the geophone. A second geophone furnishes similar information. Taken together, the two geophones determine a circle along which the image must lie. If the speed is known, the circle is unique. If the speed is unknown, we know only that the image must lie on one of the circles of a family whose centers lie on the straight line through the two geophones and whose planes are perpendicular to that line. If the speed is known, a third geophone, not collinear with the other two, determines the image point exactly and uniquely. If the speed is not known, the third geophone will specify a curve along which the image must lie, each point of this curve corresponding uniquely to a particular value of the speed. A fourth geophone, properly placed, locates the image when the speed is not known. Each geophone beyond the necessary minimum number increases the probable accuracy of the position as determined from the observations.

In the preceding paragraph, we have assumed a single shot and several geophones. The situation is similar if we have a single geophone and a sufficient number of shots. Each shot point has an image as viewed from each geophone and each geophone has an image as viewed from each shot point. Hence, two or more shot points should not be considered together unless they are so closely spaced as to be considered a single point, or unless due consideration is given to their relative positions. As will be seen later, it is possible and even desirable to group shots and geophones. But in such cases, the calculations must be made with a clear appreciation of the problem.

Section 2 (*Part* IV). *Formulation and Solution*

Let the origin of coordinates be taken at the shot point and the origin of time as the instant of the explosion. (See Fig. 2.) Let the z axis be

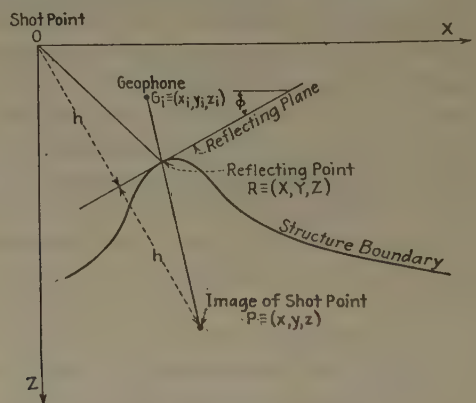

FIG. 2.

vertically downward and the x axis and the y axis in convenient perpendicular directions at the surface of the earth. Let the i-th geophone be located at $G_i \equiv (x_i, y_i, z_i)$ and the corresponding observed time be t_i. Let the image of the origin in the plane tangent to the structure at the point of reflection be $P \equiv (x, y, z)$ and the speed of propagation of the impulse in the upper medium be v. Then the calculated time of travel from shot to geophone will be

$$\frac{1}{v}\sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}$$

and the discrepancy between the observed and calculated times will be

$$f_i(x, y, z, v) = \frac{1}{v}\sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2} - t_i. \qquad [14]$$

The observation equations are

$$f_i = 0.$$

After determining the values of (x, y, z, v), we may determine the direction of dip, the angle of dip and the depth of the structure. Let the direction of dip be θ, measured horizontally from the positive x axis toward the positive y axis. Then

$$\tan\theta = \frac{y}{x}. \tag{15}$$

Let the angle of dip be ϕ, taken as acute. Then

$$\tan\phi = \frac{\sqrt{x^2 + y^2}}{z}. \tag{16}$$

The perpendicular distance from the origin to the plane tangent to the structure at the reflecting point is

$$h = \frac{1}{2}\sqrt{x^2 + y^2 + z^2}. \tag{17}$$

In the exact case, with no independent information, the observation equations are given by setting the function in equation 14 equal to zero, for each of the four observations. These may be written

$$x^2 + y^2 + z^2 - 2x_ix - 2y_iy - 2z_iz - t_i^2v^2 + (x_i^2 + y_i^2 + z_i^2) = 0. \tag{18}$$

One method of solution is to subtract one equation from each of the other three, treating v^2 as a parameter. This leads to three equations linear in x, y and z, which may be solved in terms of v^2. After they are found, one of the original equations determines v, after which the solution is simple. Signs are chosen so that v and z are both positive.

In the excess case, we use the method of least squares. The function considered is of the form

$$\left.\begin{aligned} f_i(x, y, z, v) &= (f_i)_0 + (x - x_0)\left(\frac{\partial f_i}{\partial x}\right)_0 + (y - y_0)\left(\frac{\partial f_i}{\partial y_0}\right) \\ &\quad + (z - z_0)\left(\frac{\partial f_i}{\partial z}\right)_0 + (v - v_0)\left(\frac{\partial f_i}{\partial v}\right)_0 + \cdots \\ \text{where}\quad & \\ (f_i)_0 &= f_i(x_0, y_0, z_0, v_0) \\ \left(\frac{\partial f_i}{\partial x}\right)_0 &= \frac{\partial}{\partial x} f_i(x_0, y_0, z_0, v_0) \end{aligned}\right\}. \tag{19}$$

etc.

The approximate solution (x_0, y_0, z_0, v_0) is assumed to be so nearly correct that all powers of the corrections

$$\left.\begin{aligned} \alpha &= x - x_0 \qquad & \gamma &= z - z_0 \\ \beta &= y - y_0 \qquad & \nu &= v - v_0 \end{aligned}\right\} \tag{20}$$

beyond the first may be neglected. For convenience, we write

$$\left.\begin{aligned} \xi_i &= x_i - x_0 \qquad & \zeta_i &= z_i - z_0 \\ \eta_1 &= y_i - y_0 \qquad & \rho_i &= \sqrt{\xi_i^2 + \eta_i^2 + \zeta_i^2} \end{aligned}\right\}. \tag{21}$$

Then, direct calculation shows that

$$\left.\begin{aligned}\left(\frac{\partial f_i}{\partial x}\right)_0 &= \frac{-\xi_i}{v_0\rho_i} & \left(\frac{\partial f_i}{\partial y}\right)_0 &= \frac{-\eta_i}{v_0\rho_i}\\ \left(\frac{\partial f_i}{\partial z}\right)_0 &= \frac{-\zeta_i}{v_0\rho_i} & \left(\frac{\partial f_i}{\partial v}\right)_0 &= \frac{-\rho_i}{v_0^2}\end{aligned}\right\}. \qquad [22]$$

Hence

$$f_i(x, y, z, v) = \left(\frac{\rho_i}{v_0} - t_i\right) + \alpha\left(\frac{-\xi_i}{v_0\rho_i}\right) + \beta\left(\frac{-\eta_i}{v_0\rho_i}\right) + \gamma\left(\frac{-\zeta_i}{v_0\rho_i}\right) + \nu\left(\frac{-\rho_i}{v_0^2}\right), \qquad [23]$$

and the observation equations are reduced to the linear form

$$\xi_i\alpha + \eta_i\beta + \zeta_i\gamma + \frac{\rho_i^2}{v_0}\nu = v_0\rho_i\left(\frac{\rho_i}{v_0} - t_i\right). \qquad [24]$$

If we write

$$\lambda = \frac{\nu}{v_0} \quad \text{and} \quad \epsilon_i = \frac{\rho_i}{v_0} - t_i, \qquad [25]$$

the observation equations may be written

$$\xi_i\alpha + \eta_i\beta + \zeta_i\gamma + \rho_i^2\lambda = v_0\rho_i\epsilon_i. \qquad [26]$$

The normal system is

$$\left.\begin{aligned}(\Sigma\xi_i^2)\alpha + (\Sigma\xi_i\eta_i)\beta + (\Sigma\xi_i\zeta_i)\gamma + (\Sigma\xi_i\rho_i^2)\lambda &= v_0\Sigma\xi_i\rho_i\epsilon_i\\ (\Sigma\xi_i\eta_i)\alpha + (\Sigma\eta_i^2)\beta + (\Sigma\eta_i\zeta_i)\gamma + (\Sigma\eta_i\rho_i^2)\lambda &= v_0\Sigma\eta_i\rho_i\epsilon_i\\ (\Sigma\xi_i\zeta_i)\alpha + (\Sigma\eta_i\zeta_i)\beta + (\Sigma\zeta_i^2)\gamma + (\Sigma\zeta_i\rho_i^2)\lambda &= v_0\Sigma\zeta_i\rho_i\epsilon_i\\ (\Sigma\xi_i\rho_i^2)\alpha + (\Sigma\eta_i\rho_i^2)\beta + (\Sigma\zeta_i\rho_i^2)\gamma + (\Sigma\rho_i^4)\lambda &= v_0\Sigma\rho_i^3\epsilon_i\end{aligned}\right\}. \qquad [27]$$

These four equations may be solved in any convenient manner for α, β, γ and λ, after which

$$x = x_0 + \alpha, \quad y = y_0 + \alpha, \quad z = z_0 + \gamma, \quad v = (1 + \lambda)v_0. \qquad [28]$$

If the corrections are small, the corrected values of x, y, z and v may be accepted as the best. If they are not small enough to neglect, the corrected values may be used as the approximate solution for a new calculation.

As was indicated above, four geophones do not always furnish a unique solution. Let the four geophones be located on the surface of the earth at the four corners of a square. We may, without loss of generality, write

$$\left.\begin{aligned}G_1 &\equiv (-a, a, 0) & G_2 &\equiv (a, a, 0)\\ G_3 &\equiv (a, -a, 0) & G_4 &\equiv (-a, -a, 0)\end{aligned}\right\}. \qquad [29]$$

Then the observation equations may be written

$$\left.\begin{aligned}(x + a)^2 + (y - a)^2 + z^2 &= v^2t_1^2\\ (x - a)^2 + (y - a)^2 + z^2 &= v^2t_2^2\\ (x - a)^2 + (y + a)^2 + z^2 &= v^2t_3^2\\ (x + a)^2 + (y + a)^2 + z^2 &= v^2t_4^2\end{aligned}\right\}. \qquad [30]$$

If we multiply these equations by $+1$, -1, $+1$, -1, respectively and add, we have

$$0 = v^2(t_1^2 - t_2^2 + t_3^2 - t_4^2). \qquad [31]$$

Hence, the system is inconsistent, unless equation 31 is satisfied. If this relation holds, one of the equations is obtainable from the other three and each arbitrary value of v determines a set of values for x, y and z.

Geometrically, the same conclusion is reached. The spheres from two adjacent corners meet in a plane circle which is perpendicular to the edge connecting these two corners; similarly, the spheres from the other two corners. These planes are parallel and intersect only when they coincide. If they do coincide, each value of v determines a solution. Hence, there is no determinate solution.

If, as is usually the case, all the geophones are at or near the surface of the earth, we have $z_i = 0$ so that $\zeta_i = -z_0$. If we write

$$\omega = -z_0\gamma, \qquad [32]$$

the normal system becomes

$$\left.\begin{array}{l} (\Sigma\xi_i^2)\alpha + (\Sigma\xi_i\eta_i)\beta + (\Sigma\xi_i)\omega + (\Sigma\xi_i\rho_i^2)\lambda = v_0\Sigma\xi_i\rho_i\epsilon_i \\ (\Sigma\xi_i\eta_i)\alpha + (\Sigma\eta_i^2)\beta + (\Sigma\eta_i)\omega + (\Sigma\eta_i\rho_i^2)\lambda = v_0\Sigma\eta_i\rho_i\epsilon_i \\ (\Sigma\xi_i)\alpha + (\Sigma\eta_i)\beta + n\omega + (\Sigma\rho_i^2)\lambda = v_0\Sigma\rho_i\epsilon_i \\ (\Sigma\xi_i\rho_i^2)\alpha + (\Sigma\eta_i\rho_i^2)\beta + (\Sigma\rho_i^2)\omega + (\Sigma\rho_i^4)\lambda = v_0\Sigma\rho_i^3\epsilon_i \end{array}\right\}. \qquad [33].$$

For the exact case, we have

$$(x - x_i)^2 + (y - y_i)^2 + z^2 = v^2t_i^2 \qquad (i = 1, 2, 3, 4) \qquad [34]$$

The examples will all assume $z_i = 0$.

Section 3 (*Part* IV). *Examples*

Example 2

As an example of the calculations, when the speed is unknown, consider the data of Table 2. For the exact case, consider the first four observations, the unit of distance being 1000 ft.:

$$\begin{array}{lll} x_1 = 0 & y_1 = 1 & t_1 = 1.237 \\ x_2 = 1 & y_2 = 0 & t_2 = 1.242 \\ x_3 = 1 & y_3 = 1 & t_3 = 1.239 \\ x_4 = -1 & y_4 = -1 & t_4 = 1.280 \end{array}$$

The observation equations may be written

$$\begin{array}{l} x^2 + y^2 + z^2 \qquad\quad - 2y - t_1^2v^2 = -1 \\ x^2 + y^2 + z^2 - 2x \qquad\quad - t_2^2v^2 = -1 \\ x^2 + y^2 + z^2 - 2x - 2y - t_3^2v^2 = -2 \\ x^2 + y \ + z^2 + 2x + 2y - t_4^2v^2 = -2. \end{array}$$

TABLE 2

Approximate Solution						$x_0 = +0.4019$			$y_0 = +0.6830$			$z_0 = 7.9879$		$v_0 = 6.4710$		$z_0^2 = 63.8065$		
i	x_i	y_i	t_i	ξ_i	η_i	ξ_i^2	η_i^2	$\xi_i\eta_i$	ρ_i^2	ρ_i	ϵ_i	$\xi_i\rho_i^2$	$\eta_i\rho_i^2$	ρ_i^4	$\rho_i\epsilon_i$	$\xi_i\rho_i\epsilon_i$	$\eta_i\rho_i\epsilon_i$	$\rho_i^3\epsilon_i$
1	0	1	1.237	−0.4019	+0.3170	0.1615	0.1005	−0.1274	64.0685	8.0043	−0.0001	− 25.749	+ 20.310	4104.8	−0.0008	+0.0003	−0.0003	−0.0513
2	1	0	1.242	+0.5981	−0.6830	0.3577	0.4665	−0.4085	64.6307	8.0393	+0.0004	+ 38.656	− 44.143	4177.1	+0.0032	+0.0019	−0.0022	+0.2068
3	1	1	1.239	+0.5981	+0.3170	0.3577	0.1005	+0.1896	64.2647	8.0165	−0.0002	+ 38.437	+ 20.372	4130.0	−0.0016	−0.0010	−0.0005	−0.1028
4	−1	−1	1.280	−1.4019	−1.6830	1.9653	2.8325	+2.3594	68.6043	8.2828	=	− 96.177	−115.461	4706.5	=	=	=	=
5	0	−1	1.263	−0.4019	−1.6830	0.1615	2.8325	+0.6764	66.8005	8.1732	+0.0001	− 26.847	−112.425	4462.3	+0.0008	−0.0003	−0.0013	+0.0534
6	−1	0	1.258	−1.4019	−0.6830	1.9653	0.4665	+0.9575	66.2383	8.1387	−0.0003	− 92.860	− 45.241	4387.5	−0.0024	+0.0034	+0.0016	−0.1590
7	1	−1	1.265	+0.5981	−1.6830	0.3577	2.8325	−1.0132	66.9967	8.1852	−0.0001	+ 40.071	−112.755	4488.5	−0.0008	−0.0005	+0.0013	=0.0536
8	−1	1	1.254	−1.4019	+0.3170	1.9653	0.1005	−0.1975	65.8723	8.1162	+0.0002	− 92.347	+ 20.881	4339.2	+0.0016	−0.0022	+0.0005	+0.1054
			Σ	−3.2152	−5.4640	7.2920	9.7320	+2.4363	527.476			−216.816	−368.462	34795.9	=	+0.0016	−0.0009	−0.0011

$$
\begin{aligned}
+7.2920\alpha + 2.4363\beta - 3.2152\omega - 216.816\lambda &= +0.0104 \\
+2.4363\alpha + 9.7320\beta - 5.4640\omega - 368.462\lambda &= -0.0058 \\
-3.2152\alpha - 5.4640\beta + 8\omega + 527.476\lambda &= 0 \\
-216.816\alpha - 368.462\beta + 527.476\omega + 34795.9\lambda &= -0.0071
\end{aligned}
$$

$$
\begin{aligned}
\alpha &= +0.0017 \\
\beta &= -0.0010 \\
\omega &= 0 \\
\lambda &= 0 \\
\gamma &= 0 \\
\nu &= 0
\end{aligned}
\qquad
\begin{aligned}
x &= +0.4036 \\
y &= +0.6820 \\
z &= 7.9879 \\
v &= 6.4710
\end{aligned}
\qquad
\begin{aligned}
\tan\theta &= 1.6898 \\
\tan\phi &= 0.0992 \\
\theta &= 59^\circ\, 23' \\
\phi &= 5^\circ\, 40' \\
h &= 4.0136
\end{aligned}
$$

which reduce to

$$
\begin{aligned}
2x + 4y + (t_1^2 - t_4^2)v^2 &= -1 \\
4x + 2y + (t_2^2 - t_4^2)v^2 &= -1 \\
4x + 4y + (t_3^2 - t_4^2)v^2 &= 0 \\
z = \sqrt{v^2t_1^2 - x^2 - (y-1)^2}
\end{aligned}
$$

Thus

$$
\begin{aligned}
2x &= 1 + (t_1^2 - t_3^2)v^2 \\
2y &= 1 + (t_2^2 - t_3^2)v^2 \\
v &= \frac{2}{\sqrt{t_4^2 - 2t_2^2 + 3t_3^2 - 2t_1^2}}.
\end{aligned}
$$

Numerically,

$$
\begin{aligned}
v &= 6.3791 \\
x &= +0.3992 \\
y &= +0.6514 \\
z &= 7.8731.
\end{aligned}
$$

If we use these values for the approximate solution and proceed as in Table 2, we find the corrected values

$$x = +0.4019, \quad y = +0.6830, \quad z = 7.9879, \quad v = 6.4710$$

Using these values for the approximate solution, we have the calculations shown in Table 2. The azimuth of dip is $\theta = 59° \ 23'$, the angle of dip is $\phi = 5° \ 40'$, and the depth to the reflecting plane is 4014 ft. The best speed is 6471 ft. per second.

Example 3

If we know the speed, or assume it, we have $\lambda = 0$ and the normal equations 33 become

$$
\left.
\begin{aligned}
(\Sigma\xi_i^2)\alpha + (\Sigma\xi_i\eta_i)\beta + (\Sigma\xi_i)\omega &= v\Sigma\xi_i\rho_i\epsilon_i \\
(\Sigma\xi_i\eta_i)\alpha + (\Sigma\eta_i^2)\beta + (\Sigma\eta_i)\omega &= v\Sigma\eta_i\rho_i\epsilon_i \\
(\Sigma\xi_i)\alpha + (\Sigma\eta_i)\beta + n\omega &= v\Sigma\rho_i\epsilon_i
\end{aligned}
\right\}. \qquad [35]
$$

For the exact case, the observation equations are reducible to

$$
\left.
\begin{aligned}
2(x_2 - x_1)x + 2(y_2 - y_1)y &= v^2(t_1^2 - t_2^2) + (x_2^2 - x_1^2) \\
&\qquad + (y_2^2 - y_1^2) \\
2(x_3 - x_2)x + 2(y_3 - y_2)y &= v^2(t_2^2 - t_3^2) + (x_3^2 - x_2^2) \\
&\qquad + (y_3^2 - y_2^2) \\
z &= \sqrt{v^2t_1^2 - (x - x_1)^2 - (y - y_1)^2}
\end{aligned}
\right\}. \qquad [36]
$$

Numerically, consider the data of example 2 with the assumption that the speed is $v = 6.5$. For the exact case, consider the first three observations

$$
\begin{array}{lll}
x_1 = 0 & y_1 = 1 & t_1 = 1.237 \\
x_2 = 1 & y_2 = 0 & t_2 = 1.242 \\
x_3 = 1 & y_3 = 1 & t_3 = 1.239
\end{array}
$$

TABLE 3

Approximate Solution						$x_0 = +0.4061$		$y_0 = +0.6902$		$z_0 = 8.0242$		$v = 6.5$		$z_0^2 = 64.3878$
i	x_i	y_i	t_i	ξ_i	η_i	ξ_i^2	η_i^2	$\xi_i\eta_i$	ρ_i^2	ρ_i	e_i	$\rho_i e_i$	$\xi_i\rho_i e_i$	$\eta_i\rho_i e_i$
1	0	1	1.237	−0.4061	+0.3098	0.1649	0.0960	−0.1258	64.6487	8.0404	=	=	=	=
2	1	0	1.242	+0.5939	−0.6902	0.3527	0.4764	−0.4099	65.2169	8.0757	+0.0004	+0.0032	+0.0019	−0.0022
3	1	1	1.239	+0.5939	+0.3098	0.3527	0.0960	+0.1840	64.8365	8.0521	−0.0002	−0.0016	−0.0010	−0.0005
4	−1	−1	1.280	−1.4061	−1.6902	1.9771	2.8568	+2.3766	69.2217	8.3200	=	=	=	=
5	0	−1	1.263	−0.4061	−1.6902	0.1649	2.8568	+0.6864	67.4095	8.2103	+0.0001	+0.0008	−0.0003	−0.0014
6	−1	0	1.258	−1.4061	−0.6902	1.9771	0.4764	+0.9705	66.8413	8.1757	−0.0002	−0.0016	+0.0022	+0.0011
7	1	−1	1.265	+0.5939	−1.6902	0.3527	2.8568	−1.0038	67.5973	8.2218	−0.0001	−0.0008	−0.0005	+0.0014
8	−1	1	1.254	−1.4061	+0.3098	1.9771	0.0960	−0.4356	66.4609	8.1524	+0.0002	+0.0016	−0.0022	+0.0005
			Σ	−3.2488	−5.5216	7.3192	9.8112	+2.2424				+0.0016	+0.0001	−0.0011

$$+7.3192\alpha + 2.2424\beta - 3.2488\omega = +0.0006$$
$$+2.2424\alpha + 9.8112\beta - 5.5216\omega = -0.0072$$
$$-3.2488\alpha - 5.5216\beta + 8\omega = +0.0104$$

$\omega = +0.0016$
$\beta = 0$
$\alpha = +0.0008$
$\gamma = -0.0002$

$x = +0.4069$
$y = +0.6902$
$z = 8.0240$

$\tan\theta = 1.6962$
$\tan\phi = 0.0998$

$\theta = 59° 29'$
$\phi = 5° 42'$
$h = 4.0321$

which lead to

$$x = +0.3954, \quad y = +0.6572, \quad z = 8.0234.$$

Considering all the observations and using these values for the approximate solution, we have, by calculations similar to those in Table 3, the corrected values:

$$x = +0.4061, \quad y = +0.6902, \quad z = 8.0242.$$

Using these values for the approximation, we have the calculations for Table 3. The corrected values are:

$$x = +0.4069, \quad y = +0.6902, \quad z = 8.0240.$$

The azimuth of dip is 59° 29′ as compared with 59° 23′ in the preceding case. The angle of dip is 5° 42′ as compared with 5° 40′ in the preceding example. The depth is 4032 ft. as compared with 4013 ft. Thus, an error of 29 ft. per second in the assumed speed causes an error of 19 ft. in the calculated depth of the structure.

Example 4

If the direction of dip is known, but the speed of the upper layer is not known, we may place the shot and geophones along this direction, which may be chosen as the x axis. Then $y = \beta = 0$ and the normal equations become

$$\left.\begin{aligned} (\Sigma\xi_i^2)\alpha + (\Sigma\xi_i)\omega + (\Sigma\xi_i\rho_i^2)\lambda &= v_0\Sigma\xi_i\rho_i\epsilon_i \\ (\Sigma\xi_i)\alpha + n\omega + (\Sigma\rho_i^2)\lambda &= v_0\Sigma\rho_i\epsilon_i \\ (\Sigma\xi_i\rho_i^2)\alpha + (\Sigma\rho_i^2)\omega + (\Sigma\rho_i^4)\lambda &= v_0\Sigma\rho_i^3\epsilon_i \end{aligned}\right\}. \quad [37]$$

For the exact case, the observation equations are reducible to

$$\left.\begin{aligned} 2(x_2 - x_1)x + (t_2^2 - t_1^2)v^2 &= x_2^2 - x_1^2 \\ 2(x_3 - x_2)x + (t_3^2 - t_2^2)v^2 &= x_3^2 - x_2^2 \\ z &= \sqrt{v^2t_1^2 - (x - x_1)^2} \end{aligned}\right\}. \quad [38]$$

For a numerical illustration, consider the data of Table 4. For the exact case, consider the three points

$$x_1 = 0.1 \qquad x_5 = 2.0 \qquad x_8 = 5.0$$
$$t_1 = 0.61 \qquad t_5 = 0.67 \qquad t_8 = 0.95$$

The observation equations are

$$\begin{aligned} (x - 0.1)^2 + z^2 - (0.61v)^2 &= 0 \\ (x - 2)^2 + z^2 - (0.67v)^2 &= 0 \\ (x - 5)^2 + z^2 - (0.95v)^2 &= 0, \end{aligned}$$

which lead to

$$x = +0.1560, \quad v = 6.6507, \quad z = 4.0565.$$

For the excess case, calculations similar to those in Table 4 lead to the corrected values

$$x = +0.2817, \quad v = 6.5203, \quad z = 4.0120.$$

TABLE 4

Approximate Solution							$x_0 = +0.2817$	$z_0 = 4.0120$		$v_0 = 6.5203$		$z_0^2 = 16.0961$
i	x_i	t_i	ξ_i	ξ_i^2	ρ_i^2	ρ_i	ϵ_i	$\xi_i\rho_i^2$	ρ_i^4	$\rho_i\epsilon_i$	$\xi_i\rho_i\epsilon_i$	$\rho_i^3\epsilon_i$
1	0.1	0.61	−0.1817	0.0330	16.1291	4.0161	+0.0059	− 2.9307	260.148	+0.0237	−0.0043	+0.3823
2	0.5	0.62	+0.2183	0.0477	16.1438	4.0179	−0.0038	+ 3.5242	260.622	−0.0153	−0.0033	−0.2470
3	1.0	0.63	+0.7183	0.5160	16.6121	4.0758	−0.0049	+ 11.9325	275.962	−0.0200	−0.0144	−0.3322
4	1.5	0.64	+1.2183	1.4843	17.5804	4.1959	+0.0031	+ 21.4182	309.071	+0.0130	+0.0158	+0.2285
5	2.0	0.67	+1.7183	2.9526	19.0487	4.3645	−0.0006	+ 32.7314	362.853	−0.0026	−0.0045	−0.0495
6	3.0	0.74	+2.7183	7.3892	23.4853	4.8462	+0.0033	+ 63.8401	551.559	+0.0160	+0.0435	+0.3758
7	4.0	0.84	+3.7183	13.8258	29.9219	5.4701	−0.0011	+111.2586	895.320	−0.0060	−0.0223	−0.1795
8	5.0	0.95	+4.7183	22.2624	38.3585	6.1934	−0.0001	+180.9869	1471.375	−0.0006	−0.0028	−0.0230
		Σ	14.8464	48.5110	177.280			+422.761	4386.91	+0.0082	+0.0077	+0.1554

$$48.5110\alpha + 14.8464\omega + 422.761\lambda = +0.0502$$
$$14.8464\alpha + 8\omega + 177.280\lambda = +0.0535$$
$$422.761\alpha + 177.280\omega + 4386.91\lambda = +1.0133$$

$\lambda = +0.0015$
$\omega = -0.0095$
$\alpha = -0.0091$
$\gamma = +0.0024$
$\nu = +0.0098$

$x = +0.2726$
$z = 4.0144$
$v = 6.5301$

$\tan\phi = 0.0679$
$\phi = 3° 53'$
$h = 2.0118$

These are corrected in Table 4 to

$$x = +0.2726, \quad v = 6.5301, \quad z = 4.0144.$$

Thus the speed is 6530 ft. per second, the dip is 3° 53′ and depth perpendicular to the bed at the shot point is 2012 feet.

Example 5

If the direction of dip and the speed of the upper layer are both known, the normal system becomes

$$\left.\begin{aligned} (\Sigma\xi_i^2)\alpha + (\Sigma\xi_i)\omega &= v\Sigma\xi_i\rho_i\epsilon_i \\ (\Sigma\xi_i)\alpha + \eta\omega &= v\Sigma\rho_i\epsilon_i \end{aligned}\right\}. \tag{39}$$

For the exact case, we have

$$\left.\begin{aligned} x &= \frac{x_1 + x_2}{2} - \frac{v^2(t_2^2 - t_1^2)}{2(x_2 - x_1)} \\ z &= \sqrt{v^2t_1^2 - (x - x_1)^2} \end{aligned}\right\}. \tag{40}$$

Numerically, we may consider the data of example 4 with the assumption of $v = 6.5$. The two observations $x_1 = 0.1$, $t_1 = 0.61$ and $x_8 = 5$, $t_8 = 0.95$ determine

$$x = +0.2633 \quad \text{and} \quad z = 3.9616,$$

which are corrected as in Table 5 to

$$x = +0.2928 \quad \text{and} \quad z = 3.9985,$$

and these are corrected in Table 5 to

$$x = +0.2930 \quad \text{and} \quad z = 3.9983.$$

The dip is calculated as 4° 11′ instead of 3° 53′ and the depth is 2004 ft. The decrease of 30 ft. per second in the speed corresponds to a decrease of 8 ft. in the calculated depth.

Example 6

If the layer is known to be horizontal, but the speed is unknown, the normal system reduces to

$$\left.\begin{aligned} n\omega + (\Sigma\rho_i^2)\lambda &= v_0\Sigma\rho_i\epsilon_i \\ (\Sigma\rho_i^2)\omega + (\Sigma\rho_i^4)\lambda &= v_0\Sigma\rho_i^3\epsilon_i \end{aligned}\right\}, \tag{41}$$

and for the exact case, we have

$$v = \sqrt{\frac{x_2^2 - x_1^2}{t_2^2 - t_1^2}} \quad \text{and} \quad z = \sqrt{\frac{x_2^2t_1^2 - x_1^2t_2^2}{t_2^2 - t_1^2}}. \tag{42}$$

TABLE 5

Approximate Solution					$x_0 = +0.2928$	$z_0 = 3.9985$		$v = 6.5$	$z_0^2 = 15.9880$
i	x_i	t_i	ξ_i	ξ_i^2	ρ_i^2	ρ_i	ϵ_i	$\rho_i\epsilon_i$	$\xi_i\rho_i\epsilon_i$
1	0.1	0.61	−0.1928	0.0372	16.0252	4.0031	+0.0059	+0.0236	−0.0046
2	0.5	0.62	+0.2072	0.0429	16.0309	4.0039	−0.0040	−0.0160	−0.0033
3	1.0	0.63	+0.7072	0.5001	16.4881	4.0606	−0.0053	−0.0215	−0.0152
4	1.5	0.64	+1.2072	1.4573	17.4453	4.1768	+0.0026	+0.0109	+0.0132
5	2.0	0.67	+1.7072	2.9145	18.9025	4.3477	−0.0011	−0.0048	−0.0082
6	3.0	0.74	+2.7072	7.3289	23.3169	4.8288	+0.0029	+0.0140	+0.0379
7	4.0	0.84	+3.7072	13.7433	29.7313	5.4526	−0.0011	−0.0060	−0.0222
8	5.0	0.95	+4.7072	22.1577	38.1457	6.1762	+0.0002	+0.0012	+0.0056
		Σ	14.7576	48.1819				+0.0014	+0.0032

$$48.1819\alpha + 14.7576\omega = +0.0208$$
$$14.7576\alpha + 8\omega = +0.0091$$

$\alpha = +0.0002$
$\omega = +0.0008$
$\gamma = -0.0002$

$x = +0.2930$
$z = 3.9983$

$\tan\phi = 0.0733$
$\phi = 4° 11'$
$h = 2.0044$

TABLE 6

Approximate Solution						$z_0 = 4.0114$	$v_0 = 6.4692$	$z_0^2 = 16.0913$
i	x_i	t_i	ρ_i^2	ρ_i	ϵ_i	ρ_i^4	$\rho_i\epsilon_i$	$\rho_i^3\epsilon_i$
1	0.1	0.62	16.1013	4.0126	+0.0003	259.2519	+0.0012	+0.0193
2	0.5	0.63	16.3413	4.0424	−0.0051	267.0381	−0.0206	−0.3366
3	1.0	0.64	17.0913	4.1342	−0.0009	292.1125	−0.0037	−0.0632
4	1.5	0.66	18.3413	4.2827	+0.0020	336.4033	+0.0086	+0.1577
5	2.0	0.69	20.0913	4.4823	+0.0029	403.6603	+0.0130	+0.2612
6	3.0	0.77	25.0913	5.0091	+0.0043	629.5733	+0.0215	+0.5395
7	4.0	0.88	32.0913	5.6649	−0.0043	1029.8515	−0.0244	−0.7830
8	5.0	0.99	41.0913	6.4102	+0.0009	1688.4949	+0.0058	+0.2383
		Σ	186.2404			4906.39	+0.0014	+0.0332
$8\omega + 186.2404\lambda = +0.0091$ $186.2404\omega + 4906.39\lambda = +0.2148$						$\lambda = 0$ $\omega = +0.0011$	$\nu = 0$ $\gamma = -0.0003$	$v = 6.4692$ $z = 4.0111$

Numerically, consider the data of Table 6. The two points

$$x_1 = 0.1, \quad t_1 = 0.62 \quad \text{and} \quad x_8 = 5, \quad t_8 = 0.99$$

determine

$$v = 6.4770 \quad \text{and} \quad z = 4.0144.$$

These are corrected as in Table 6 to

$$v = 6.4692 \quad \text{and} \quad z = 4.0114$$

and these are checked in Table 6. The best speed is 6469 ft. per second and the best depth is 2006 feet.

Example 7

If the bed is known to be horizontal and the speed is known, the observation equations are reducible to

$$\left.\begin{aligned} \omega &= -z_0\gamma = \frac{v}{n}\Sigma\rho_i\epsilon_i \\ &\text{or} \\ z &= z_0 - \frac{v}{nz_0}\Sigma\rho_i\epsilon_i \end{aligned}\right\}. \qquad [43]$$

In most cases, this value will agree with the arithmetic mean of the separate depths determined by the relation

$$z = \frac{1}{n}\Sigma z_i = \frac{1}{n}\Sigma\sqrt{v^2t_i^2 - x_i^2}. \qquad [44]$$

TABLE 7

i	x_i	t_i	z_i
1	0.1	0.62	4.0288
2	0.5	0.63	4.0644
3	1.0	0.64	4.0380
4	1.5	0.66	4.0192
5	2.0	0.69	4.0144
6	3.0	0.77	4.0062
7	4.0	0.88	4.0888
8	5.0	0.99	4.0508
$z_0 = 4.0388$			

TABLE 8

Approximate Solution				$z_0 = 4.0388$	$v = 6.5$	$z_0^2 = 16.3119$
i	x_i	t_i	ρ_i^2	ρ_i	ϵ_i	$\rho_i\epsilon_i$
1	0.1	0.62	16.3219	4.0400	+0.0015	+0.0061
2	0.5	0.63	16.5619	4.0696	−0.0039	−0.0158
3	1.0	0.64	17.3119	4.1608	+0.0001	+0.0004
4	1.5	0.66	18.5619	4.3084	+0.0028	+0.0121
5	2.0	0.69	20.3119	4.5069	+0.0034	+0.0153
6	3.0	0.77	25.3119	5.0311	+0.0040	+0.0201
7	4.0	0.88	32.3119	5.6844	−0.0055	−0.0313
8	5.0	0.99	41.3119	6.4274	−0.0012	−0.0077
$\Sigma\rho_i\epsilon_i = -0.0008$				$\gamma = +0.0002$		$z = 4.0390$

If we consider the data of example 6 with an assumed speed, $v = 6.5$, Table 7 shows the calculation of z from the separate observations. The mean value is $z_0 = 4.0388$. Using this value as the approximation, Table 8 checks this value. Comparison with example 6 shows that the choice of 6500 instead of 6469 ft. per second in the speed leads to an increase of 14 ft. in the depth.

Section 4 (*Part* IV). *Comments*

In the examples, the assumption was made that a single plane was effective in all of the reflections. Each pair (shot—geophone) has its own reflection point and its reflection plane. It was assumed that all the planes coincided to a degree of approximation sufficient for the obser-

vations. If they do not agree, the method determines a plane of best fit. The actual point of reflection for a particular pair is the intersection of this best plane with the line from the image to the geophone. The method is valid whenever the reflecting surface does not depart appreciably from the reflecting plane over the region covered by the reflecting points. Usually, the structure may be considered to be plane over the entire region of the survey.

To illustrate this, consider example 4. The depth to the plane is $h = 2.0118$ and the dip is $\phi = 3° 53'$, up in the x direction. The trace of the reflecting plane on the xz plane has the equation

$$Z = h \sec \phi - X \tan \phi. \qquad [45]$$

The image of the origin is at,

$$x = +0.2726, \; z = 4.0144.$$

The line from the image to the geophone, G_i, has the equation

$$zX + (x_i - x)Z = x_i z. \qquad [46]$$

This line intersects the reflecting plane at

$$\left.\begin{aligned} X &= \frac{(h \sec \phi - z)x_i - (hx \sec \phi)}{(\tan \phi)x_i - (x \tan \phi + z)} \\ Z &= \frac{(z \tan \phi)x_i - (hz \sec \phi)}{(\tan \phi)x_i - (x \tan \phi + z)} \end{aligned}\right\}. \qquad [47]$$

For the present case,

$$\left.\begin{aligned} X &= \frac{0.5497 + 1.9980x_i}{4.0329 - 0.0679x_i} \\ Z &= \frac{8.0946 - 0.2726x_i}{4.0329 - 0.0679x_i} \end{aligned}\right\}. \qquad [48]$$

For

$$x_1 = 0.1, \quad X_1 = 0.1862, \quad Z_1 = 2.0038$$

For

$$x_8 = 5, \quad X_8 = 2.8537, \quad Z_8 = 1.8226.$$

These two points mark the limits of the structure which are needed, and the surface is assumed plane between these two points.

The discussion has considered a single shot point and several geophones. Since the time of travel is independent of which point is the geophone, we may consider a single geophone with several shot points, locating the image of the geophone instead of the image of the shot point. In regions where the survey is desired with considerable detail, a "dual" system may be used. In the exact case, we may use four shot points and four geophone positions. The four geophones determine the image of each shot point and the four shot points determine the image of each geophone. Thus eight image points are determined from four shots. In the excess case, we use each shot point with all the geophones to determine the image of that shot point. Similarly, we use each geophone

with all the shot points to determine the image of that geophone point. Each separate determination is made as in the examples.

There is no essential gain in standardization of the method of observation. After the initial calculations, the advantages of standardization disappear. In fact, a random distribution of shot and receiver positions will avoid difficulties of symmetry.

The least square analysis is valuable in detecting false readings of the records. The value of ϵ_i furnishes a check on the observations. If ϵ_i is abnormally large, a new choice of the reflection record is indicated, if there is a choice. Discarding a reading solely because of a large value of ϵ_i is preferred by some persons, but the writer is not in sympathy with such a procedure. An apparently better solution is afforded when discordant values are discarded, without comments, but scientific honesty requires the retention of all observations not ruled out for some specific and independent reason. In such cases, the record should be included in the reports along with a complete statement of reasons for discarding it. Assuming accurate data, the least square method furnishes a direct method for its interpretation. With unreliable data, no method can be expected to produce reliable conclusions.

The tables included will show convenient forms for the calculations. Where routine observations are to be reduced, blank forms should be printed with sufficient lines to cover all cases likely to arise in the work. In such cases, a collection of the controlling formulas should be available to the computer in a form for ready reference.

Part V. Interpretation of Magnetometer and Torsion Balance Surveys

Section 1. *The Problem and Its Solution*

The torsion balance measures the difference in the horizontal force of gravity at its two weights. By proper operation, it is a simple matter to calculate the vector gradient of gravity at the station under observation. This gradient is usually plotted on a map of the region under exploration, as an arrow of which the length represents the magnitude of the gradient and which has the direction of the fastest increase in gravity. The gradient is merely the vector horizontal derivative of the force of vertical gravity, or the directed rate at which the acceleration of gravity is increasing. It is usually measured in Eötvös, one Eötvös being a change of 10^{-9} dynes per centimeter.

For many purposes, a gradient map is sufficient, but for some purposes an isogam map is desirable. The isogam map is a contour map of the gravity surface. Each isogam line is a line of constant gravity value. The purpose of this paper is not to discuss the interpretation of the

isogam map, but to show how to reduce the gradient map to an isogam map, when the survey covers an area and not merely a few profiles.

In this paper, the term "gravity" will be used in several senses, but no confusion is likely. The most frequent interpretation will be "the value of the acceleration of gravity."

To obtain the difference in the value of gravity at two stations, we assume that the change in the gradient in going from one station to the other is linear in the component parallel to the line through the two stations. The component perpendicular to the common line does not enter the calculations for that particular pair of stations. With this assumption, we may, for that particular pair of stations, replace the actual gradients by a single, constant gradient of which the magnitude is the average of the two components of the actual gradients along the common line.

For definiteness, consider two near-by stations, A and B, a distance l apart. Assume that the x axis passes through A and B, postively from A toward B. If the gradient at A is G_1, in a direction α_1 with the x axis, while that at B is G_2 in a direction α_2 with the x axis, we assume that the difference in gravity from A to B is given by

$$g_2 - g_1 = \tfrac{1}{2}(G_1 \cos \alpha_1 + G_2 \cos \alpha_2)l. \qquad [49]$$

For some purposes, it is important to know the absolute value of the gravity differences. If a structure is to be interpreted as to shape, as well as to position, this may be necessary, but for this purpose, gradients are usually to be preferred to isogams. The relation between structure and gradients is so complex that many interpreters have discontinued trying to outline structures, except in very simple cases, such as some salt domes in a homogeneous region.[5] In fact, the attempt to use the torsion balance for structural outlining has been largely responsible for the discarding of all torsion balance work by some petroleum engineers. Many executives, engineers, and even some geophysicists, seem to follow an "all or none" policy. If the purpose of a torsion balance survey is not so pretentious, but is merely for locating suspicious positions, and of occasionally furnishing more detailed information, an arbitrary unit is as useful as the absolute one. When needed, the results may be converted into absolute units by the map factor. In reading the map, an arbitrary unit is usually more convenient.

For definiteness, consider a scale used satisfactorily by the writer. A sheet of ordinary thin celluloid in the form of a square measures about 6 in. on a side. The diagonals are scratched with a fine knife so as to make a scratch deep enough to hold india drawing ink, but not deep

[5] D. C. Barton: The Eötvös Torsion Balance Method of Mapping Geologic Structure. Geophysical Prospecting, A. I. M. E. (1929) 416.

enough to cause the sheet to crack. Beginning at the intersection of the diagonals, a scale of inches divided into tenths, in the usual manner of scales, is cut along each of the four diagonal lines. The cuts are filled with india ink and allowed to dry. When thoroughly dry, the surplus ink is wiped off with a damp cloth. We now have a cross of perpendicular scales, graduated into units of $\frac{1}{10}$ inch.

To use this scale, we place one of the lines over the pair of stations considered. Then we slide the scale along the common line until the tip of the arrow to be measured just touches the second line. It is a simple matter to read the projection of the arrow on the common line, through the celluloid. Inspection tells the sign to be used. If the arrow points in the direction from the first station to the second station, the sign is positive. Furthermore, the distance between the two stations is read directly through the first line. To obtain the difference in gravity in this arbitrary unit, we merely add the two components, with signs, and multiply by the separation. Thus the factor $\frac{1}{2}$ which enters in equation 49 is absorbed in the arbitrary unit. While few maps and scales are accurate to $\frac{1}{100}$ in., the writer has found it convenient to estimate the measures to that degree of fineness, in order to protect the first decimal from forcing errors, which are nearly always present in calculations.

For the analysis of a single map or comparison of maps using the same scales, this method is convenient. To reduce the values of gravity to dynes, we may proceed as follows: Suppose that the distances are plotted in the scale of one mile to the inch and that the gradients are plotted in the scale of one Eötvös to the millimeter. Then one arbitrary unit of distance is the equivalent of $\frac{1}{10}$ mile, or 1.609×10^4 cm., while one unit of gradient is 2.54 mm. and is equivalent to 2.54×10^{-9} dynes per centimeter. If the gradient components are C_1 and C_2 and the distance is L, all in units of $\frac{1}{10}$ in., the difference in gravity is calculated as $(C_1 + C_2)L$. But C_1 and C_2 are the equivalents of $2.54 \times 10^{-9}C_1$, and $2.54 \times 10^{-9}C_2$, respectively. The average component of gradient is

$$1.27 \times 10^{-9}(C_1 + C_2).$$

Since the distance is $1.609 \times 10^4 L$ cm., the actual difference in gravity is

$$[1.27 \times 10^{-9}(C_1 + C_2)][1.609 \times 10^4 L] = 2.04 \times 10^{-5}(C_1 + C_2)L$$

measured in dynes. Thus the map factor is 2.04×10^5 for this case. For other scales, the map factor may be computed as needed.

The calculation of the gravity difference for two stations is a fundamental problem in any attempt to reduce a survey to isogams. While it is simple and direct, it usually represents a large expenditure of time and energy. The map must be drawn and each individual difference needed must be determined. Some linking of stations is necessary, the particular method used depending upon the particular needs and the

choice of the interpreter.[6] The greater the linking of stations, the finer is the grain of the results. This point will be discussed later.[7]

In the case of the magnetometer, the intensity at each station of a group is measured by its variation from that at a base station of the group. The instrument measures the difference between the magnetic intensities at the two stations, usually one station entering all the pairs of a group. If the magnetometer survey contains just the differences as observed, the net is determined.

Frequently, such surveys are made by profiles with an occasional tying in of stations, by a cross profile. For more accurate surveys, it would be preferable to have a network of profiles, having as many reference stations as possible, especially at the crossing of lines of the net. With a slight increase in the cost of the survey, such a net could be followed to advantage.

With the torsion balance, the interpreter has a wide choice of linkings. No definite rules can or need be made. Usually, the survey has been run for some special purpose or in accordance with local conditions, such as roads. Thus, if a fault is suspected, the stations are apt to be taken in lines across the suspected fault. If a salt dome is suspected, the lines probably will radiate from the suspected center, the pattern being irregular. In sectionized areas, the stations are likely to be made at, or near, the section corners, so far as convenient. Each map has its own peculiarities, which will make a separate study desirable. To try to standardize the interpretation often means to sacrifice the very peculiarities that have governed the survey. The present paper removes the needs for such a standardization.

Experience, judgment and information as to the general features likely to be of interest must guide the interpreter of a torsion balance survey. Usually, for practical reasons, stations are placed in profiles. Roads affect the actual placing of the stations. Thus there is usually a natural start for the network. The writer studies the map before starting any linking. For this purpose, it is convenient to make a tracing of the gradient map. This tracing contains nothing but the station positions, the gradient arrows, three or four orientation identifications and a simple designation, such as the name of the survey. It contains no station numbers, gradient magnitude figures, landmarks, roads or other data likely to prejudice the interpreter. These extraneous data may be used after the isogams have been drawn, to check the conclusions. The writer has found them of doubtful value. From this tracing, base maps are made for use in the calculations. These maps are blue-line (white prints, or positives) and serve very well for the purpose. In addition to other advantages, this extra map makes it possible to file the original

[6] See footnote 2.

[7] See Section 2 following.

map without disfiguring it, or to pass it to another interpreter without inconvenience.

One of the base maps is studied to select as many profiles as convenient. The ideal least-square net would probably have the stations at the lattice points of a square network. In practice, this simple distribution is seldom found. We try to approximate some such distribution on a larger scale. Thus, if stations are placed every ½ mile on profiles running north-south and if the profiles are 2 miles apart, we draw the linking lines along each profile, and east-west lines about 2 miles apart. Thus, we have a system of squares with three secondary stations on the east and the west sides and none on the north or south sides. If stations are omitted, a change in the system is necessary, but this does not increase the necessary calculations. In regions of small extent, but close spacing of stations, triangular networks have been found satisfactory. Each map will suggest its own network, which is not unique. A different net will result in different calculations and in different values for the gravity, but experience has shown that the final results are remarkably reliable, if the net is not on too large a scale; *i. e.*, if the primary stations are not too far apart. Usually, the length of calculations increases rapidly with the number of primary stations selected, and practical considerations limit the number chosen. On the contrary, the accuracy of interpretation increases and the grain of the net chosen will be an important factor in the final results.

The writer has found it convenient to draw the net on one of the base maps in a colored ink, other than the blue of the map lines. By selecting the longer profiles first and connecting stations progressively, the work of drawing the net proceeds smoothly. The lines should connect stations as close together as practicable, because of the assumption that the gradient changes linearly between the two stations of a link.

After the network has been drawn, to the satisfaction of the interpreter, the primary stations are numbered for purposes of calculation. We define a primary station as one entering the solution by way of the unknowns of the least square system. Except for corners, and exceptions of a special nature, the primary stations are those having more than two links in the net; that is, those entering more than two of the observation equations. The numbering is important in determining the length of the calculations. A guiding principle is that each station should be kept out of the calculations as long as possible. For this effect, we select one corner of the map as a starting station. This may be the corner with the fewest links, or the one with apparently the smallest value of gravity, or may be chosen for some local reason. This station is numbered 1, in a colored ink not yet used on the map. A direction of rotation is also chosen, say clockwise. All the stations connected with station 1 are numbered consecutively, proceeding from the left

limit of the map. After numbering all the stations connected with station 1, we proceed to station 2, numbering all the stations linked with it and not previously numbered. This is continued until all of the primary stations have been numbered.

To illustrate the numbering of the stations, consider the net shown in Fig. 3. All stations of this group are primary and we assume that the differences have been determined before we start. Station 1 has been selected at the southeast corner. It is linked with three stations,

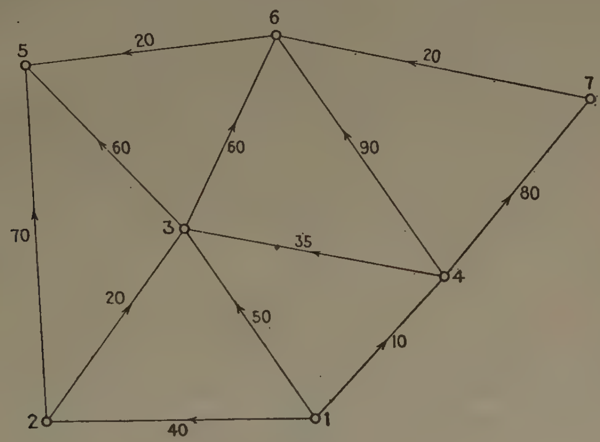

FIG. 3.

which are numbered 2, 3 and 4. Station 2 is linked with 1 and 3, already numbered. Hence, the third station linked with 2 is numbered 5. Station 3 has one free link, the station being numbered 6. Finally, station 4 brings in station 7. The method is the same for any number of stations, but for a large and complicated map, a little care is advisable.

After the map has been numbered, the gravity difference is calculated for each link in the net, both primary and secondary. This number is written on the map beside the link line, and an arrow on the line shows the direction of increase of gravity. In Fig. 3, the gradient arrows have been omitted. The measuring of the map is a long and tedious task, but is inherent in the net chosen, not a part of the least square reduction. The least square part of the interpretation starts after the observation equations have been formed and stops after the gravity value of each station has been determined. This point is often overlooked and the entire time of interpretation is charged to least squares, especially when the method is being condemned.

After the map has been measured and numbered, the observation equations are formed. In all arrangements of stations, a decreasing order is preserved. The observation equations are arranged in three columns, as shown for the example of Fig. 3, in Table 9. The first column contains

TABLE 9.—*Observation Equations*

From	To	$g_2 - g_1$
7	6	+20
	4	−80
6	5	+20
	4	−90
	3	−60
5	3	−60
	2	−70
4	3	+35
	1	−10
3	2	−20
	1	−50
2	1	−40

the number of the station from which the difference in gravity is measured; the second column contains the number of the station to which the difference in gravity is measured; the third column contains the difference in gravity as measured. In Table 9, the first line tells us that the difference in gravity from station 7 to station 6 is +20, or that station 6 has a gravity 20 units greater than that of station 7. Written as an equation, this observation is

$$x_6 - x_7 = +20,$$

where x_i is the gravity at station i. If the observation equation involves two stations linked directly, the difference in gravity is taken directly from the corresponding link of the map. If secondary stations separate the two primary stations, a simple addition is needed, signs being considered.

We are now ready for the least square part of the interpretation. The normal equations are formed in the usual manner, but are simpler. In the normal system of equations 2, the bracket $[aa]$ is simply the number of equations involving x_a, the gravity at station a. If x_a and x_b appear in the same equation, that is, if stations a and b are linked in the net, without being separated by other primary stations, the value of $[ab]$ is (-1). If x_a and x_b do not appear in the same equation, $[ab]$ is zero. To obtain $[ak]$, the procedure is slightly more involved. We add the differences in gravity for all equations involving x_a, retaining the sign if station number a appears in the second or "to" column of the observation equations and reversing the sign if station number a appears in the first or "from" column.

If the normal equations were written in the usual manner, they would be very inconvenient for large groups of stations. For this reason, we use a schematic form, which becomes familiar after a short period of use. The coefficients are grouped in sets. Since $[ab] = [ba]$, we consistently restrict $[ab]$ to the group $b < a$. Each group has a "key" number, representing the station involved, and "secondary" numbers, representing connected stations. For the first set, all the coefficients appear. For subsequent sets, some may be missing because of appearance in a previous set. In each set, the key coefficient and the secondary coefficients are written without sign, the former being always positive and the latter always negative. The constant is written with its proper sign. Where no confusion can arise, it will be convenient to include the constant in the term coefficient, for many of the same relations apply to constants as to the actual coefficients.

TABLE 10.—*Normal System and Reductions*

7	2	
6	1	
4	1	✓
k	+ 60	7
6	4	
5	1	
4	1	×
3	1	7
k	+150	
5	3	
3	1	×
2	1	6
k	+150	
4	4	
3	1	×
1	1	7
k	−195	
3	5	
2	1	×
1	1	6
k	− 15	
2	3	
1	1	×
k	− 50	5
1	3	×
k	−100	4

Code 3k2		
7) 6	3 500	
5	1 000	
4	1 500	✓
3	1 000	6
k	+1 8000	
4	3 500	
3	1 000	×
1	1 000	6
k	−16 500	
6) 5	2 714	
4	429	
3	1 285	✓
2	1 000	5
k	+ 20 143	
4	2 858	
3	1 429	×
1	1 000	5
k	− 8 786	
3	4 714	
2	1 000	×
1	1 000	5
k	+ 3 643	

5) 4	2 790	
3	1 632	
2	158	✓
1	1 000	4
k	− 5 602	
3	4 106	
2	1 474	×
1	1 000	4
k	+13 180	
2	2 632	
1	1 000	×
k	+ 2 422	4
4) 3	3 151	
2	1 566	✓
1	1 585	3
k	+ 9 903	
2	2 623	
1	1 057	×
k	+ 2 105	3
1	2 642	×
k	−12 008	3
3) 2	1 845	
1	1 845	
k	+ 7 027	
1	1 845	
k	− 7 027	

In the example (Table 10), set 7 represents the equation

$$2x_7 - x_6 - x_4 = +60,$$

and set 1 represents the equation

$$-x_4 - x_3 - x_2 + 3x_1 = -100.$$

In set 7, all the coefficients appear. In set 1, we obtain the secondary coefficients from sets 4, 3 and 2.

In writing these equations, as later for all reduced equations, we use three columns. In the first column, we write the station numbers for each set, retaining decreasing order consistently. We also add k for the constant term. In the second column, we write the values of the coefficients. The third column is to be used in the solution. In each set, the first coefficient is the key coefficient, the final is the constant term and the intervening coefficients are the negatives of those secondary coefficients not appearing in previous sets. The writer has found colored pencils a convenience in this solution. The normal system, and later, each reduction, is bordered in a prominent color, such as dark brown or blue. Within each group, the sets are bordered in a duller color, such as orange or green. The color is a matter of personal reaction, but assists the eye in grouping the numbers. In Table 10, the prominent color is indicated by a double line and the duller color by a single line, but in actual computation, the colored pencils are more convenient to use and the lines are easier to read.

After the normal system is formed, it is advisable to check the system. The first check is a comparison of the normal system with the map, the system having been formed from the observation equations. The key coefficient of each set must agree with the number of lines radiating from that station on the map. Thus, in the example, the key coefficient of set 3 must be 5, since station 3 is linked with the five stations 6, 5, 4, 2 and 1. Further, 3 must be a secondary number in sets 6, 5 and 4 (greater than 3) and must have the secondary numbers 2 and 1 (less than 3). The number of secondary numbers must be the same as the key coefficient. Thus, there must be five for set 3 (there are three in previous sets and two in the set 3). Each secondary coefficient in the original normal system is minus unity, written as 1. The check on the constants consists in the fact that the sum must vanish, since each observation constant appears once with a variable in the "from" column and once with a variable in the "to" column, the signs being respectively reversed and retained. These checks should be applied and may prevent much useless calculation.

If the number of primary stations is small, the usual methods of algebra may be used to solve the normal system. However, for a large number of primary stations, the method of Gauss[8] is preferable. With a little practice, it proceeds systematically.

[8] See footnote 3.

The system of equations 2 may be written in the form

$$\left.\begin{aligned} [nn]x_n + \sum_{j=1}^{n-1}[nj]x_j &= [nk] \\ [ni]x_n + \sum_{j=1}^{n-1}[ij]x_j &= [ik] \quad (i = 1, 2, 3, \ldots, n-1) \end{aligned}\right\}. \qquad [50]$$

From the first equation, we have

$$x_n = \frac{[nk]}{[nn]} - \sum_{j=1}^{n-1}\frac{[nj]}{[nn]}x_j. \qquad [51]$$

Substituting this value in the second equation, we have

$$\sum_{j=1}^{n-1}\left\{[ij] - \frac{[nj][nj]}{[nn]}\right\}x_j = [ik] - \frac{[ni][nk]}{[nn]}. \qquad [52]$$

If we define

$$\left.\begin{aligned} [ij]' &= [ij] - \frac{[ni][nj]}{[nn]} \\ [ik]' &= [ik] - \frac{[ni][nk]}{[nn]} \end{aligned}\right\}, \qquad [53]$$

we may write equation 52 in the form

$$\sum_{j=1}^{n-1}[ij]'x_j = [ik]' \quad (i = 1, 2, 3, \ldots, n-1). \qquad [54]$$

Thus, we have x_n expressed by equation 51 in terms of x_j for $j < n$ and we have replaced the original normal system of order n by another normal system of order $(n - 1)$. Equations 53 constitute reduction formulas which make the calculation simple, especially when a computing machine is available. We shall call the calculations involved in eliminating x_j, "reduction j." We shall also, without confusion, consider the sets involved in this elimination by the same term.

In applying the Gauss method, the calculations proceed systematically. We shall call the third column of the equations, not used until now, the "transfer" column, and shall use it to trace the set into the next reduction in which it appears. In complicated nets, this tracing is very convenient in the correction of errors or checking the calculations.

We draw a short slanting line like the slanting line of the capital letter N in the transfer column of each set of which the key number appears in the set eliminated in the reduction. Thus, if we are eliminating x_a, the process is "reduction a," and we draw the slant in each set having a key number which appears in set a. Under this line we write the number a of the reduction. This tells us that we are computing with this group. After the reduction calculations have been checked, we add another slanting line in the opposite direction. For set a, we convert the slanting line into a check ($\overset{\checkmark}{a}$). This set will determine x_a during the reversal, to

be discussed later, but will not appear again in the reduction, or elimination of the variables. For all other sets, we convert the slanting line into a cross ($\underset{a}{\times}$) indicating that the set has been transferred to reduction a. The purpose of this transfer column and the tracing data is to eliminate copying of sets not involved in the reduction being made.

In the example, reduction 7 will involve sets 7, 6 and 4, which we indicate during the calculation of this particular reduction by the southeast line with a figure 7 under it. After the calculations for the reduction have been checked, set 7 will have a check over the 7, while sets 6 and 4 will each have a cross over the 7 and will be found transformed in reduction 7. The remaining sets, 5, 3, 2 and 1, will not have been changed, and will have the transfer space clear.

Having assembled the reduction a, we transfer as much of the data as can be transferred without calculation. We indicate the reduction a by a small figure a in the fourth quadrant of a circle at the upper left corner of the reduction group. For reduction 7 in the example, see the top set in the second group of columns of Table 10. Each new set b must have all the elements of the old set b and also all the elements of set a. If the element c does not appear in set a, we transfer the coefficient $[bc]$ to the new set b without change. If the element c does appear in set a, we must supply the new value of $[bc]$ after further calculating. Because of the decreasing order maintained in writing the elements, this transferring is done with little difficulty and should be accomplished before any attempt is made to calculate the changes in the coefficients.

Before starting calculations, it is well to choose a code for the calculations and indicate it near the top of the normal system. This code tells the number of decimals retained. In the example, the code is $3k2$ and tells us that three decimals are retained in the station coefficients while two are retained in the constants. This code is followed throughout the reducing calculations. It is usually desirable to select a code that will make the constants and the station coefficients have the same number of significant figures. The code is not used in writing the normal equations or after the reductions have been made. In the reduction, it must be remembered that all key coefficients are positive and all secondary coefficients are negative.

To facilitate calculations, auxiliary forms have been made for use when a computing machine is available. These forms may be mimeographed, multigraphed or printed, according to the frequency with which they will be needed. Forms with spaces for 12 stations are convenient, although occasionally more spaces may be needed. When sufficient use is made of the method, it may be desirable to have several sets of forms, say for 3, 6, 9, 12, 15, 18, etc., stations. In this case, the computer should select, for each reduction, the smallest form with a sufficient number of spaces. About as many blanks will be used as there are primary stations in the

net. For economy and ease of handling, several forms may be arranged on the same sheet. The writer has standardized on three forms of twelve spaces each on a single sheet. In Table 11, the forms are on a basis of three spaces, but for large nets, this size would be inadequate.

TABLE 11.—*Auxiliary Calculations*

Sta 7	[aa] = 2 000			1/[aa] = 500 000	
[ac]/[aa]	Σ [ab] Sta	= 1 000 6	= 1 000 4		+ 1 500 + 6 000 K
50 000 50 000	6 4	500	500 500		+ 3 000 + 3 000
Check		=	=		=
Sta 6	[aa] = 3 500			1/[aa] = 285 714	
[ac]/[aa]	Σ [ab] Sta	= 1 000 5	= 1 500 4	= 1 000 3	+15 000 +18 000 K
28 571 42 857 28 571	5 4 3	286	429 642	285 429 286	+ 5 143 + 7 714 + 5 143
Check		+1	+1	+1	=
Sta 5	[aa] = 2 714			1/[aa] = 368 460	
[ac]/[aa]	Σ [ab] Sta	= 429 4	= 1 285 3	= 1 000 2	+10 000 +20 143 K
15 807 47 347 36 846	4 3 2	68	203 608	158 474 368	+ 3 184 + 9 537 + 7 422
Check		=	−1	−1	=
Sta 4	[aa] = 2 790			1/[aa] = 358 423	
[ac]/[aa]	Σ [ab] Sta	= 1 632 3	= 158 2	= 1 000 1	= − 5 602 K
58 495 5 663 35 842	3 2 1	955	92 9	585 57 358	− 3 277 − 317 − 2 008
Check		=	=	=	=
Sta 3	[aa] = 3 151			1/[aa] = 317 360	
[ac]/[aa]	Σ [ab] Sta	= 1 566 2	= 1 585 1		= + 9 903 K
49 699 50 301	2 1	778	788 797		+ 4 922 + 4 981
Check		=	=		=

The use of the form is direct. The top line is filled directly with the number of the reduction, the coefficient [*aa*] and the reciprocal of this coefficient, if the computer is not trained in the addition method of division. The writer has discarded this number, but has found it convenient for beginners. After, and below the abbreviation "Sta" in the form, are entered the numbers of the secondary elements of set a. After [*ab*], in the second line, is entered the value of the coefficient [*ab*] for the station b in each column. In the first column is entered the quantity [*ac*]/[*aa*] for the station c of that line. This value is obtainable by putting [*aa*] in the computing machine and adding to obtain each [*ac*] as needed. The sum in this column must be unity. A slight deviation from unity is not serious and need not be adjusted, but a large discrepancy indicates a checking of the calculations for this column. This check should be applied before the rest of the table is calculated. After Σ, is written the sum of all the b coefficients appearing in the reduction. This must also equal the negative of the sum of all b coefficients not appearing in this reduction, since, after each reduction the system is normal and the sum of all the coefficients involving a particular variable must vanish. In obtaining the sum Σ, the signs must be kept in mind.

The portion of the form for $c < b$ is not needed because of symmetry. In the upper right half of the form, the number in each space is the product of the [*ab*] at the head of its column and the [*ac*]/[*aa*] at the left of its line. The constant k is treated in the same manner as a variable, but independently. These values are checked by addition. The sum in each column down to the diagonal and thence along the line, excluding the k column, must equal the value [*ab*] for the column. The sum in the k column must equal [*ak*]. If the check is perfect, we write = in the check line for that column. If the discrepancy is small, we write in the check line the amount by which the sum exceeds [*ab*]. Then we adjust the values in the table by one unit at a time until the sums check, after which we encircle the errors to show that they have been corrected. If the discrepancy is large, an error is indicated and should be corrected. If this large discrepancy occurs in just one column, other than the k column, an error is indicated in the diagonal term for that column. If it occurs in two columns, the error is indicated in that number appearing in the column containing the error the second time (to the right) and in the line whose first entry is in the column containing the error the first time (to the left). This apparently complicated analysis becomes simple if we remember that symmetry has enabled us to omit nearly one-half of the table. If the error is in the constant column, it can often be located by a quick inspection. If it cannot be located readily, a recalculation of the column is indicated.

When this auxiliary table is completed and checked, we have merely to subtract from each value [*bc*] in the replaced set the value in column

b and line c of the auxiliary form, as may be seen from equations 53. This will be the value of $[bc]'$ from which the prime may be dropped. The sums of the b coefficients after reduction must equal the values of Σ for their respective columns, without discrepancy. All discrepancies due to forcing errors have been adjusted in the auxiliary from. After checking a sum of coefficients, we encircle the station number at the head of that column of the auxiliary form. After all sums in the reduction have been checked, we "check" set a and "cross" all sets b in the replaced group. Then the next reduction proceeds similarly.

A careful study of the example will make this clear. The reader interested in using the method should calculate the example in detail, step by step. While there are only seven stations in this case, the essentials are the same for any number of stations. Lack of space forbids our including the appearance of the calculations after each of the various steps. In carrying out the calculations, the work is simplified by the absence of numbers to be obtained later.

After completing reduction 3, two identical equations result. In the example, these equations are

$$+1.845x_2 - 1.845x_1 = +70.27$$
$$-1.845x_2 + 1.845x_1 = -70.27.$$

If we assign an arbitrary value to x_1, the value of x_2 is determined. After x_i is determined for all values of i below a particular value a, the value of x_a is determined by the set $\checkmark_a$. A simple way to make this calculation is to use an auxiliary "reversal slip," as shown in Table 12.

TABLE 12.—*Reversal*

g	Sta	Links	Linked with					k
87.42	7	2	6	4				+ 60
103.31	6	4	7	5	4	3		+150
113.92	5	3	6	3	2			+150
11.54	4	4	7	6	3	1		−195
50.36	3	5	6	5	4	2	1	− 15
38.09	2	3	5	3	1			− 50
0	1	3	4	3	2			−100

The first column is left blank until the values of the variables have been calculated. The second column contains as many integers as the slip will permit, with number 1 at the bottom. The third column contains the number of links radiating from the station whose number is in column two. The final column contains the value of k in the normal equation for that number. The intervening columns, as many as needed, contain the numbers of the stations linked with the particular station. This

slip should be prepared from the map and checked from the original set of normal equations. It is merely a convenient way of writing this normal system.

Having prepared the reversal slip, we write in the first column the value of each variable as it is calculated, starting with the assumed value for x_1. After the variables have been calculated for all the links in a particular line, we check that line. This is done by multiplying the gravity of that station by the number of links and subtracting the gravity of each station linked with it. The result should be the value of k, except for the forcing error, which should be small. When a particular line has been checked, we draw a line over the station number of that line and over that same station number wherever it appears in the links. Usually it will appear irregularly. After the station number and all of its links have been overlined, we complete the line across the slip, for that station will not be needed again in the reversal. The checking is multiple, since each gravity value is checked in each equation in which the station number appears. In Table 12, these overlines are omitted.

When all of the gravity values have been calculated and the equations checked as far as possible for this slip, we make a new reversal slip. We transfer to the new slip all lines not completely overlined and then extend the slip upward as far as the length of the slip will permit. To avoid using more than one slip at a time, it is best to make each additional one as needed. These slips furnish a convenient list of the gravity values for the primary stations. When a standardized calculation paper is used, the calculation of the gravity values is simple. By placing the reversal slip along the right edge of set $\overset{\checkmark}{a}$, we may evaluate x_a directly on the computing machine. We enter the value of $[ak]$ with the proper sign, from the set into the machine, which has been previously cleared. Then we add into the machine the product of $[ab]$ as written in the set and the value of x_b as written on the slip, until all the secondary terms have contributed. Then we divide the sum by $[aa]$ from the set. This is in accordance with the equation

$$[aa]x_a = [ak] - \sum_{b=1}^{a-1}[ab]x_b \qquad [55]$$

recalling that the numbers actually appearing in set $\overset{\checkmark}{a}$ are

$$[aa], - [ab] \text{ and } [ak].$$

After all of the variables have been calculated and the equations checked, the gravity values are written in a new color on the base map used for the net. The least square part of the solution is now completed. While the method may seem complicated and involved to the casual reader, it is soon mastered and can be used by an ordinary computer who does not understand the underlying theory.

Having the gravity values on the net for each primary station, we have now to interpolate the value for each secondary station. For this purpose, we may use either of two convenient methods. Consider each group of stations terminated by primary stations in the net as a separate chain and apply one of the two following rules:

Rule I.—Calculate the gravity of the intermediate station from each end of the chain as if no other station were present. Giving each value a weight equal to the number of links to the opposite end, find the weighted mean of the two values.

Rule II.—Distribute equally over all the links, the excess of the difference in the values of gravity for the two ends and the sum of the observed elements of the chain, signs considered. Proceed in any convenient manner using the adjusted difference for the links.

These two rules are equivalent[9] and easily applied. The interpolation of secondary values is often made erroneously.

For a single secondary station between stations a and b, with observed steps Δ_1 and Δ_2 in the direction from station a to station b, the rules lead to

$$x_c = \tfrac{1}{2}[(x_a + \Delta_1) + (x_b - \Delta_2)]. \qquad [56]$$

For two intermediate stations, c and d with observed steps Δ_1, Δ_2, and Δ_3, they lead to

$$\left.\begin{aligned} x_c &= \tfrac{1}{3}[2(x_a + \Delta_1) + (x_b - \Delta_2 - \Delta_3)] \\ x_d &= \tfrac{1}{3}[(x_a + \Delta_1 + \Delta_2) + 2(x_b - \Delta_3)] \end{aligned}\right\}. \qquad [57]$$

With a computing machine, this is rapid and simple. Thus, to calculate x_c in equation 57, we add, successively,

$$x_a,\ x_a,\ \Delta_1,\ \Delta_1,\ x_b,\ -\Delta_2,\ -\Delta_3,$$

and divide the resulting sum by 3. The necessary data are obtained directly from the map, and the results are written directly on the map. After the gravity is calculated for every station, the values are transferred to a new map made from the tracing. The isogams are drawn by the usual methods of contouring, except that the arrows indicate the direction of greatest increase in gravity, in addition to the gravity values. As far as possible, the contours should be perpendicular to the arrows at stations near them.

After the isogams are drawn, the map is completed by adding whatever map data and information the interpreter may desire. Any irregularity in the isogams should be investigated. If there is a regional gradient, this may be removed. However, for a large survey, a constant gradient is not satisfactory over the entire map. Experience furnishes

[9] For a proof of the two rules by the method of least squares, see the appendix. This proof will not be of interest to the untrained mathematician, but is included for the benefit of those desiring a proof of these apparently arbitrary and different rules. For illustration, the appendix contains a comparison of the present method with a conflicting one.

the only reliable guide to the interpretation after the isogam map has been completed.

If the map is to be adjusted to a previous map, or to a base line, we introduce a fictitious station S_0, whose gravity will eventually be taken as zero. In this case, the map is numbered with reference to the known base. Station numbering begins with those stations linked to the base map, after which it proceeds as for the independent survey. The observation equation for a station tied to a base station is of the form

$$x_a - x_0 = M_a \tag{58}$$

where M_a is the gravity of station a as calculated from the base station. Put into other words, we link each tied station to the value zero by way of the reference station of the base survey. In this case, we proceed with the reducing process until after reduction 2, when we have two identical equations:

$$\left.\begin{aligned} +[11]x_1 - [11]x_0 &= k \\ -[11]x_1 + [11]x_0 &= -k \end{aligned}\right\} \tag{59}$$

The value of x_0 must now be taken as zero, so that x_1 is determined uniquely instead of being arbitrary. Obviously, no link may connect two stations of the base survey.

Section 2 (*Part* V). *Comments.*

In the present method, a primary station has been considered as a "junction" station in the net, while a secondary station has been considered as a "way" station. A secondary station lies on the network in such a position that it is on only one chain of the net terminated by primary stations. A primary station is at a meeting of the separate chains. The net is the same for all stations and includes every station, so that no gradients are discarded in the primary reduction. On the other hand, the primary net method as used by Eötvös and discussed by Barton[10] uses a net selected from the entire group of stations. A few stations are selected as primary stations and these are connected as simply as possible. This net is adjusted by least squares. Then, other stations are tied in with this primary net by a secondary net. It is evident that no station not lying on the primary net can have an effect in the adjustment of that net. The method involves the ranking of stations as of primary, secondary, etc., importance. The present method makes no such choice. Every gradient makes a contribution to the network of primary stations. While primary stations are emphasized slightly, they have only a slight importance over the secondary ones.

In contrast to the primary net method is the method of simultaneous adjustment. In this method, every station of the field is made a primary

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[10] See footnote 5.

station of a single net. Each station is linked with as many neighboring stations as convenient. This is the method used for most of the fields mentioned by Barton. It is this method which Barton describes as "extremely tedious" and "somewhat treacherous."

The method discussed in the present paper is a compromise between the primary net and the simultaneous adjustment methods. It is flexible and is not handicapped by the dangers of either extreme. In the earlier torsion balance work, simultaneous adjustment was possible without using more than 150 stations. However, recent surveys have had many more stations. In one case, a survey involved nearly 400 stations. Later, this survey was extended to include about 300 more stations. The first survey was adjusted with about 150 stations primary and the rest as secondary on the same net. The second survey was adjusted to the first with a slightly smaller number of primary stations. Surveys of more than one hundred stations are not a rarity, or, at least, were not before the recent curtailments.

There is always an intangible estimate of importance in making or interpreting a set of measurements of any type. To make the data of too coarse grain will often render the conclusions useless by masking the effects sought. To make the data of too fine grain will increase the difficulty of making and reducing it and will introduce a false feeling of security and accuracy. Just what grain to use must be left to the judgment of the individual. Experience, expense, purpose of the investigation, reliability of the instrument, trustworthiness of the observer, and many other factors enter the choice. These general conclusions apply to torsion balance and magnetometer surveys with full force because of the time and expense involved in making the survey. To fail to utilize available information is imprudent: to inject extraneous importance into data is foolhardy.

For definiteness, consider a fictitious survey as follows: There are roads each mile in both coordinate directions, *i. e.*, north-south and east-west. The stations are placed at each section corner and on the roads midway between each two section corners. The region is 10 miles square. Then there will be 341 stations in all. To select 5, 10, or even 20 stations as primary would be to discard much valuable information, especially if the gradients showed large local irregularities. If we recall that 12 torsion balance stations a week is a fair average over a long time, we have a survey requiring nearly seven months of observations. To treat this in an elementary manner by selecting only a few stations as primary is equivalent to wasting several months in the field. In this fictitious case, the writer would favor a net following the roads, with each of the 121 section corners as a primary station and the remaining 220 stations as secondary. To treat the 341 stations as primary would involve many months of calculations. This example illustrates the need of choosing a

compromise between the two extremes. Practical applications have justified the general features of the choice.

Part VI. Summary

In the foregoing parts, we have shown how to apply the method of least squares to three distinct problems of practical geophysics. While the problems have arisen in geophysical exploration in the petroleum field, it is apparent that the solutions will apply to other lines of activity, and that the methods may be extended to other problems.

The three problems have been taken from methods of (1) sound surveying, (2) reflection seismograph surveying, (3) torsion balance or magnetometer surveying. In each case, we have discussed the source of the problem, its formulation, its solution and numerical examples of the method. Each problem has its own particular details, but, in each case, the reduction of more observations than necessary enables us to select such a solution that the sum of the squares of the discrepancies between the observed and the adjusted values of some quantity is a minimum. This method is usually taken as furnishing the best interpretation of data not completely self-consistent. If the data are consistent, the method applies but is unnecessary, since we have only to select enough observations to furnish the solution and all choices lead to the same values. In practice, this case arises so infrequently that we may neglect it.

The numerical examples indicate convenient forms for routine solution of each problem, and suggest the manner of arranging the forms when the process is applied often enough to justify the use of printed forms.

The writer has been criticized for his use of the Greek symbols, on the basis that most geophysicists and computers are not familiar with them. However, the symmetry and consistency of the notation seem to justify their retention. Those readers who are interested in the details can acquire the Greek alphabet from various sources. To the trained reader, the letters ξ, η, ζ, ρ are associated definitely with x, y, z, r. When forms are being arranged for the untrained computer, the notation may be changed to suit the individual group.

Appendix

In section 1 of Part V., two interpolation rules were stated without proof. We shall refer to them as rule I and rule II without repeating the statements. We shall prove that both are direct consequences of the principle of least squares and thus show that they are equivalent. The rules will be applied to a case discussed by a conflicting method.

Let it be desired to interpolate $(n - 1)$ values x_i between $x_0 = x_a$ and $x_n = x_b$, in accordance with observed differences Δ_i. Let the values x_i be determined so that

$$x_i - x_{i-1} = \Delta_i' = \Delta_i + Q_i \quad (i = 1, 2, 3, \ldots, n)$$

where Q_i are to be determined so that

$$\sum_{i=1}^{n} \Delta_i' = x_b - x_a$$

and $\sum_{i=1}^{n} Q_i^2$ shall be a minimum with respect to all choices of x_i. This quantity ΣQ_i^2 depends upon a particular x_i by two terms:

$$\left.\begin{aligned} Q_i &= x_i - x_{i-1} - \Delta_i \\ Q_{i+1} &= x_{i+1} - x_i - \Delta_{i+1} \end{aligned}\right\} \quad (i = 1, 2, 3, \ldots, n-1)$$

and no others. Thus

$$\frac{\partial}{\partial x_i}\left(\sum_{i=1}^{n} Q_i^2\right) = 2Q_i - 2Q_{i+1},$$

whence

$$Q_{i+1} = Q_i \quad (i = 1, 2, 3, \cdots, \quad n-1).$$

Thus, all the values of Q_i are the same and we may write them as Q. The value of this constant is determined by

$$\sum_{i=1}^{n} \Delta_i' = x_b - x_a = \sum_{i=1}^{n} (\Delta_i + Q_i) = \sum_{i=1}^{n} \Delta_i + nQ,$$

so that

$$Q = \frac{1}{n}\left[(x_b - x_a) - \sum_{i=1}^{n} \Delta_i\right].$$

This value of Q is that used in rule II, which is thus proved.

To prove rule I, we have

$$\begin{aligned} x_j &= x_a + \sum_{i=1}^{j} \Delta_i' = x_a + \sum_{i=1}^{j} \Delta_i + jQ \\ &= x_a + \sum_{i=1}^{j} \Delta_i + \frac{j}{n}\left[(x_b - x_a) - \sum_{i=1}^{n} \Delta_i\right] \\ nx_j &= n\left(x_a + \sum_{i=1}^{i} \Delta_i\right) - j\left[\left(x_a + \sum_{i=1}^{i} \Delta_i\right) - \left(x_b - \sum_{i=j+1}^{n} \Delta_i\right)\right] \\ &= (n-j)\left(x_a + \sum_{i=1}^{j} \Delta_i\right) + j\left(x_b - \sum_{i=j+1}^{n} \Delta_i\right). \end{aligned}$$

But $\left(x_a + \sum_{i=1}^{j} \Delta_i\right)$ is the value of x_j as calculated from x_a without adjustment or relation to intermediate values. Furthermore, the quantity $\left(x_b - \sum_{i=j+1}^{n} \Delta_i\right)$ is the value of x_j as calculated from x_b without adjustment. Finally, there are j links between x_a and x_j and $(n-j)$ links

between x_j and x_b. Thus, rule I is proved. Since both rules lead to the same choices of intermediate values, they are equivalent.

With a computing machine, they are applied without difficulty. As an illustration, consider the example cited by Barton, with the notation changed to that of the present paper. We have

$$x_a = 100,\ x_b = 200,\ \Delta_1 = +60,\ \Delta_2 = +40,\ \Delta_3 = -20,\ \Delta_4 = +40.$$

By rule I, we have

$$x_1 = \tfrac{1}{4}[3(100 + 60) + (200 - 40 + 20 - 40)] = 155$$
$$x_2 = \tfrac{1}{2}[(100 + 60 + 40) + (200 - 40 + 20)] = 190$$
$$x_3 = \tfrac{1}{4}[(100 + 60 + 40 - 20) + 3(200 - 40)] = 165.$$

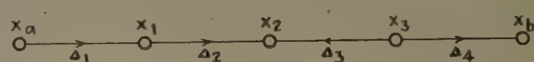
By rule II, we increase each link by

$$\tfrac{1}{4}[(200 - 100) - (+60 + 40 - 20 + 40)] = -5,$$

making the adjusted differences: +55, +35, −25, +35; and this rule leads to the same interpolated values as rule I.

TABLE 13

Barton adjusted values		150	183	167	
Roman adjusted values		155	190	165	
Base values	100				200



Barton adjusted links	50	33	16	33
Observed links	60	40	20	40
Roman adjusted links	55	35	25	35
Barton discrepancies	10	7	4	7
Roman discrepancies	5	5	5	5
Barton sum of squares			214	
Roman sum of squares			100	

A comparison of Barton's results with those calculated in the present paper is shown schematically in Table 13. The sums of the squares of the discrepancies between the observed and calculated values of the differences are 214 and 100, respectively. In addition to conforming to the principle of least squares, the present rules are much easier to apply than is the method of proportional distribution of the excess.

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(Note: In this index the names of authors of papers and discussions and of men referred to are printed in SMALL CAPITALS, and the titles of papers in *italics*.)